Modelling the Volatility of Cryptocurrencies using Markov-Switching GARCH models, estimated via MCMC

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Abstract

Recent research show single-regime GARCH models may fail to capture structural breaks when modelling volatility. Regime changes are evident in cryptocurrencies, with periods of high volatility followed by areas of low volatility, therefore Markov-Switching models may be more suitable. The aim of the following is to look at the volatility dynamics of five popular cryptocurrencies: Ether, EOS, Litecoin, Ripple and Binance Coin, with their prices denominated in Bitcoin. Markov-Switching GARCH models are applied to these cryptocurrencies, estimated via Markov Chain Monte Carlo, to forecast a one-step ahead prediction of Value-at-Risk. Backtesting methods, such as the conditional coverage (CC) test and the dynamic quantile (DQ) test, are applied to find which models yield more accurate Value-at-Risk forecasts for each cryptocurrency. A mixture of single-regime and Markov-Switching models are chosen as the most suitable model for each of the five cryptocurrencies.

Keywords: Cryptocurrency, Bitcoin, Ether, Ripple, Litecoin, EOS, Binance Coin, Volatility, GARCH, Markov-Switching GARCH, MCMC, Value-at-Risk, Backtesting.

Word Count (excluding References and Appendix): 6759

1 Introduction

The first decentralised cryptocurrency Bitcoin was created by Nakamoto (2008). Bitcoin is the largest cryptocurrency, built upon blockchain technology and is designed to enable a system for electronic transactions without relying on trust. Its popularity over recent years, has led to the development of over 1000 cryptocurrencies, including: Ether, Ripple and Litecoin. The characteristics that make cryptocurrencies so attractive include its anonymity, lower transaction costs, quicker processing times and no need for intermediaries like banks. This makes the virtual currency advantageous over other traditional payment methods and are seen as assets, that have a clear place in financial markets and in portfolio management. Nonetheless, due to the development of newer technologies and applications, cryptocurrencies will continue to grow.

The price of Bitcoin over a short period of time can unpredictability increase or decrease, so there is no surprise that cryptocurrencies behave differently to traditional currencies. Cheah and Fry (2015) concluded that cryptocurrency markets have a vulnerability to speculative bubbles, their results indicating a bubble phase beginning around January 2013. This perspective is supported by Bitcoin exhibiting a bubble in 2017. Therefore, it is evident that cryptocurrencies are extremely volatile. Risk is commonly associated with increasing volatility in the financial market, and the appropriate selection of the distribution of asset returns is a major challenge of risk management. Therefore, the modelling and forecasting of volatility has become a very crucial area of research among academics and practitioners.

Research has mainly focused on the volatility of Bitcoin returns, despite the immense growth of the cryptocurrency market. So the motivation for the following is to look at the volatility dynamics of five cryptocurrencies and to account for the volatility clustering that is evident in their log returns (Figure 1). Clearly, there are durations of low volatility followed by areas of high volatility in each time series. Therefore, during times of financial crisis, the right methodology for risk predictions is important in accurate risk management. Volatility is also an essential tool in the calculation of other risk metrics such as Value-at-Risk (VaR). Value-at-Risk is a standard measure of market risk embraced by banks, mutual funds, trading firms etc. Regulators enforce capital requirements, in order for financial institutions to be prepared to incur losses. For example, the United States and European Union adopt one of the most important (well known) regulations, The Basel Accords. Therefore, there is pressure in measurement of their risk via backtesting, as failure to meet the validity requirements, leads to the financial institution being penalised. VaR can provide an accurate, credible and reliable measure of risk exposure and I use VaR since it is the most well know risk measure in finance.

The aim is to investigate the forecasting performance of different volatility models and find which models yield more accurate VaR forecasts for each cryptocurrency, using backtesting methods. These include the conditional coverage (CC) test (Christoffersen (1998)) and the dynamic quantile (DQ) test (Engle and Manganelli (2004)). In the following: Section 2 explores the literature underlying volatility models and estimation techniques; Section 3 describes the data used; Section 4 presents the methodology; Section 5 discusses the results and Section 6 concludes.

2 Literature Review

The Autoregressive Conditional Heteroskedasticity (ARCH) model, introduced by Engle (1982), is used to analyse the statistical volatility of financial time series. Bollerslev (1986) extends this to the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. GARCH specifies variance as a linear function of past variance and squared residual past shocks. In the literature, GARCH is most traditionally used for modelling volatility and extra extensions are added and modified to develop many other models. For example, further symmetric models include the Student t-GARCH model of Bollerslev et al. (1987), and asymmetric models, such as the exponential GARCH (EGARCH) model of Nelson (1991), GJRGARCH model of Glosten et al. (1993) and the Power GARCH (PGARCH) model, proposed by Ding et al. (1993). These models can capture the asymmetry in the conditional volatility process. Papers mainly rely on GARCH models when modelling the volatility using cryptocurrency data. Out of several competing GARCH-type models, Katsiampa (2017) finds that the optimal model to explain Bitcoin price volatility is the AR-CGARCH, highlighting the importance of having both a short-run and long-run component of conditional variance. In contrast, when looking at the most popular and largest cryptocurrencies, Chu et al. (2017) find that that the IGARCH and GJRGARCH models provide the best fit. However, this good fit using IGARCH for numerous cryptocurrencies may stem from structural breaks, that may not be accounted for.

Further studies find that structural breaks can lead to poor volatility forecasts and biased estimations, that is evident in traditional GARCH models (Bauwens et al. (2014)). It is important for estimation and risk forecasting to be as accurate as possible for understanding volatility. Therefore, key for GARCH models to estimate accordingly if regime changes are present. Bariviera (2017) finds evidence of some form of regime change in Bitcoin returns. Similarly, Ardia et al. (2019) find Bitcoin daily log-returns exhibit regime changes in their volatility dynamic. Therefore, it may be more suitable to use Markov Switching models, where parameters take different values in different states over time and can anticipate structural breaks in the conditional variance process.

Early studies into Markov-switching focus on ARCH models for modelling volatility. Including Cai (1994), who introduced the switching process to the constant term in the conditional variance and Hamilton and Susmel (1994), who applied the switching parameter to the coefficients of the conditional variance equation. Hamilton and Susmel (1994) find that a Markov-switching process provides a more suitable statistical fit on stock market returns and financial data, than GARCH models without switching. In recent years, there has been interest in regime-switching GARCH models. These models parameters can change over time according to a latent (i.e, unobservable) variable. In particular, papers have used Markov-Switching GARCH (MSGARCH) models when applied to cryptocurrency data. Ardia et al. (2018) performed a a large-scale empirical analysis comparing the risk forecast performances of single-regime and MSGARCH models on stocks, foreign exchange rates and equity indices and extended their work to Bitcoin, (Ardia et al. (2019)). Both papers find an inverted leverage effect in both high and low volatility regimes and the best-in sample performance is when using a two regime MSGARCH model, than single-regime GARCH models.

Moreover, in many of the studies above and in other research, Maximum Likelihood estimation (MLE) is mainly used when calibrating GARCH models. However, when estimating MSGARCH models by MLE, the conditional variance depends on all the past history of the state variable, due to the framework of GARCH. Therefore, for K number of states (regimes) and T number of observations, K^T cases would need to be considered to get the likelihood function. This makes estimation very difficult to achieve for MSGARCH models. However, these problems can be fixed using Bayesian methods (Gray (1996); Dueker (1997); Das et al. (2004); Ardia (2008)). Bauwens et al. (2014) recommends the use of Bayesian methodology, such as the Markov Chain Monte Carlo (MCMC) procedure. MCMC simulation can explore the model parameters joint posterior distribution and avoids the local maxima encountered when estimating regime switching GARCH models via MLE (Ardia (2008)). The MSGARCH R package (Ardia et al. (2016)) enables you to compare the two estimation techniques. Ardia et al. (2018) use both the MLE and MCMC estimation methods and find that performances obtained by Bayesian estimation are either similar or better than those obtained by MLE, for both the single regime and two regime models. Therefore, Ardia et al. (2019) estimated the model parameters using Bitcoin returns with a Bayesian approach, via MCMC simulation. Caporale and Zekokh (2019) extended the work of Ardia et al. (2019), by also using MCMC and analysing Bitcoin and three other cryptocurrencies (Ethereum, Litecoin and Ripple). They also used Value-at-Risk (VaR) and Expected Shortfall backtesting, as well as a MCS procedure to select the best fit or best set of models for each of the cryptocurrencies. Conclusions from both, find that two-regime GARCH models produce better results of Value-at-Risk and Expected Shortfall predictions than single-regime models.

My analysis will extend the work of Ardia et al. (2019) and Caporale and Zekokh (2019), but by modelling the volatility dynamics of five popular cryptocurrencies: Ether (ETH), EOS, Ripple (XRP), Binance Coin (BNB) and Litecoin (LTC) using two-regime MSGARCH models, estimated via MCMC. Ardia et al. (2018) tested four different conditional distributions: standard normal, Student-t and their skewed versions. They find MSGARCH models are not able to jointly account for the switch in the parameters, as well as for the excess of kurtosis exhibited in the data, with a (skew) normal distribution. Therefore, I follow their recommendation of a skew Student-t specification for MSGARCH models. Furthermore, I estimate a 1-step ahead prediction of VaR on a rolling window basis and backtest the performance of the VaR forecasts using the conditional coverage (CC) test and dynamic quantile (DQ) test. Since I only focus on MSGARCH models and not compare the performance to single regime GARCH models, I consider an Exponentially Weighted Moving Average (EWMA) model, with $\lambda = 0.94$.

3 Data

The data analysed are the cryptocurrencies daily closing prices expressed in Bitcoin, e.g. ETH/BTC. The data of all five cryptocurrencies is retrieved from CryptoCompare,[1] a reliable source (Alexander and Michael (2019)). The end dates of all the data are the same, which is the 17th July 2019, whereas

[1]www.cryptocompare.com

the start dates for each cryptocurrency differ. For Litecoin the start date is the 29th September 2013, Ripple is from the 20th January 2015, Ether from the 7th August 2015, EOS from the 29th June 2017 and Binance Coin from the 8th September 2017. The prices are used to compute the daily log-return series denoted by $r_t \equiv \log(p_t/p_{t-1})$, where p_t is the price on day t and p_{t-1} is the price on day t - 1. By using an AR(1)-filter, the returns r_t are de-meaned and the filtered returns, y_t, are used to estimate.

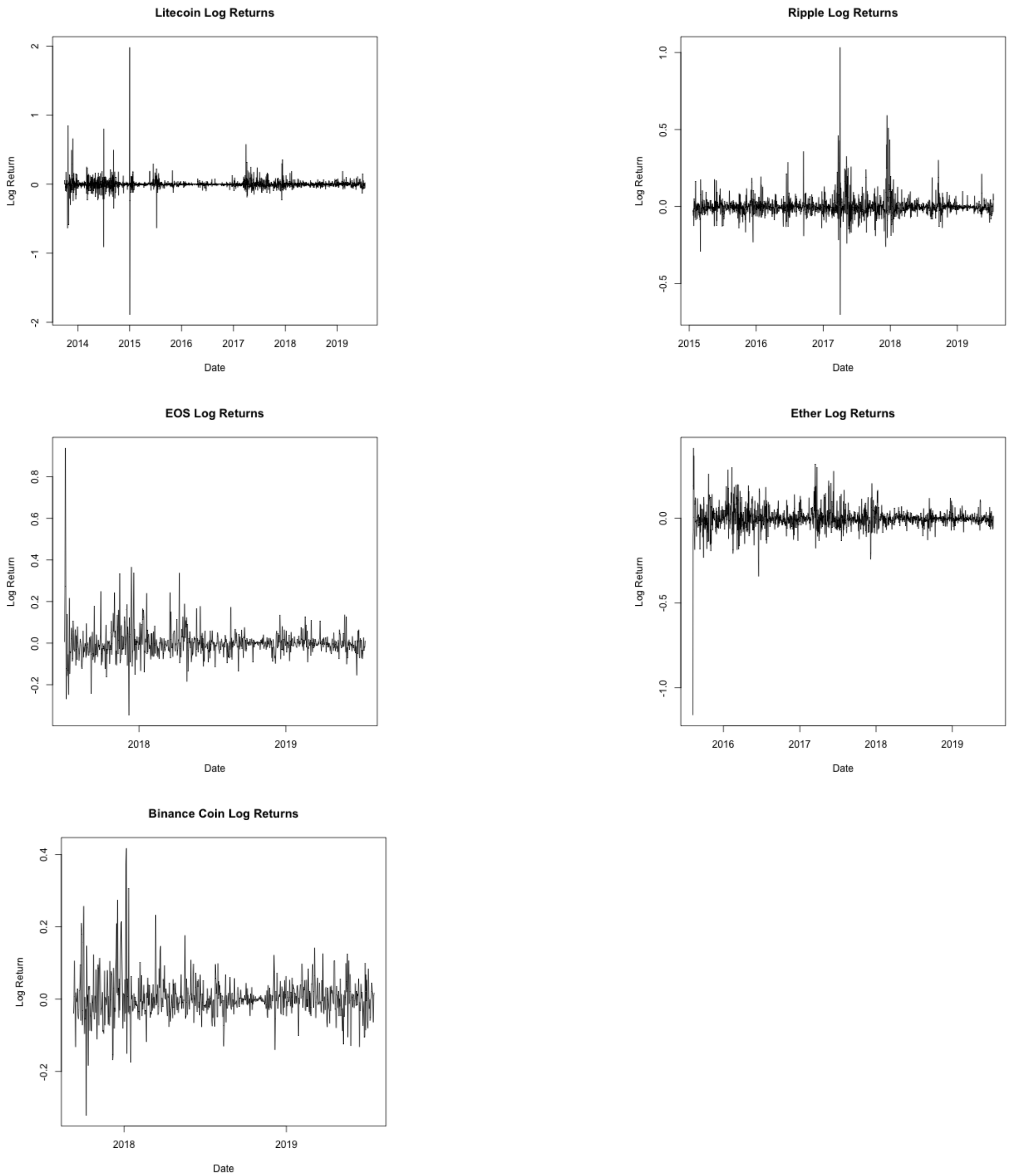

Figure 1: Log Returns

4 Methodology

4.1 Model Specification

Let y_t denote the daily log-return of each of the cryptocurrencies at time t. The log-returns have been de-meaned and therefore, not autocorrelated and have a zero mean. Markov-Switching GARCH (MSGARCH) models allow-time varying skewness contrary to traditional GARCH-type models. The MSGARCH specification, as denoted by Ardia et al. (2016), can be expressed as:

$$y_t|(s_t = k, I_{t-1}) \sim D(0, h_{k,t}, \xi_k),$$

where $D(0, h_{k,t}, \xi_k)$ is a continuous distribution with zero mean, a vector ξ_k of additional shape parameters (e.g. tail and asymmetry) and a time varying conditional variance $h_{k,t}$ in regime k. Additionally, the state variable s_t evolves accordingly to a 1st-order homogeneous Markov Chain with a finite number of states K, with transition probability matrix **P**:

$$\mathbf{P} \equiv \begin{bmatrix} p_{1,1} & \cdots & p_{1,K} \\ \vdots & \ddots & \vdots \\ p_{K,1} & \cdots & p_{K,K} \end{bmatrix}$$

where $p_{i,j} \equiv \mathbf{P}[s_t = j|s_{t-1} = i]$, which denotes the probability of transitioning to state s_t from state s_{t-1}. Lastly, I_{t-1} denotes the information set up to t-1.

The conditional variance of y_t is assumed to follow a GARCH process, as by Haas et al. (2004) and can be specified as:

$$h_{k,t} \equiv h(y_{t-1}, h_{k,t-1}, \boldsymbol{\theta}_k),$$

where $h_{k,t}$ is a function of past returns y_{t-1}, past variance $h_{k,t-1}$ and conditional on regime $s_t =$ k, with a regime-dependent vector of parameters $\boldsymbol{\theta}_k$. Also, to ensure the conditional variance is positive, h($\cdot$) is a I_{t-1}, which is a measurable function, that defines the conditional variance filter.

Three different specifications are considered for the conditional variance, including:

The **GARCH(1,1)** model of Bollerslev et al. (1987)

$$h_{k,t} \equiv \omega_k + \alpha_k y_{t-1}^2 + \beta_k h_{k,t-1}$$

Here $\boldsymbol{\theta}_k \equiv (\omega_k,\ \alpha_k,\ \beta_k)'$, where $\omega_k > 0$, α_k, $\beta_k \geq 0$ and $\alpha_k + \beta_k < 1$ to ensure positivity and covariance stationarity in each regime.

The **EGARCH** model of Nelson (1991)

$$ln(h_{k,t}) \equiv \omega_k + \alpha_k(|\eta_{k,t-1}| - E[|\eta_{k,t-1}|]) + \gamma_k y_{t-1} + \beta_k ln(h_{k,t-1})$$

Here $\boldsymbol{\theta}_k \equiv (\omega_k,\ \alpha_k,\ \eta_k,\ \beta_k)'$, with $\beta_k < 1$ to ensure covariance stationarity in each regime. The expectation $E[|\eta_{k,t-1}|]$ is taken with respect to the distribution conditional on regime k. Also, leverage effect is taken into account in this specification.

The **GJRGARCH(1,1)** model of Glosten et al. (1993)

$$h_{k,t} \equiv \omega_k + (\alpha_k + \gamma_k \mathbb{1}\{y_{t-1} < 0\})y_{t-1}^2 + \beta_k h_{k,t-1}$$

Again, here $\boldsymbol{\theta}_k \equiv (\omega_k,\ \alpha_k,\ \eta_k,\ \beta_k)'$, with $\omega_k > 0$, $\alpha_k > 0$, $\eta_k \geq 0$ and $\beta_k \geq 0$, which ensures positivity and $\omega_k + \alpha_k \mathrm{E}[\eta_{k,t}^2 \mathbb{1}\{\eta_{k,t} < 0\}] + \beta_k < 1$, required for covariance stationarity in each regime. Also, the indicator function $\mathbb{1}\{\cdot\}$ takes the value of one if the condition holds and zero if it does not hold. This model is able to, in the conditional volatility process, capture the asymmetry, which is controlled by the parameter γ_k.

The standardised Student-t distribution can be denoted as:

$$f_S(\eta;\nu) \equiv \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{(v-2)\pi}\Gamma\frac{\nu}{2}}\left(1 + \frac{\eta^2}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \eta \in \mathbb{R} \quad (1)$$

An additional parameter, $\xi > 0$, is added to the standardised distribution to incorporate skewness, where the distribution is symmetric if $\xi = 1$. This can be denoted as sk$N$.

4.2 Estimation

The models are calibrated using a Bayesian approach via Markov Chain Monte Carlo (MCMC), which requires the evaluation of the likelihood function. Let the vector of the model parameters be denoted as $\Psi \equiv (\boldsymbol{\theta}_k,\ \boldsymbol{\xi}_1, \cdots,\ \boldsymbol{\theta}_k,\ \mathbf{P})$. The likelihood function can be expressed as:

$$L(\Psi|I_T) = \prod_{t=1}^{T} f(y_t|\Psi, I_{t-1}),$$

where I_{t-1} are the past observations and f($y_t \mid \Psi,\ I_{t-1}$) the density of y_t.
The conditional density of y_t for the MSGARCH model is:

$$f(y_t|\Psi, I_{t-1}) \equiv \sum_{i=1}^{K}\sum_{j=1}^{K} p_{i,j} z_{i,t-1} f_D(y_t|s_t = j, \Psi, I_{t-1}),$$

where $z_{i,t-1} \equiv$ P[s_{t-1} = i — Ψ, I_{t-1}], which represents at time t-1 the filtered probability of state i. The filtered probabilities are obtained via Hamiltons filter. By maximising the logarithm in Eq. (1), the ML estimator $\widehat{\boldsymbol{\Psi}}$ is obtained. For Bayesian estimation, in order to build the kernel of the posterior distribution f($\widehat{\boldsymbol{\Psi}}$ — I_T), a prior f($\widehat{\boldsymbol{\Psi}}$) is used and combined with the likelihood function.

From diffuse independent priors, the prior is built as follows (Ardia et al. (2016)):

$$f(\boldsymbol{\Psi}) \propto f(\boldsymbol{\theta}_1, \boldsymbol{\xi}_1) \cdots f(\boldsymbol{\theta}_K, \boldsymbol{\xi}_K) f(\mathbf{P})$$

$$f(\boldsymbol{\theta}_k, \boldsymbol{\xi}_k) \propto f(\boldsymbol{\theta}_k) f(\boldsymbol{\xi}_k) \mathbb{1}\{(\boldsymbol{\theta}_k, \boldsymbol{\xi}_k) \in CSC_k\} \quad (k = 1, \cdots, K)$$

$$f(\boldsymbol{\theta}_k) \propto f_N(\boldsymbol{\theta}_k; \boldsymbol{\mu}_{\boldsymbol{\theta}_k}, diag(\boldsymbol{\sigma}^2_{\boldsymbol{\theta}_k}) \mathbb{1}\{\boldsymbol{\theta}_k \in PC_k\} \quad (k = 1, \cdots, K)$$

$$f(\boldsymbol{\xi}_k) \propto f_N(\boldsymbol{\xi}_k; \boldsymbol{\mu}_{\boldsymbol{\xi}_k}, diag(\boldsymbol{\sigma}^2_{\boldsymbol{\xi}_k})) \mathbb{1}\{\xi_{k,1} > 0, \xi_{k,2} > 2\} \quad (k = 1, \cdots, K)$$

$$f(\mathbf{P}) \propto \prod_{i=1}^{K} (\prod_{j=1}^{K} p_{i,j}) \mathbb{1}\{0 < p_{i,j} < 1\},$$

where the covariance stationarity is denoted by CSK_k and the positivity condition in the regime k is denoted by PC_k. The asymmetry parameter is $\xi_{k,1}$ and the tail parameter is $\xi_{k,2}$ in regime k of the skewed Student-t distribution. The multivariate Normal density from above is $f_N(\boldsymbol{\theta}_k; \boldsymbol{\mu}_{\boldsymbol{\theta}_k}, diag(\boldsymbol{\sigma}^2_{\boldsymbol{\theta}_k}))$ and $f_N(\boldsymbol{\xi}_k; \boldsymbol{\mu}_{\boldsymbol{\xi}_k}, diag(\boldsymbol{\sigma}^2_{\boldsymbol{\xi}_k}))$, with covariance matrix $\boldsymbol{\Sigma}$ and mean vector $\boldsymbol{\mu}$. $\boldsymbol{\sigma}^2_{\boldsymbol{\theta}_k}$ and $\boldsymbol{\sigma}^2_{\boldsymbol{\xi}_k}$ are vectors of prior variances, with entries set to 1,000 by default and $\boldsymbol{\mu}_{\boldsymbol{\theta}_k}$ and $\boldsymbol{\mu}_{\boldsymbol{\xi}_k}$ are vectors of prior means, with entries set to 0 by default.

Overall, since I only consider Markov-Switching in a two state regime, the number of regimes is K=2 and following Ardia et al. (2019), the conditional distribution $D(\cdot)$ considered is the skewed Student-t distribution. Therefore in total, the model set includes three specifications each with k $\in \{1, 2\}$, a conditional distribution $D \in \text{sk}N$, estimated via MCMC, but a different conditional variance specification in each. These include GARCH(1,1), EGARCH and GJRGARCH(1,1). Throughout, the MSGARCH R package (Ardia et al. (2016)) is used to implement MSGARCH models in the R statistical language with efficient C++ code, which provides regulators and risk managers the methodologies to improve risk forecasts of their portfolios.

4.3 Risk Metric

The risk metric used is Value-at-Risk (VaR). Given a risk level $\alpha \in$ (0,1), the VaR is the loss such that the probability of losses equaling or exceeding VaR in a given trading period is equal to α. The VaR forecast at risk level α in T+1 is defined as:

$$VaR^{\alpha}_{T+1} \equiv inf\{y_{T+1} \in \mathbb{R} | F(y_{T+1}|I_T) = \alpha\},$$

where $F(y_{T+1}|I_T)$ is the the one-step ahead CDF evaluated in y.

The one-day ahead VaR is forecasted and considered at 1% and 5% risk levels.

Exponentially Weighted Moving Average (EWMA) is also used to graphically compare the VaR forecasts of the model specification, chosen for each cryptocurrency. It is effectively a restricted integrated GARCH (iGARCH) model, with ω equal to zero. More weight is assigned to the most recent observations and the smoothing parameter λ is traditionally set to 0.94. The model is denoted as:

$$\sigma_{t+1}^2 = (1-\lambda)y_t^2 + \lambda\sigma_t^2$$

4.4 Back-testing

The R package **GAS** is used to evaluate the accuracy of the VaR forecasts, in terms of correctly predicting the α-quantile loss, given that we expect to have a proportion α of exceedances. The package is able to compute the p-values for two backtesting hypothesis tests of correct conditional coverage of the VaR. The first is the conditional coverage (CC) test (Christoffersen (1998)). It is achieved via a likelihood ratio test, which is asymptotically chi-squared distributed, with two degrees of freedom. The second is the dynamic quantile (DQ) test (Engle and Manganelli (2004)), which tests if the VaR violations are independent over time. Under the correct model specification both of the tests should be fulfilled.

Following Ardia et al. (2018), to carry out the VaR forecast tests, a hit variable is defined, that indicates a loss that exceeds the VaR level:

$$I_t^\alpha \equiv \mathbb{1}\{y_t \leq VaR_t^\alpha\},$$

where $\mathbb{1}\{\cdot\}$ is the indicator function. If the condition holds the function is equal to one and zero otherwise. In other words, when $I_t = 1$, we say that a violation has occurred. The VaR prediction at a risk level of α is denoted as VaR_t^α. If $\{I_t^\alpha; t = 1, ..., H\}$ is an independent and identically distributed sequence of Bernoulli random variables with parameter α, then at risk level α a sequence of VaR forecasts has conditional coverage. For the DQ test, if $\{I_t^\alpha - \alpha; t = 1, ..., H\}$ has the correct model specification, the following moment conditions are satisfied:

$$\mathbb{E}[I_t^\alpha - \alpha] = 0,$$

$$\mathbb{E}[I_t^\alpha - \alpha \,|\, \mathcal{I}_{t-1}] = 0,$$

$$\mathbb{E}[(I_t^\alpha - \alpha)(I_{t'}^\alpha - \alpha)] = 0 \quad for\, t \neq t'$$

The risk levels tested are $\alpha = 0.01$ and $\alpha = 0.05$.

5 Results

The results from Table 1 report the DIC and IC values for each model specification, estimated via MCMC, with a skewed Student-t conditional distribution. For each cryptocurrency, the table highlights the optimal model, which is the smallest DIC and IC value. The DIC and IC values indicate that the optimal model is EGARCH for all cryptocurrencies, as well GARCH for Litecoin.

Table 1: DIC and IC values of two state Markov-Switching GARCH models, for each cryptocurrency. Highlighted in red are the smallest (optimal) values.

	LTC	XRP	ETH	EOS	BNB
			GARCH		
DIC	-8227.569	-5801.579	-4890.661	-2357.56	-2232.507
IC	-8211.931	-5793.004	-4878.452	-2341.869	-2216.065
			EGARCH		
DIC	-8223.578	-5805.114	-4891.516	-2369.202	-2256.315
IC	-8212.732	-5798.421	-4881.444	-2360.389	-2247.378
			GJR GARCH		
DIC	-8216.299	-5794.868	-4887.597	-2357.238	-2244.912
IC	-8203.231	-5783.169	-4873.806	-2343.567	-2237.034

However, additional results indicate otherwise. Tables 5, 6 and 7 in the appendix show the parameter estimates of the two state Markov Switching models, for each cryptocurrency. The median, 25th and 75th percentiles are reported. Since the cryptocurrencies are denominated in Bitcoin, the highlighted values of the parameter medians in the tables, indicate EOS and Ether only need two states and Binance Cash, Litecoin and Ripple only need one state. Therefore, I estimate these three cryptocurrencies again, with model specifications that have K=1 for the number of regimes. Table 2 reports the DIC and IC values for these three cryptocurrencies. The smallest (optimal) DIC and IC values are highlighted. The DIC and IC values indicate that the optimal model is EGARCH for Ripple and Binance Coin and GJR GARCH for Litecoin.

Table 2: DIC and IC values of one-state GARCH models for Litecoin, Ripple and Binance Cash. Highlighted in red are the smallest (optimal) values.

	LTC	XRP	BNB
		GARCH	
DIC	-8113.488	-5742.664	-2224.822
IC	-8107.311	-5738.148	-2215.922
		EGARCH	
DIC	-8111.276	-5773.351	-2243.922
IC	-8105.709	-5767.588	-2239.112
		GJR GARCH	
DIC	-8121.326	-5476.415	-2231.524
IC	-8116.417	-5739.852	-2226.303

Table 3 reports the p-values for the backtesting procedures: the conditional coverage (CC) test (Christoffersen (1998)) and dynamic quantile (DQ) test (Engle and Manganelli (2004)), for EOS and Ether. Table 4 presents the results of the same procedures, but for Litecoin, Ripple and Binance Coin, as they use single-regime models. The model backtested for each of the latter three cryptocurrencies is the optimal model, highlighted in Table 2. A one-step ahead VaR prediction at 1% and 5% levels are applied to each cryptocurrency. Since Ether and EOS use Markov-Switching specification, a backtest is run on all three models one-step ahead VaR forecasts (GARCH, EGARCH, GJR

GARCH). For the estimates and predictions, a rolling window is fitted at every step. Following Caporale and Zekokh (2019), the rolling window size is 70% of the total number of observations. The model parameters are updated every tenth observation to speed up the computations, since Ardia et al. (2018) find similar results when the model parameters are updated daily for a subset of stocks. Furthermore, given the results from the in-sample analysis in Table 1, it's surprising the one-step ahead VaR forecasts could not be computed for the EGARCH specification and consequently the CC and DQ tests either, therefore results for EGARCH are not in Table 3. This could imply that EGARCH is not a suitable model specification for the two cryptocurrencies: Ether and EOS.

The significance levels 1% and 5% are applied for each test and for the model to be accepted the p-value needs to be above 1% and 5% respectively for the conditional coverage (CC) test. Essentially, the CC test tells us if violations are clustered at the same time, as the number of violations are correct. The alternative conditional coverage tested is the dynamic quantile (DQ) test, which is more powerful than the CC test.

Table 3: P-values of the conditional coverage (CC) test and the dynamic quantile (DQ) test for the one-step ahead 1 % and 5 % VaR. Based on two-state Markov-Switching models, for Ether and EOS.

	ETH	**EOS**
CC 1% VaR		
GARCH	0.77948	0.64308
GJR GARCH	0.95147	0.64308
CC 5% VaR		
GARCH	0.17833	0.32087
GJR GARCH	0.35067	0.32087
DQ 1% VaR		
GARCH	0.98128	0.92901
GJR GARCH	0.98857	0.97531
DQ 5% VaR		
GARCH	0.74688	0.56744
GJR GARCH	0.83725	0.65687

Table 4: P-values of the conditional coverage (CC) test and the dynamic quantile (DQ) test for the one-step ahead 1 % and 5 % VaR. Based on one-state GARCH models, for Litecoin, Ripple and Binance Cash. * denotes the failure of the test.

LTC	**XRP**	**BNB**
GJR GARCH	**EGARCH**	**EGARCH**
	CC 1% VaR	
0.14204	0.88191	0.18815
	CC 5% VaR	
0.00052*	0.21147	0.02567*
	DQ 1% VaR	
0.0.01816	0.99888	0.1660
	DQ 5% VaR	
0.00014*	0.62918	0.03883*

5.1 Ether

The total number of observations for Ether is 1440, therefore the VaR forecast is estimated using an out-of-sample size of 432 and a rolling window size of 1008. The conditional coverage (CC) test and dynamic quantile (DQ) tests at both 1% and 5% levels could not be satisfied under the EGARCH specification, conflicting with the in-sample results earlier. From Table 3, the CC test p-values for both 1% and 5% are successful for GARCH and GJR GARCH, and therefore satisfy the VaR backtesting procedures. Since both models are suitable, GJR GARCH is chosen as it has higher p-values than GARCH, for each of the tests at both significance levels. This is inline with Caporale and Zekokh (2019), who finds a GJR GARCH specification and a Student-t distribution in the first regime as the best model and a TGARCH model with a skewed Student-t distribution in the second regime. The distribution used for each of the model specifications is a skewed Student-t. Additionally, when the DQ test is applied the p-values increase, indicating the results have improved. In Figure 2. the 1% VaR EWMA (red dotted line) is plotted against the 1% VaR forecast provided by GJR GARCH (green dotted line), along with Ethers log returns (black line) in the forecast window. The 1% VaR is smoother for EWMA and reacts quicker to the higher spikes, however the 1% VaR for GJR GARCH provides a tighter fit to the actual data.

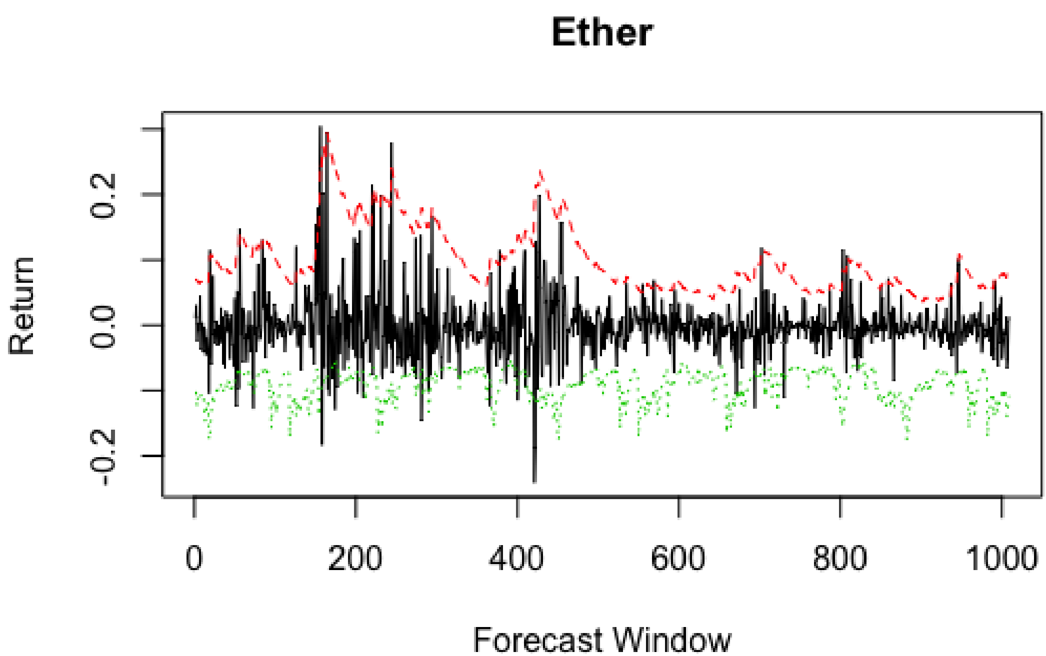

Figure 2: One-day ahead VaR forecast at 1% risk level, provided by the Markov-Switching GJR GARCH model (dotted green line), a 1% VaR EWMA (dotted red line) and log-returns (black line), for Ether.

5.2 EOS

The total number of observations is 748, therefore the VaR forecast for EOS is estimated using an out-of-sample size of 224 and a rolling window size of 524. Similarly to Ether, the VaR forecasts could not be computed using the EGARCH model and therefore, neither the conditional coverage (CC) and dynamic quantile (DQ) tests at both 1% and 5% levels were not calculated. Again, conflicting with the in-sample results earlier in Table 2. Both GARCH, GJR GARCH satisfy the CC and DQ tests at both significance levels. The CC test at both 1% and 5% produce the same p-values for GARCH and GJR GARCH, however when the DQ test is applied, GJR GARCH gives a higher p-value than GARCH, therefore GJR GARCH is chosen as the most suitable model. In Figure 3, the 1% VaR EWMA is plotted against the 1% VaR forecast provided by GJR GARCH. Nonetheless, we have similar results to Ether, with GJR GARCH illustrating a tighter fit to the data points than the 1% VaR EWMA. This is since the log returns are very volatile, so the VaR forecast increases. I struggled to find a paper looking into the volatility dynamics of EOS, however EOS was only launched in January 2018, so the cryptocurrency is still rather new.

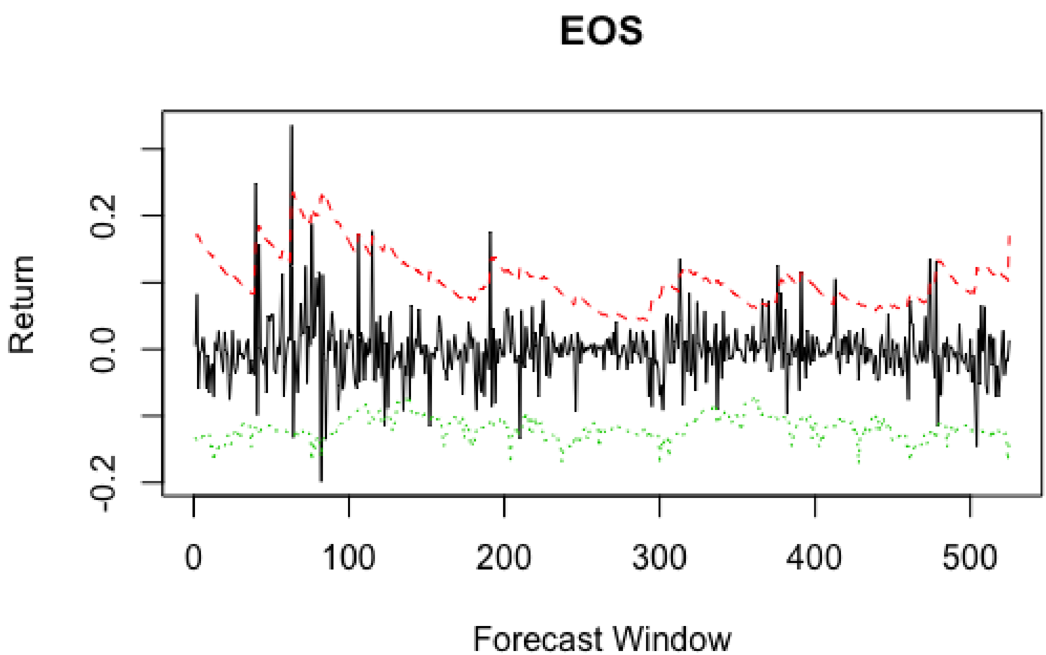

Figure 3: One-day ahead VaR forecast at 1% risk level, provided by the Markov-Switching GJR GARCH model (dotted green line), a 1% VaR EWMA (dotted red line) and log-returns (black line), for EOS.

5.3 Litecoin

The total number of observations for Litecoin is 2117, therefore the VaR forecast is estimated using an out-of-sample size of 518 and a rolling window size of 1482. Given the in-sample analysis results in

Table 2, the GJR GARCH model is chosen and backtested. In Table 4, at the 5% significance level, the conditional coverage (CC) test and dynamic quantile (DQ) test fails, since the p-value is under 5%. At the 1% significance level, the p-value for the CC test is close to 1% and when the DQ test is applied, the result worsens, by being very close to 1%. The 1% VaR using a GJR GARCH model is plotted against a 1% EWMA in Figure 4. The 1% VaR GJR GARCH model seems to overestimate the forecast in areas on the graph, as well as many violations occurring, indicated whenever the returns (black line) is lower than the VaR forecast (dotted green line). The backtest results and the plot below, indicates an inaccuracy of the VaR forecast. Consequently, the 1% VaR EWMA forecast produces a smoother plot, indicating a better prediction. In comparison to the literature, Caporale and Zekokh (2019) results suggest a symmetric GARCH model are appropriate for one regime in the case for Bitcoin and Litecoin. The DIC and IC values, in Table 2, between GARCH and GJR GARCH were not very different for Litecoin, therefore if GARCH was selected for forecasting and backtesting, similar results could have been attained. Chu et al. (2017) also finds GJR GARCH an acceptable model to provide a reliable estimate of VaR, however finds IGARCH(1,1) model gives the best fit for Litecoin.

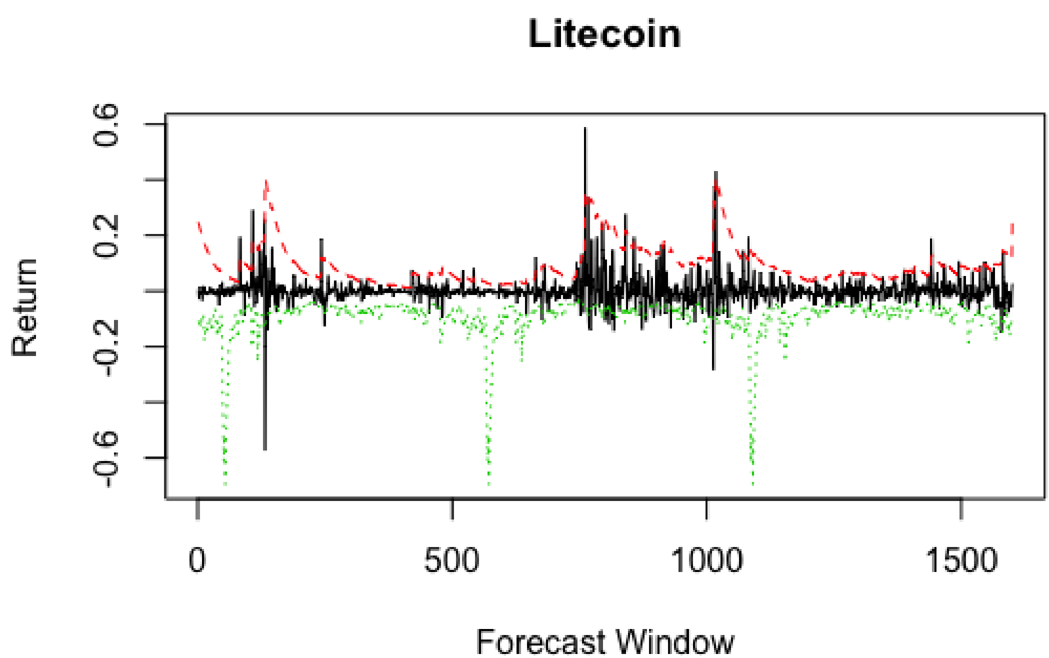

Figure 4: One-day ahead VaR forecast at 1% risk level, provided by single-regime GJR GARCH model (dotted green line), a 1% VaR EWMA (dotted red line) and log-returns (black line), for Litecoin.

5.4 Ripple

The total number of observations is 1639, therefore the VaR forecast for Ripple is estimated using an out-of-sample size of 492 and a rolling window size of 1147. Given the in-sample results in Table 2, the EGARCH model is chosen and the VaR forecast is backtested. At both 1% and 5% significance levels the EGARCH model is accepted by the conditional coverage (CC) test. When the dynamic quantile (DQ) test is applied, the p-values improve for both significance levels. The p-values are higher using the 1% level, therefore the VaR forecast at 1% using an EGARCH model is compared to a 1% VaR EWMA, as shown in Figure 6. The violations occur when the log returns (black) is lower than the 1% VaR EGARCH forecast (dotted green). This indicates the forecast may underestimate the prediction in many areas in the graph. However, the 1% VaR EGARCH is tighter around the actual data, than the 1% VaR EWMA, which is a lot smoother. Similarly, Chu et al. (2017) performed the CC test on the returns of Ripple and find EGARCH an acceptable estimate of VaR, however find GARCH(1,1) model gives the best fit for Ripple. Additionally, the choice of model in these results are a contradiction to Caporale and Zekokh (2019), who suggests a standard GARCH and TGARCH for the single regime and a TGARCH for the second regime. However, the prices used in these studies are denominated in USD.

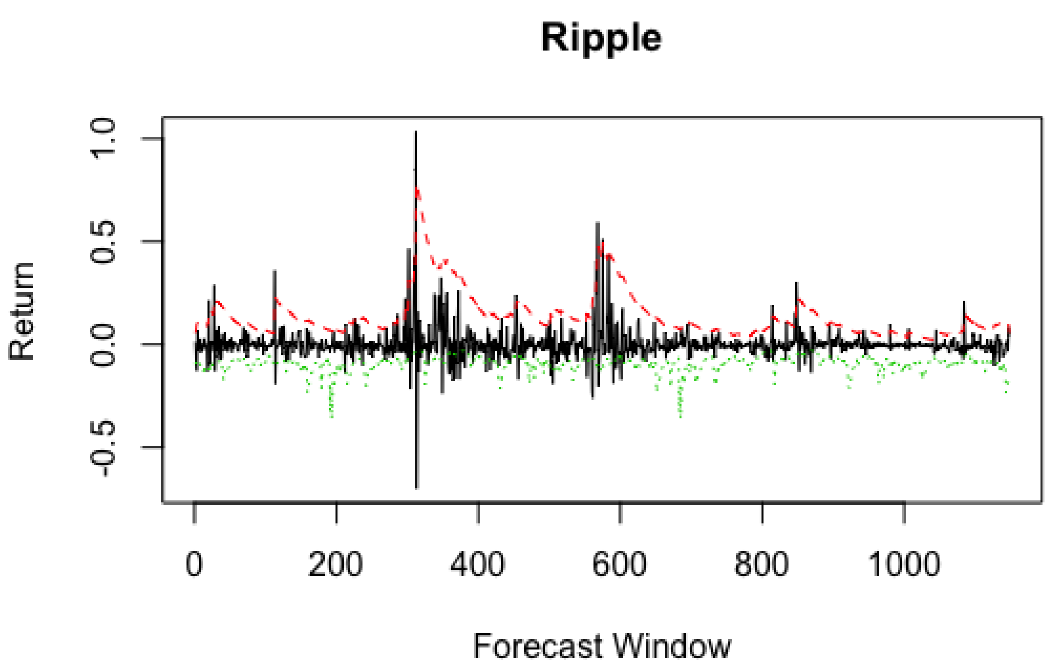

Figure 5: One-day ahead VaR forecast at 1% risk level, provided by single-regime EGARCH model (dotted green line), a 1% VaR EWMA (dotted red line) and log-returns (black line), for Ripple.

5.5 Binance Coin

The total number of observations for Binance Coin is 677, therefore the out-of-sample size is 204 and the rolling window size is 473. From the in-sample results in Table 2, the EGARCH specification is chosen and backtested. At the 1% significance level, the model is accepted by the conditional coverage (CC) test, however at the 5% significance level, the model fails both the CC test and DQ test. Also, when the DQ test is applied to the 1% VaR, the p-value worsens. This might indicate EGARCH is not the most suitable model and a model not tested could be better. The 1% VaR EGARCH forecast is compared to a 1% VaR EWMA in Figure 6. Both forecasts are not very tight to the actual returns, which are very volatile, however the 1% VaR EGARCH reacts quicker to the changes, than the 1% EWMA over time. I could not find another paper looking into the volatility dynamics of Binance Coin, but this could be due to the coin only being created in mid 2017. Therefore the data set available is a lot smaller than other cryptocurrencies. With more observations, the models may be able to analyse the behaviour of the cryptocurrency, in order to make more accurate predictions.

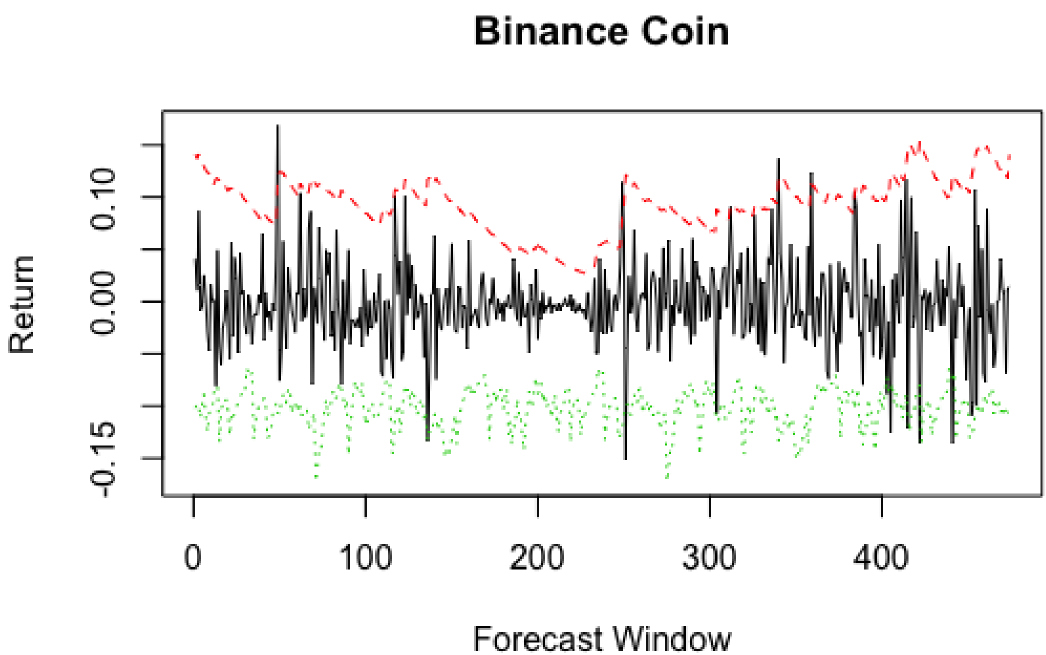

Figure 6: One-day ahead VaR forecast at 1% risk level, provided by single-regime EGARCH model (dotted green line), a 1% VaR EWMA (dotted red line) and log-returns (black line), for Binance Coin.

6 Conclusion

The use of prices denominated in Bitcoin, played a part for the models suitable for each of the cryptocurrencies, indicating that MSGARCH models may not always be fitting. I was not expecting to use single regime models, but these were more suitable for three cryptocurrencies than regime switching specifications. Caporale and Zekokh (2019) results suggest a symmetric GARCH model is appropriate for one regime in the case for Bitcoin. Therefore, since the prices are denominated in Bitcoin in this study, the single regime results may not be very surprising. For the two-state Markov-Switching models, the GJR GARCH model specification is chosen as the best model for both Ether and EOS and are inline with Ardia et al. (2019). Also, it is no surprise asymmetric models have been chosen out of the model specifications, to account for the leverage effect that are evident in cryptocurrencies. The two-state Markov-Switching models yield better results in terms of prediction of VaR for Ether and EOS, compared to the single-regime GARCH models for the three other cryptocurrencies. This is inline with Caporale and Zekokh (2019). The differences in findings to papers in the literature, could be due to these studies using prices denominated in USD, a different historical period and larger data sets used.

Overall, the models tested in this study, that are most suitable for each cryptocurrency are as follows: a two-state Markov-Switching GJR GARCH specification for both Ether and EOS. A one-state GJR GARCH from the tests was chosen, however the 1% VaR EWMA forecast gave a better prediction for Litecoin, indicating more testing of different models are needed for this cryptocurrency. Lastly, a one-state EGARCH specification for Ripple and Binance Coin.

The findings imply the understanding and modelling of cryptocurrencies is of great importance, given the highly volatile behaviour exhibited and its relationship with respect to risk. Bitcoins prices have changed drastically over the last decade and the prices fluctuations are at times erratic, even with its popularity grown worldwide. The prices collapsing and rising correspond to risky events (Rojas and Coronado (2019)). Therefore, accurate risk management, especially during times of financial crisis is key. Similarly to Caporale and Zekokh (2019), the backtesting procedures and graphs indicate that Value-at-Risk forecasts may not be accurate for single-regime models and lead to ineffective risk management. Nonetheless, these results could be useful and help others make better decisions in regards to financial investments in these particular cryptocurrencies.

Moreover, in a further study, it would be interesting to see how these results compare to their prices denominated in USD, as well as looking more closely at Bitcoin prices, since Bitcoin dominates the cryptocurrency market. Also, using regime mixture model specifications, to see if these produce different findings. Expected Shortfall is becoming more popular, with Basel III agreeing to replace Value-at-Risk with Expected Shortfall, for the internal model-based approach. Therefore a study using Expected Shortfall and/or Value-at-Risk could be looked at in the future.

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8 Appendix

Table 5: Markov- Switching GARCH parameter medians and in brackets the 25th and 75th percentiles for each cryptocurrency.

	LTC	**XRP**	**ETH**	**EOS**	**BNB**
			State 1		
$\boldsymbol{\omega}_1$	4.46E-06 (2.9E-06, 6.3E-06)	6.42E-06 (3.7E-06, 1.2E-05)	0.0002 (0.0002, 0.0003)	0.0002 (0.0001, 0.0003)	2.80E-05 (1.8E-05, 5.4E-05)
$\boldsymbol{\alpha}_1$	0.1845 (0.1687, 0.2022)	0.1895 (0.1545, 0.2072)	0.2725 (0.2199, 0.3308)	0.2803 (0.1992, 0.3398)	0.1732 (0.1355, 0.2167)
$\boldsymbol{\beta}_1$	0.8034 (0.7883, 0.8178)	0.8041 (0.7790, 0.8273)	0.6002 (0.5476, 0.6397)	0.6481 (0.5854, 0.7342)	0.7789 (0.7200, 0.8231)
$\boldsymbol{\eta}_1$	3.2322 (3.0974, 3.4157)	3.8567 (3.5618, 4.2717)	3.3293 (3.0374, 3.7011)	3.1239 (2.9110, 3.6549)	5.1551 (4.1521, 7.3254)
$\boldsymbol{\xi}_1$	1.1269 (1.1032, 1.1491)	1.2108 (1.1726, 1.2585)	1.1670 (1.1359, 1.2005)	1.1494 (1.1054, 1.2347)	1.3208 (1.2334, 1.4401)
$\mathbf{p}_{1,1}$	0.8722 (0.8469, 0.8933)	0.8831 (0.8553, 0.9050)	0.9893 (0.9860, 0.9922)	0.7876 (0.6968, 0.8395)	0.8154 (0.7454, 0.8691)
			State 2		
$\boldsymbol{\omega}_2$	0.0025 (0.0021, 0.0029)	0.0040 (0.0031, 0.0050)	0.0015 (0.0013, 0.0017)	0.0005 (0.0004, 0.0007)	0.0037 (0.0027, 0.0048)
$\boldsymbol{\alpha}_2$	**0.9987** **(0.9975, 0.9992)**	**0.8705** **(0.7777, 0.9235)**	0.3001 (0.2517, 0.3605)	0.0999 (0.0755, 0.1402)	**0.1216** **(0.0448, 0.1995)**
$\boldsymbol{\beta}_2$	**0.0002** **(0.0001, 0.0004)**	**0.0002** **(9.1E-05, 0.0010)**	0.5308 (0.4892, 0.5815)	0.8776 (0.8282, 0.8974)	**0.0002** **(8.5E-05, 0.0012)**
$\boldsymbol{\eta}_2$	2.8668 (2.7600, 3.0201)	2.6171 (2.4696, 2.8123)	4.3643 (4.0075, 4.7332)	15.5026 (10.2114, 20.9657)	10.3279 (4.9323, 18.6991)
$\boldsymbol{\xi}_2$	1.4172 (1.3524, 1.4776)	1.2706 (1.2270, 1.3147)	1.2079 (1.1698, 1.2499)	2.3057 (1.7125, 3.0618)	1.2521 (1.1479, 1.3757)
$\mathbf{p}_{2,2}$	0.3213 (0.2751, 0.3786)	0.1519 (0.1191, 0.1881)	0.0102 (0.0073, 0.0148)	0.6750 (0.5143, 0.7710)	0.4141 (0.2638, 0.5686)

Table 6: Markov- Switching EGARCH parameter medians and in brackets the 25th and 75th percentiles for each cryptocurrency.

	LTC	**XRP**	**ETH**	**EOS**	**BNB**
			State 1		
$\boldsymbol{\omega}_1$	-0.1608 (-0.1793, -0.1456)	-0.0142 (-3.1968, -0.0088)	-0.293 (-0.3464, -0.2336)	-0.293 (-0.3464, -0.2336)	-7.4453 (-8.0869, -6.5052)
$\boldsymbol{\alpha}_1$	0.4304 (0.3912, 0.4800)	0.6664 (0.4675, 0.9452)	0.364 (0.3118, 0.4109)	0.364 (0.3118, 0.4109)	0.345 (0.2451, 0.4529)
$\boldsymbol{\gamma}_1$	-0.1608 (-0.1883, -0.1358)	-0.0775 (-0.2010, -0.0223)	0.0135 (-0.0121, 0.0418)	0.0135 (-0.0121, 0.0418)	-0.0177 (-0.0861, 0.0424)
$\boldsymbol{\beta}_1$	0.9742 (0.9711, 0.9767)	0.9989 (0.4564, 0.9992)	0.9664 (0.9607, 0.9728)	0.9664 (0.9607, 0.9728)	**-0.219** **(-0.3117, -0.0833)**
$\boldsymbol{\eta}_1$	2.1508 (2.1395, 2.1658)	2.6996 (2.4572, 2.9703)	2.9807 (2.6780, 3.6405)	2.9807 (2.6780, 3.6405)	4.0124 (3.5643, 4.6888)
$\boldsymbol{\xi}_1$	1.1147 (1.0997, 1.1315)	1.2176 (1.1661, 1.2761)	1.0991 (1.0527, 1.1472)	1.0991 (1.0527, 1.1472)	1.2999 (1.2243, 1.3714)
$\mathbf{p}_{1,1}$	0.9804 (0.9795, 0.9812)	0.9132 (0.9075, 0.9219)	0.7734 (0.7581, 0.7858)	0.7734 (0.7581, 0.7858)	0.9912 (0.9902, 0.9922)
			State 2		
$\boldsymbol{\omega}_2$	-3.7019 (-3.9731, -3.4699)	-2.5743 (-3.2607, -0.0032)	-0.3587 (-0.4277, -0.2963)	-0.3587 (-0.4277, -0.2963)	-0.0901 (-0.1222, -0.0672)
$\boldsymbol{\alpha}_2$	**0.6934** **(0.6165, 0.7822)**	**0.7106** **(0.5184, 0.9945)**	0.1305 (0.0849, 0.1902)	0.1305 (0.0849, 0.1902)	0.5417 (0.4495, 0.6283)
$\boldsymbol{\gamma}_2$	0.0778 (0.0209, 0.1316)	-0.0885 (-0.1837, -0.0318)	0.1221 (0.0756, 0.1654)	0.1221 (0.0756, 0.1654)	0.0372 (-0.0292, 0.0893)
$\boldsymbol{\beta}_2$	0.2633 (0.2166, 0.3070)	0.5404 (0.4389, 0.9992)	0.9176 (0.9062, 0.9269)	0.9176 (0.9062, 0.9269)	0.9808 (0.9756, 0.9843)
$\boldsymbol{\eta}_2$	3.9811 (3.7684, 4.2359)	2.6411 (2.4480, 3.0190)	8.6545 (7.5035, 9.4695)	8.6545 (7.5035, 9.4695)	3.3718 (3.1763, 3.5458)
$\boldsymbol{\xi}_2$	1.2735 (1.2322, 1.3324)	1.2202 (1.1694, 1.2708)	2.103 (1.8041, 2.3653)	2.103 (1.8041, 2.3653)	1.1981 (1.3782, 1.2857)
$\mathbf{p}_{2,2}$	0.0451 (0.0424, 0.0496)	0.0874 (0.0803, 0.0946)	0.6579 (0.6203, 0.6970)	0.6579 (0.6203, 0.6979)	0.0086 (0.0074, 0.0106)

Table 7: Markov- Switching GJRGARCH parameter medians and in brackets the 25th and 75th percentiles for each cryptocurrency.

	LTC	**XRP**	**ETH**	**EOS**	**BNB**
			State 1		
$\boldsymbol{\omega}_1$	5.11E-06 (3.4E-06, 7.0E-06)	2.13E-05 (1.4E-05, 3.5E-05)	0.0002 (0.0002, 0.0003)	0.0022 (0.0002, 0.0037)	5.83E-06 (2.6E-06, 1.0E-05)
$\boldsymbol{\alpha}_1$	0.1076 (0.0963, 0.1179)	0.2119 (0.1933, 0.2369)	0.2506 (0.2179, 0.2832)	0.1816 (0.1163, 0.2396)	0.2682 (0.2183, 0.3110)
$\boldsymbol{\gamma}_1$	0.0735 (0.0508, 0.1007)	0.0308 (0.0203, 0.0434)	0.0010 (0.0007, 0.0014)	0.0001 (0.0001, 0.0069)	0.0001 (0.0001, 0.0001)
$\boldsymbol{\beta}_1$	0.8417 (0.8328, 0.8501)	0.7540 (0.7266, 0.7717)	0.5850 (0.5215, 0.6408)	0.6242 (0.4420, 0.7275)	0.6956 (0.6550, 0.7444)
$\boldsymbol{\eta}_1$	2.9502 (2.8310, 3.0775)	3.6442 (3.3188, 4.0213)	3.2605 (2.9553, 3.7775)	15.8259 (3.1214, 15.6858)	31.4153 (19.6643, 48.3352)
$\boldsymbol{\xi}_1$	1.1194 (1.0979, 1.1433)	1.2120 (1.1659, 1.2645)	1.1800 (1.1447, 1.2183)	2.5608 (1.1635, 4.0162)	1.6723 (1.5036, 1.8805)
$\mathbf{p}_{1,1}$	0.9115 (0.8979, 0.9251)	0.8771 (0.8527, 0.8972)	0.9876 (0.9836, 0.9909)	0.3360 (0.1995, 0.8099)	0.6644 (0.5620, 0.7653)
			State 2		
$\boldsymbol{\omega}_2$	0.0018 (0.0015, 0.0020)	0.0025 (0.0020, 0.0029)	0.0010 (0.0007, 0.0012)	0.0001 (0.0001, 0.0031)	0.0108 (0.0056, 0.0181)
$\boldsymbol{\alpha}_2$	**0.7202** **(0.6304, 0.7904)**	**0.5068** **(0.3780, 0.6155)**	0.2607 (0.2274, 0.2975)	0.2451 (0.1922, 0.2786)	**0.4868** **(0.2840, 0.6751)**
$\boldsymbol{\gamma}_2$	0.0001 (0.0001, 0.0001)	0.0001 (0.0001, 0.0001)	0.0003 (0.0002, 0.0004)	0.0087 (0.000q, 0.0182)	0.0011 (0.0004, 0.0036)
$\boldsymbol{\beta}_2$	**0.0944** **(0.0663, 0.1294)**	**0.1757** **(0.1265, 0.2312)**	0.6171 (0.5753, 0.6616)	0.7187 (0.5703, 0.7538)	**0.0009** **(0.002, 0.0033)**
$\boldsymbol{\eta}_2$	3.6270 (3.3216, 4.0190)	2.9992 (2.7792, 3.2451)	4.7165 (4.0442, 5.3429)	2.8897 (2.7225, 12.2582)	2.2229 (2.1272, 2.4929)
$\boldsymbol{\xi}_2$	1.4123 (1.3492, 1.4916)	1.2471 (1.1930, 1.3002)	1.1985 (1.1636, 1.2434)	1.1527 (1.1029, 2.1501)	1.1271 (1.0834, 1.1859)
$\mathbf{p}_{2,2}$	0.1811 (0.1463, 0.2185)	0.1460 (0.1144, 0.1795)	0.0104 (0.0076, 0.0148)	0.1469 (0.1080, 0.6967)	0.3614 (0.2253, 0.5183)