ulm university universität
uulm

The Kibble Zurek mechanism in ion traps

A. Retzker, A. Del Campo, M. Plenio,
G. Morigi and G. De Chiara

2010 European Conference on Trapped Ions

Del Campo, etal,
PRL 105, 075701

ulm university universität uulm

Collaborators

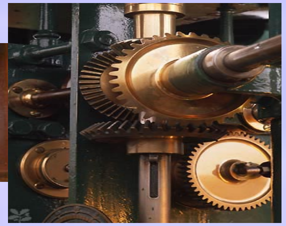
D. Segal, R. Thompson

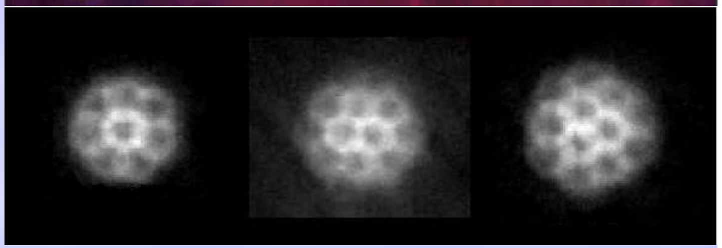
H. Landa, S. Markovich, B. Reznik

G. Morigi and G. De Chiara

Cosmology in the lab







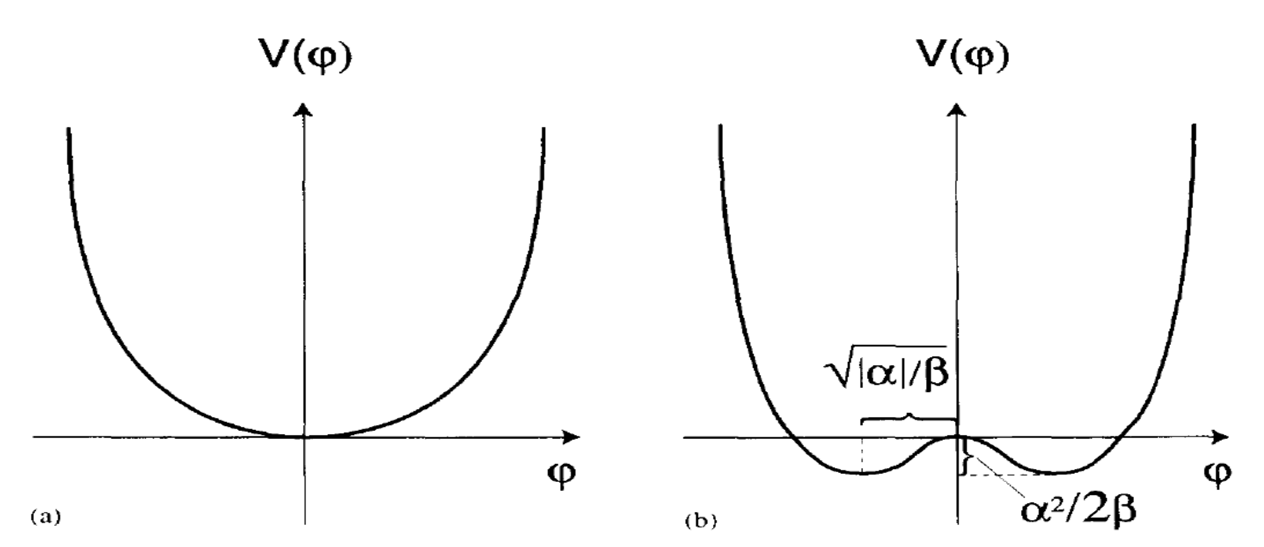
- *Cosmology : symmetry breaking during expansion and cooling of the early universe*
- *Condensed matter:*
  - *Vortices in Helium*
  - *Liquid crystals*
  - *Superconductors*
  - *Superfluids*

Similar free-energy landscape near a critical point

T. W. B. Kibble, JPA 9, 1387 (1976); Phys. Rep. 67, 183 (1980)
W. H. Zurek, Nature (London) 317, 505 (1985); Acta Phys. Pol. B. 1301 (1993)

Second order phase transition

Landau theory: Free energy landscape, changes across a 2PT from single to double well potential, parametrized by a relative temperature



$$V(\varphi) = \alpha(T - T_c)|\varphi|^2 + \frac{1}{2}\beta|\varphi|^4$$

Second order phase transition

Universal behaviour of the order parameters: divergence of correlation/healing length dynamical relaxation time

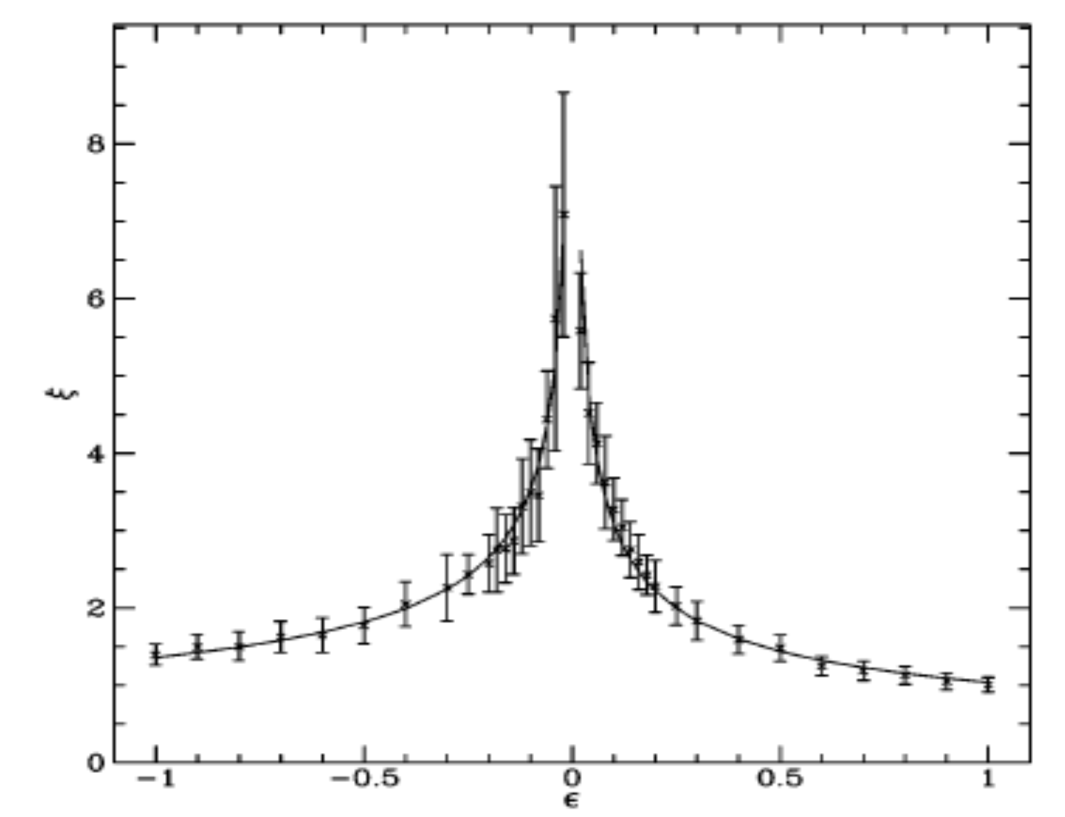
$$\varepsilon = \frac{T_C - T}{T_C}$$

Correlation length

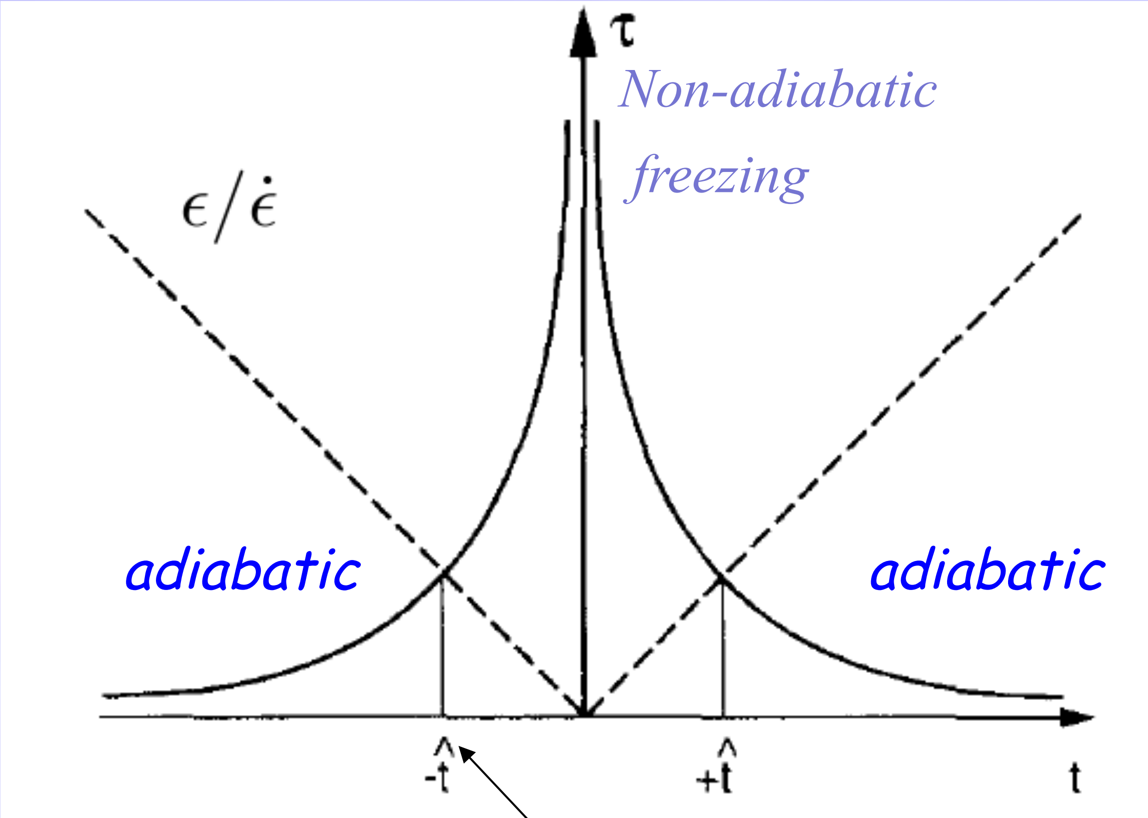
$$\xi = \frac{\xi_0}{|\varepsilon|^{\nu}}$$

Response time

$$\tau = \frac{\tau_0}{|\varepsilon|^{\mu}}$$



The Kibble Zurek mechanism



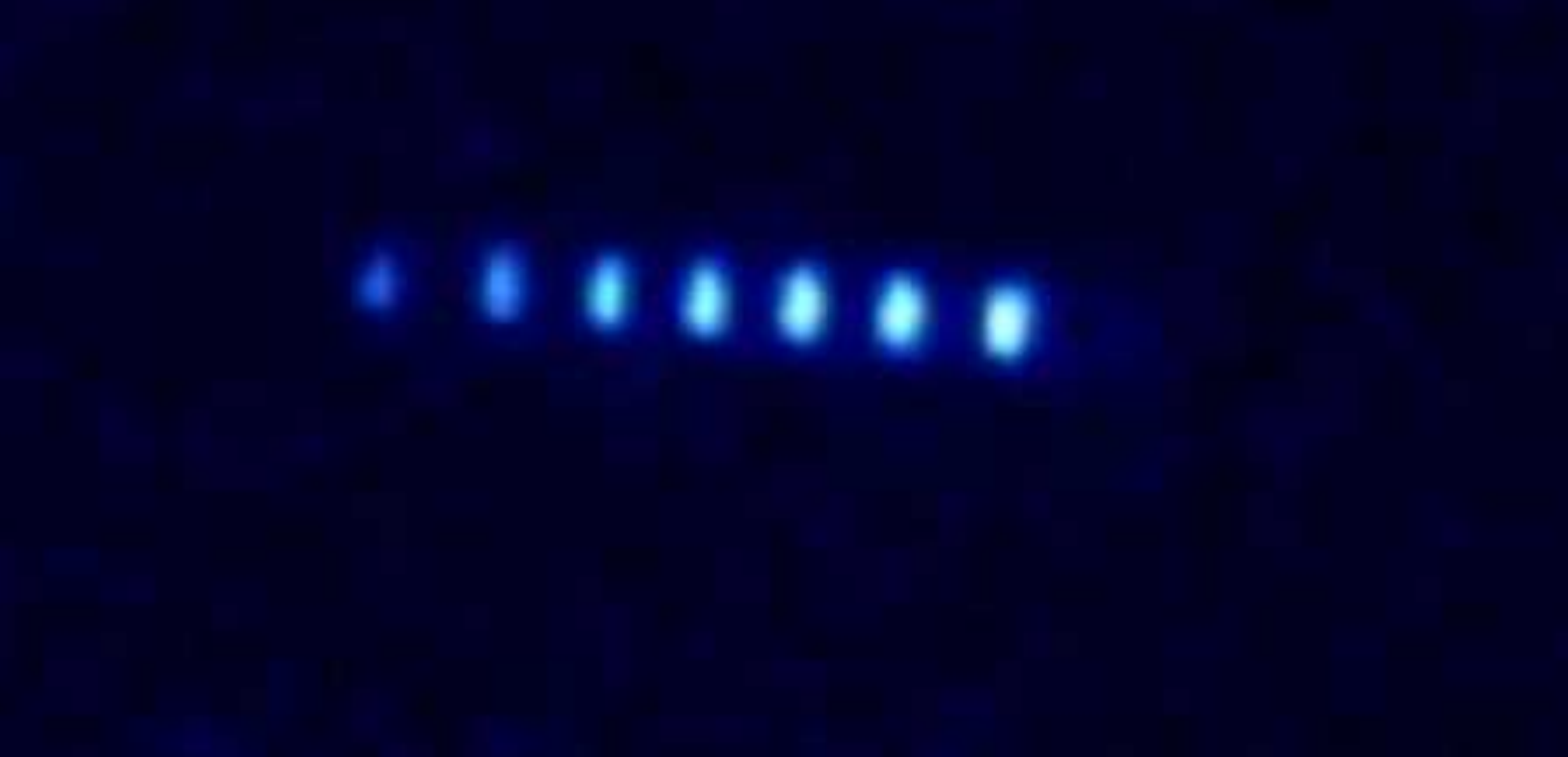
Linear quench

$$\varepsilon = \frac{t}{\tau_Q}$$

$$\tau = \frac{\tau_0}{|\varepsilon|^{\mu}}$$

The average size of a domain is given by the correlation length at the freeze out time

A test-bed: ion chains



Structural phases in ion traps

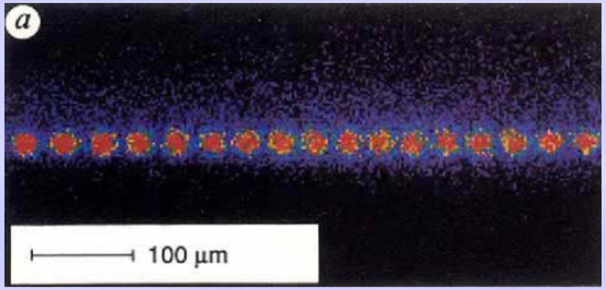
N ions on a ring with a harmonic transverse confinement

$$H = \frac{1}{2}m\sum_n \dot{\boldsymbol{r}}_n^2 + \frac{1}{2}m\sum_n(\nu_t^2 z_n^2) + \frac{Q^2}{2}\sum_{n\neq n'}\frac{1}{|\boldsymbol{r}_n - \boldsymbol{r}'_n|}$$

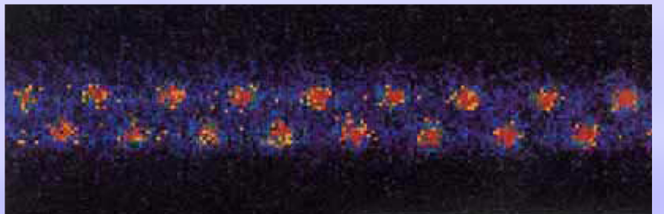
$$\upsilon_t^{(c)} = 3N\nu/4\sqrt{\log N}$$

$$\upsilon_t^{(c)2} = 4\frac{Q^2}{ma(0)^3}$$

Birkl et al. Nature 357, 310(1992)



Zig



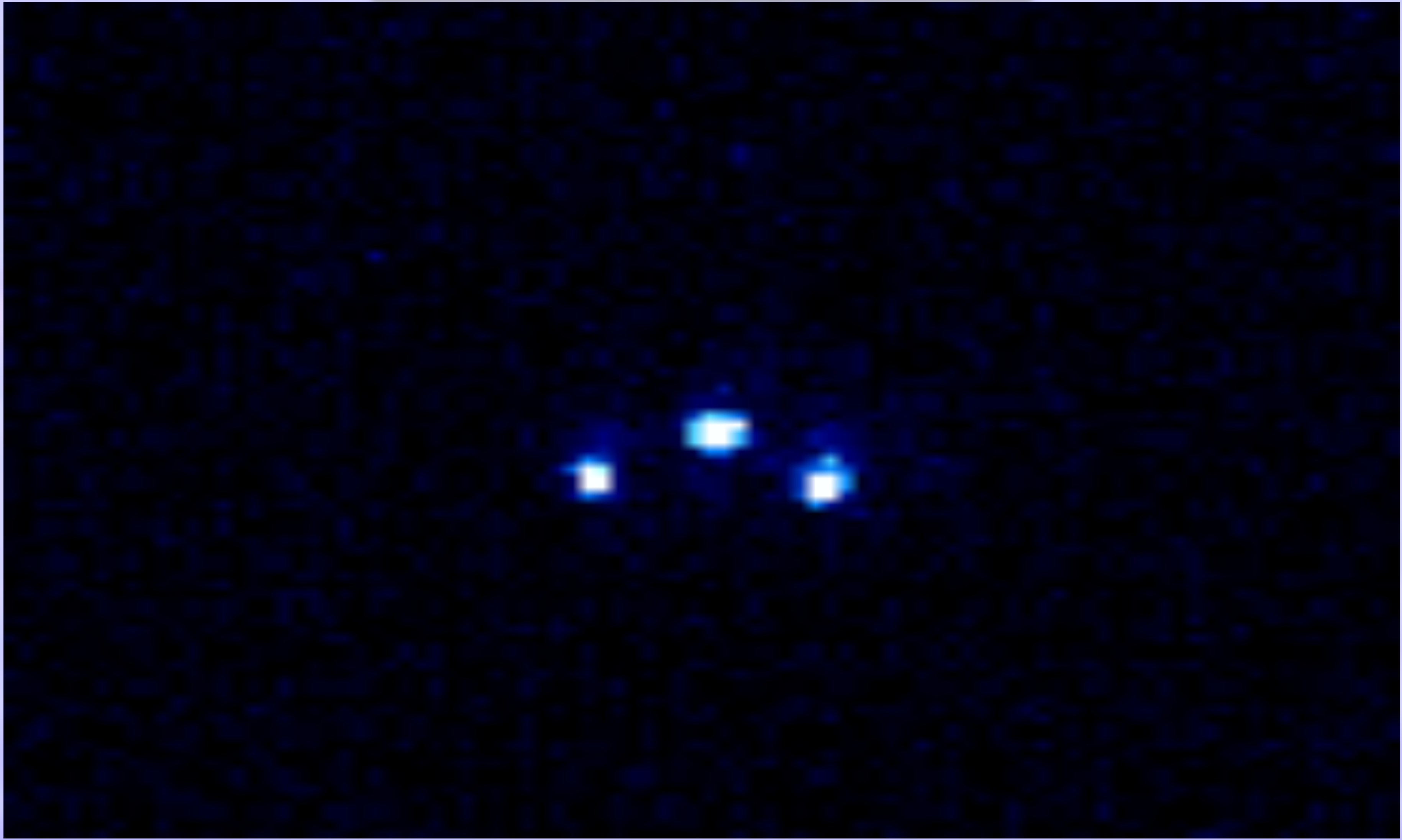
Zag

Fishman et al. PRB 77, 064111 (2008)

Morigi et al, PRL 93, 170602 (2004); Phys. Rev. E 70, 066141 (2004).

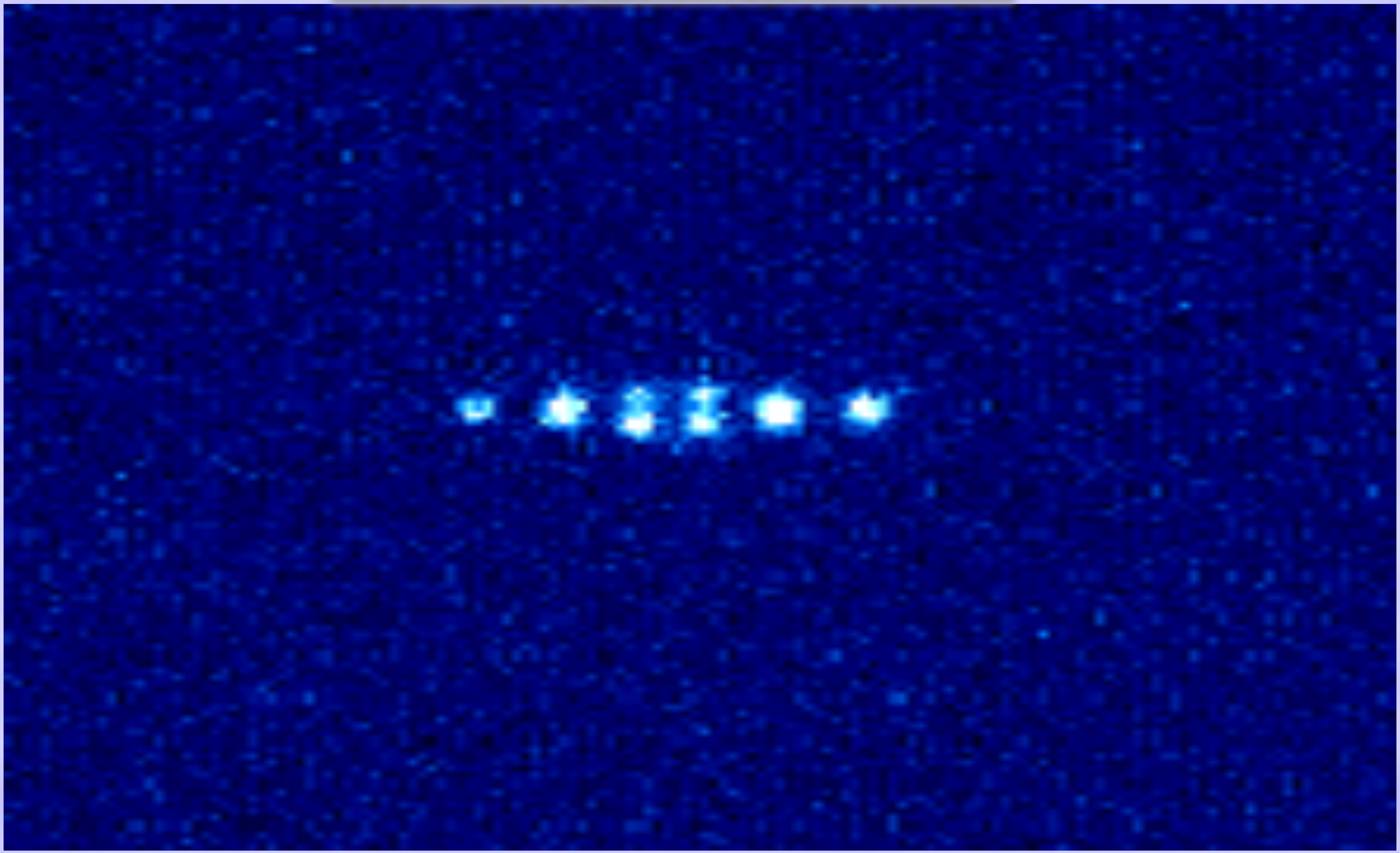
Zig Zag movie

Wolfgang Lange, Sussex
Dan Crick

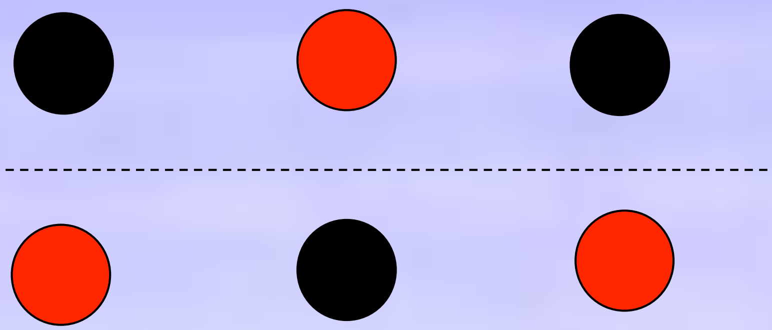


Zig Zag movie

Wolfgang Lange, Sussex
Dan Crick



Double Well



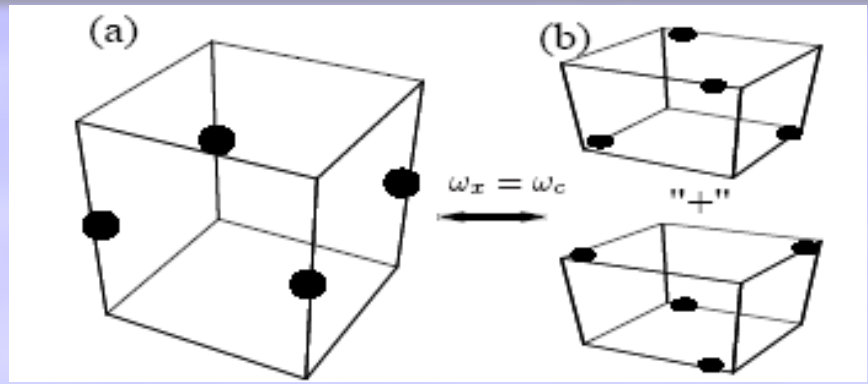


TABLE I: Double well parameters

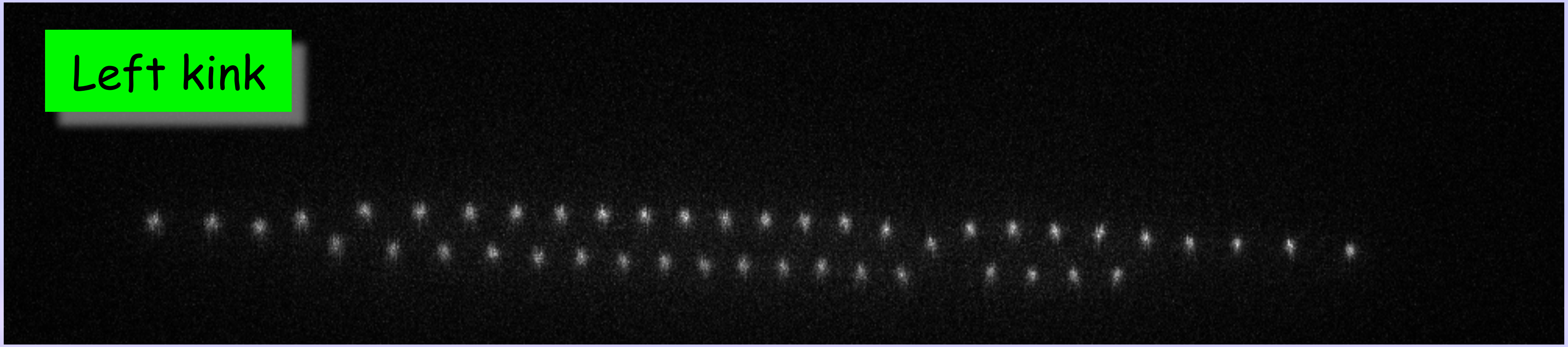
	3-ZZ	4-ZZ	5-ZZ	6-ZZ	7-ZZ	4-2D
Minimal Gap$[\frac{\nu_z}{10^3}]$	76	98	117	136	155	37
Local freq$[\frac{\nu_z}{10^3}]$	111	143	172	200	226	54
Tunneling rate$[\frac{\nu_z}{10^3}]$	26	34	41	48	55	13
Distance(nm)	594	341	339	282	270	364
Radial freq$(\omega - \omega_c)[\frac{\nu_z}{10^3}]$	2	2.4	3	3.4	3.6	0.8

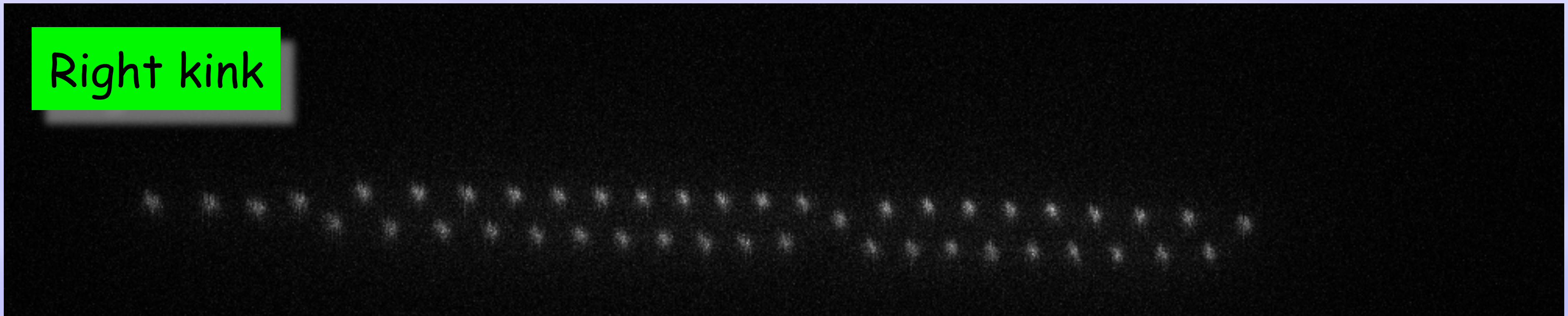
The formation parameters of the double well. The distances are calculated for $\nu_z = 1MHz$ and scale as $1/\nu^{2/3}$.

AR, R. C. Thompson, D. M. Segal, and M. B. Plenio
PRL 101, 260504 (2008)

Kinks

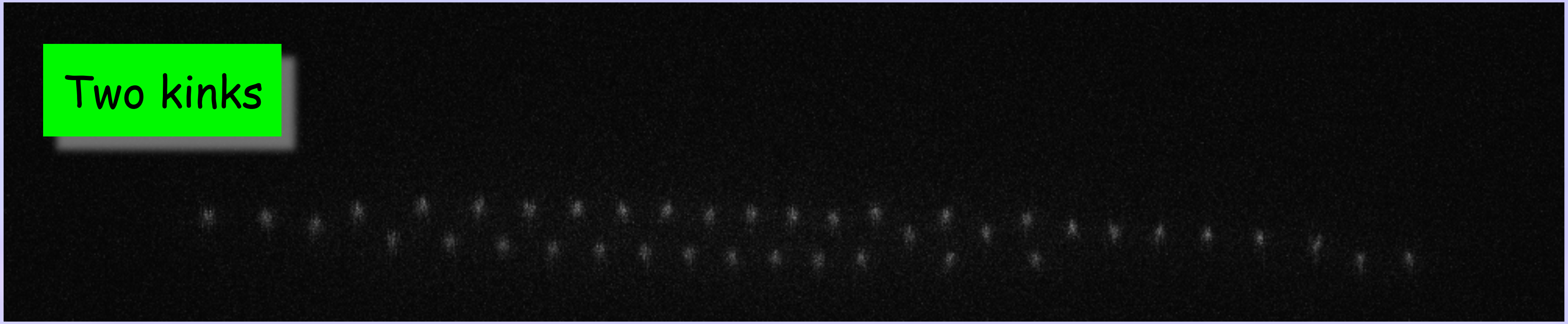
T. Schaetz MPQ
Guenther Leschhorn and Steffen Kahra

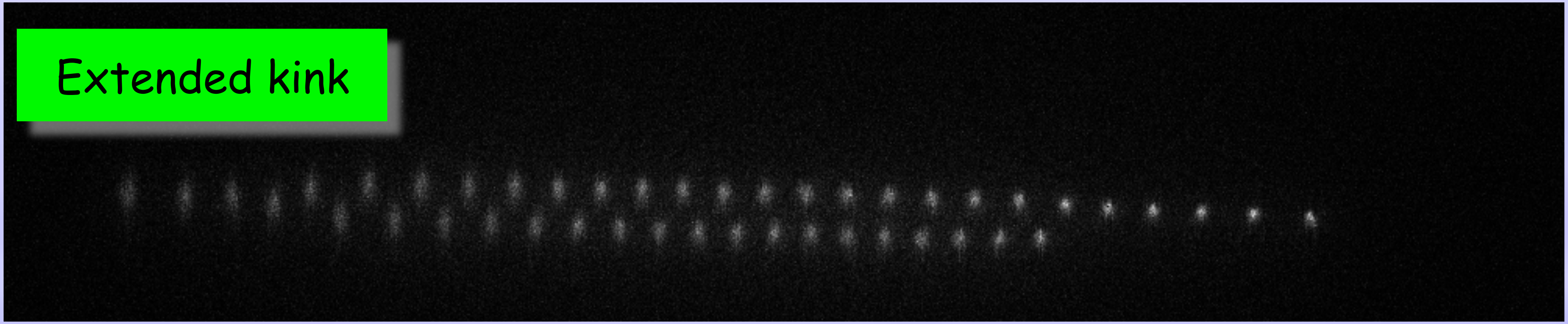




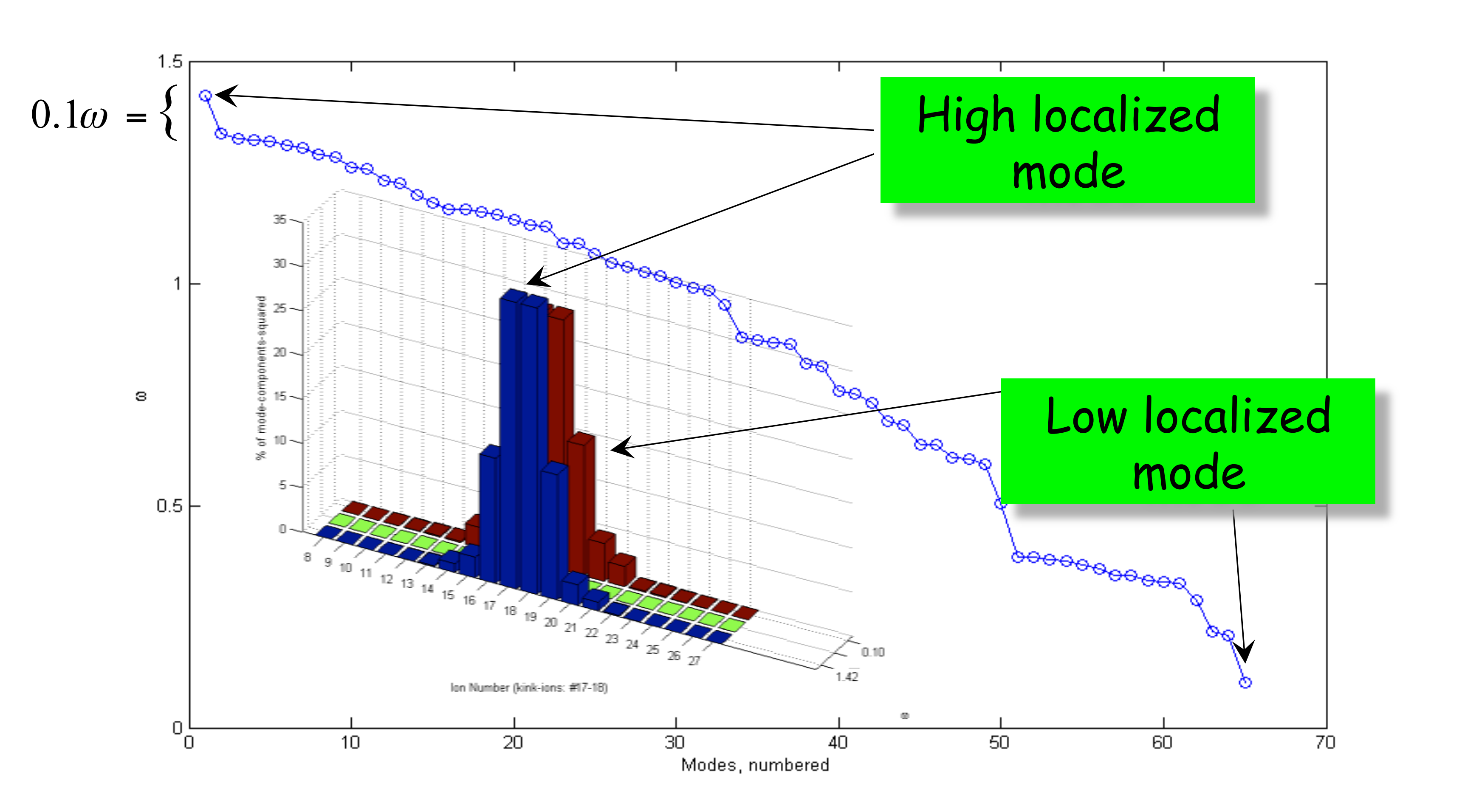
Kinks

T. Schaetz MPQ
Guenther Leschhorn and Steffen Kahra





Spectrum structure



Dispersion relation for the ion trap and profile of the two most localized states.

Landa etal, Phys. Rev. Lett. **104, 043004 (2010)**

KZM in the Ginzburg-Landau potential

Effective Ginzburg-Landau potential for a small amplitude of transverse osc.

$$V = V^{(2)} + V^{(4)}$$

Continuous description of the field for the transverse modes

$$(-1)^n \sigma_n \rightarrow \psi^\sigma(x)$$

$$x_n \rightarrow x$$

Long wavelength expansion wrt the zig-zag mode

$$L(x) = \frac{1}{2}\frac{m}{a}\sum_\sigma \left[\left(\partial_t \psi(x)\right)^2 - v_s^2 \left(\partial_x \psi(x)\right)^2 - \delta \psi(x)^2 - A\psi(x)^4 \right]$$

Control parameter

$$\delta = \upsilon_t^2 - \upsilon_t^{(c)2}$$

Linear $\delta > 0$

Zig Zag $\delta < 0$

Thermodynamic limit

Eq. of motion of the slow modes in the presence of Doppler cooling

$$\partial_t^2\psi - v_s^2\partial_x^2\psi + \eta\partial_t\psi + \delta\psi + 2\mathrm{A}\psi^3 = \varepsilon(t)$$

$$\langle\varepsilon(t)\rangle = 0, \langle\varepsilon(t)\varepsilon(t')\rangle = 2\eta k_B T\delta(t-t')$$

Get the input of the Kibble-Zurek theory

Estimate the correlation length and relaxation time

Relaxation time:

$$\tau_o \propto \eta/\delta$$

$$\tau_u \propto 1/\sqrt{\delta}$$

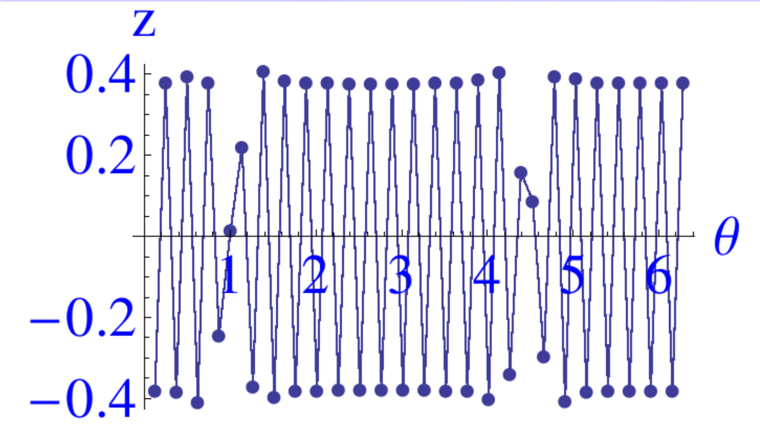
Correlation length:

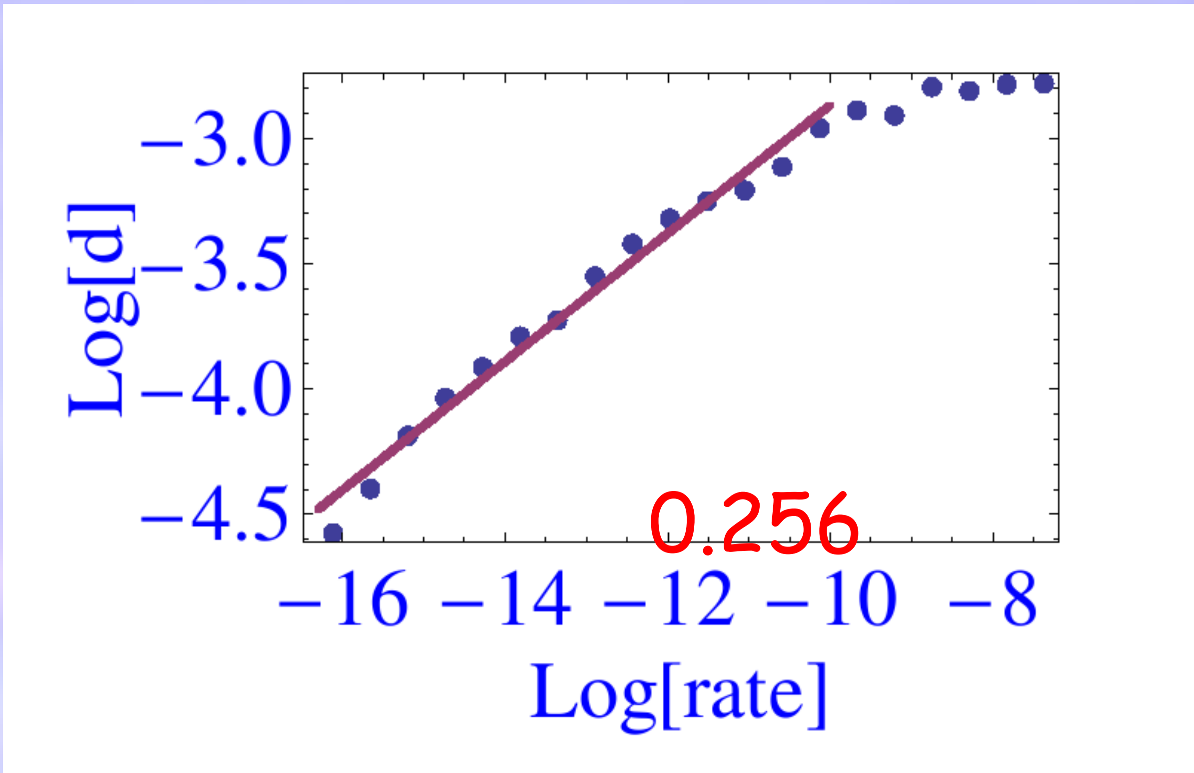
$$\xi \propto v_s/\delta^{1/2}$$

Morigi, Eschner PRA 64

On the ring

Homogeneous system:





$$d_0 = \frac{1}{v_s}\left(\frac{\delta_0 \eta}{\tau_Q}\right)^{1/4}$$

Speed of sound

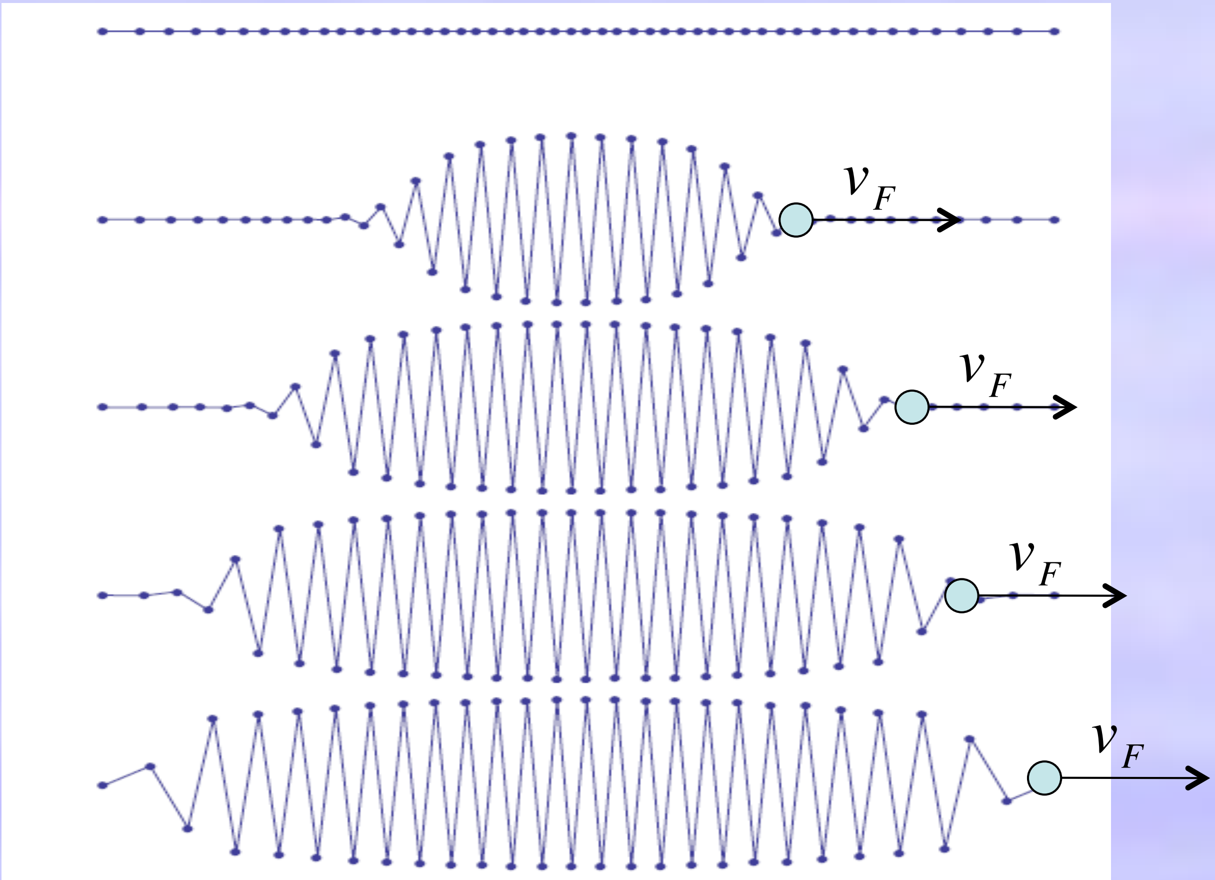
Damping rate

Quench strentgh

Inhomogeneous KZM

Axial and transverse harmonic potential (instead of a ring trap):

$$H = \frac{1}{2}m\sum_n \dot{\boldsymbol{r}}_n^2 + \frac{1}{2}m\sum_n(\nu_t^2 z_n^2 + \nu^2 x_n^2) + \frac{Q^2}{2}\sum_{n\neq n'}\frac{1}{|\boldsymbol{r}_n - \boldsymbol{r}'_n|}$$

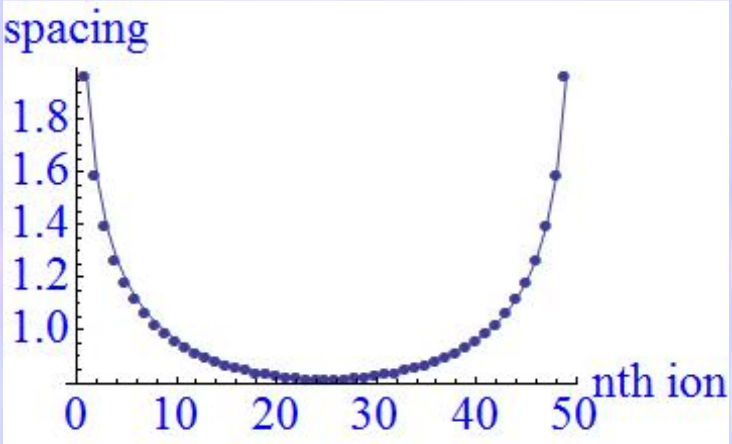


Inhomogeneous KZM

$$H = \frac{1}{2}m\sum_n \dot{\boldsymbol{r}}_n^2 + \frac{1}{2}m\sum_n(\nu_t^2 z_n^2 + \nu^2 x_n^2) + \frac{Q^2}{2}\sum_{n\neq n'}\frac{1}{|\boldsymbol{r}_n - \boldsymbol{r}'_n|}$$

Local Density Approximation (Dubin PRE 55)

$$n(x) = \frac{3}{4}\frac{N}{L}\left(1 - \frac{x^2}{L^2}\right)$$



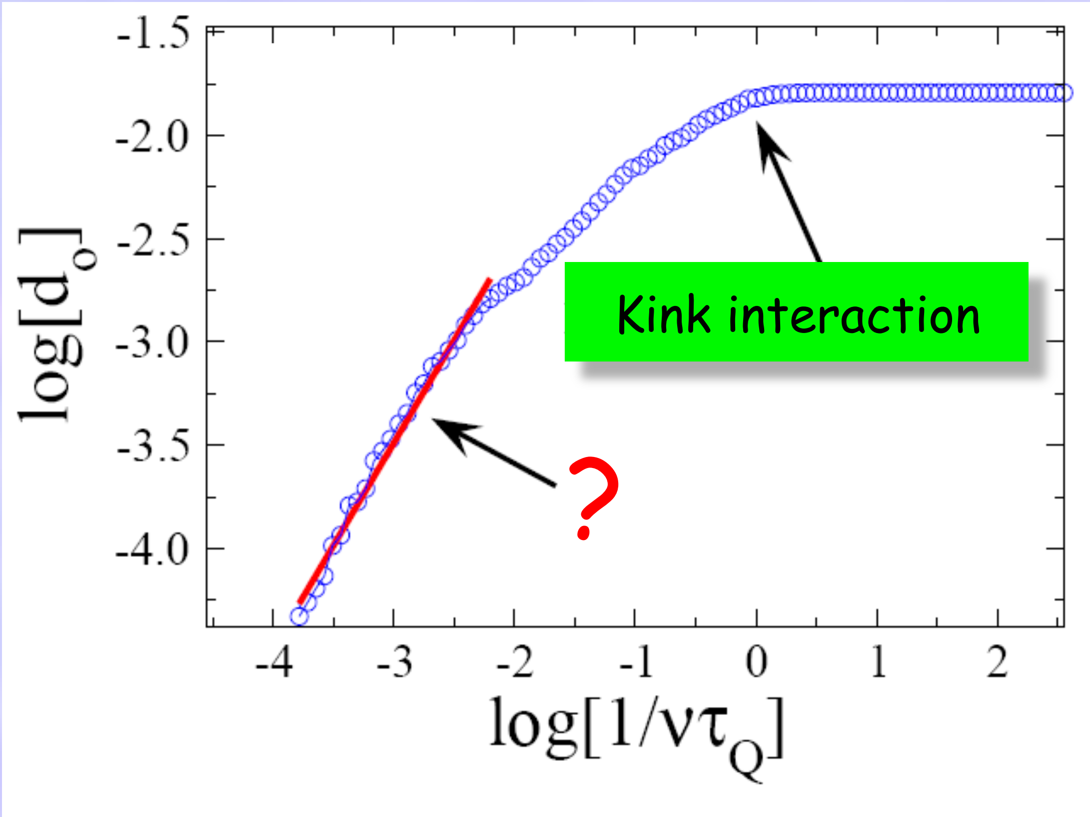
Spatially dependant critical frequency (within LDA)

$$\upsilon_t^{(c)2} = 4\frac{Q^2}{ma(0)^3} \quad \Rightarrow \quad \upsilon_t^{(c)2} = 4\frac{Q^2}{ma(x)^3}$$

Inhomogeneous transition!

On the trap KZM scaling

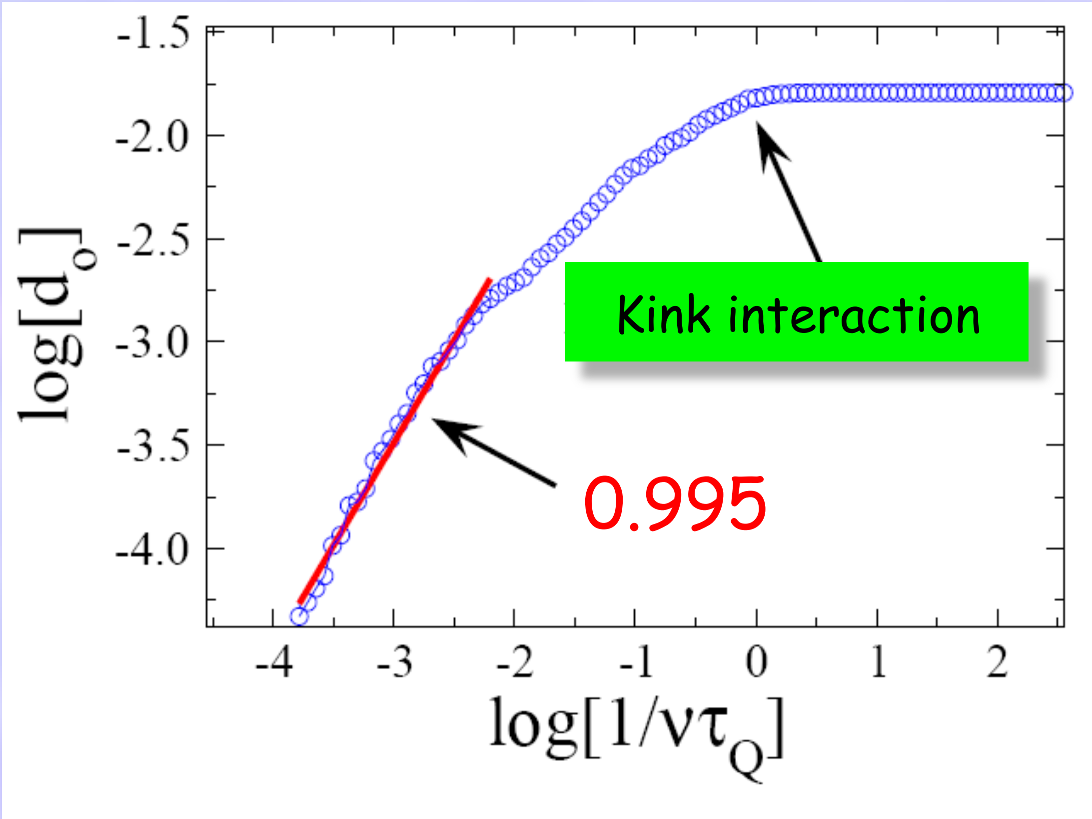
Density of defects as a function of the rate



50 ions

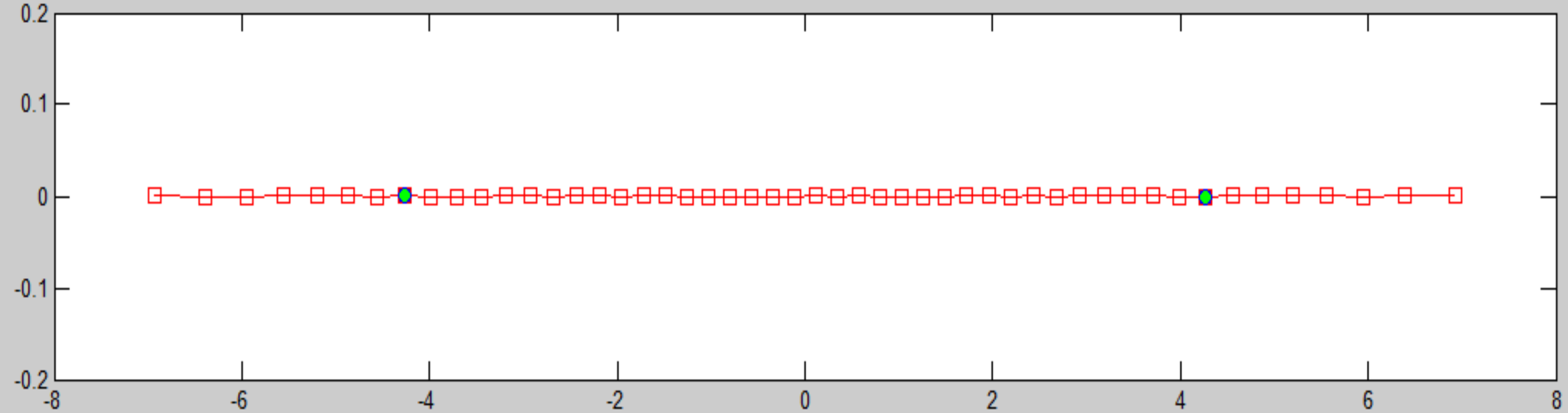
On the trap KZM scaling

Density of defects as a function of the rate

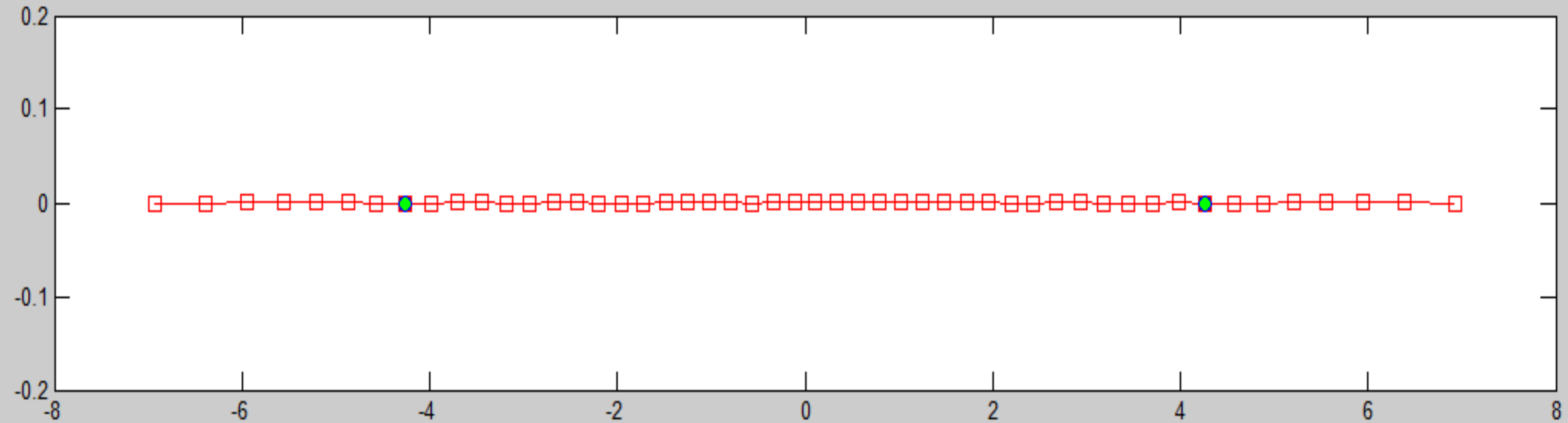


On the trap: dynamics

Slow quench: adiabatic limit



Fast quench: nucleation of kinks



Inhomogeneous KZM

Inhomogeneous density, spatially dependent critical frequency.

Linear quench: $\delta(x,t) = v_t(t)^2 - v_t^c(x)^2 = v_t(0)^2 - v_t^c(x)^2 - \delta_0 \frac{t}{\tau_Q}$

Causality restricts the effective size of the chain

Front satisfying: $\delta(x_F, t_F) = 0$

Versus sound velocity: $v_x = \xi_x / \tau_x$

Adiabatic dynamics is possible even in the thermodynamic limit when

$$v_F < v_x$$

In contrast with the homogeneous KZM

Solitons: Zurek, PRL 102 (2009)

Inhomogeneous KZM

Defects appear if: $v_F > v_x$

Effective size of the chain for KZM: $2|X_*|$

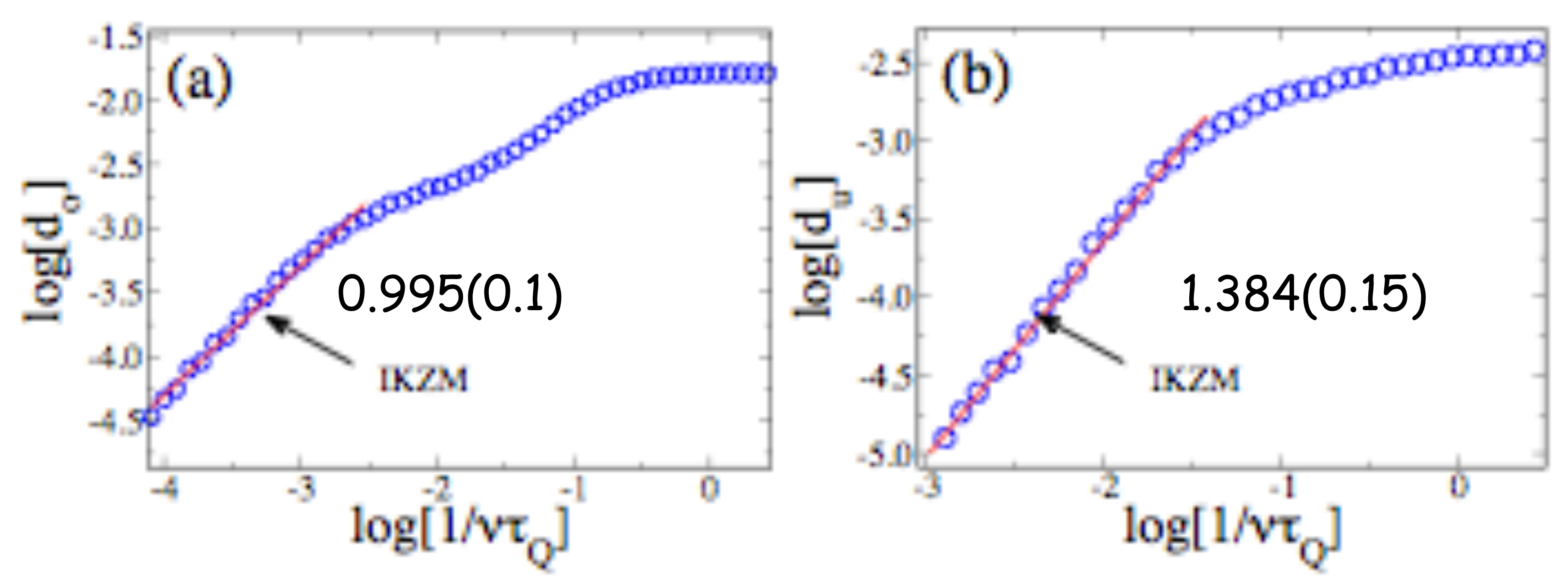
$$X_* = (v_F - v_x)t$$

NEW SCALING LAW!

$$d_0 \approx \frac{2|X_*|}{\xi} = \frac{L}{3v_t^{(c)}(0)^2 v_s^2} \frac{\eta\delta_0}{\tau_Q}$$

$$d_u \approx \frac{2|X_*|}{\xi} = \frac{L}{3v_t^{(c)}(0)^2 v_s^2} \left(\frac{\delta_0}{\tau_Q}\right)^{4/3}$$

Theory vs. "experiment"



Nice agreement

Outlook

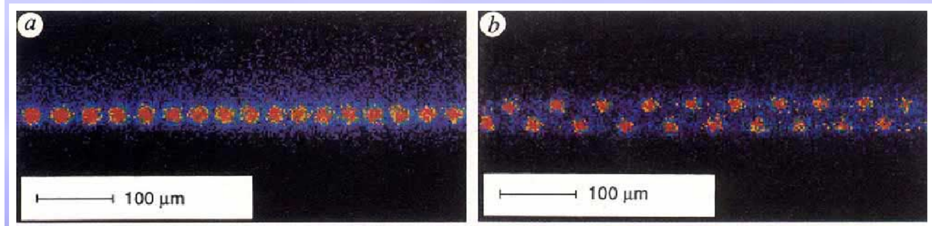
3D case

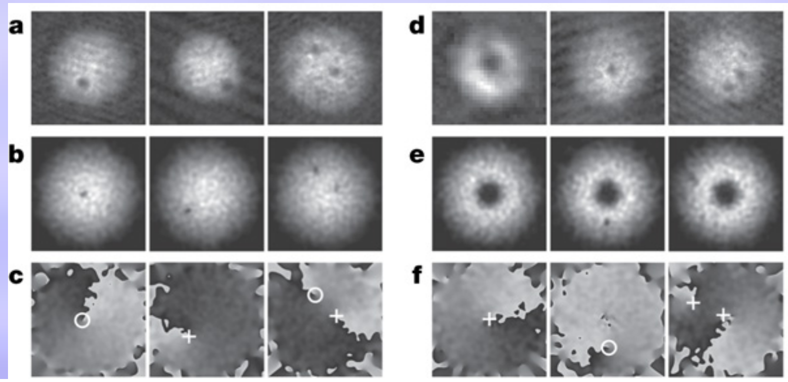
Kinks dynamics

With optical lattice:
transverse Kontorova model

Full counting statistics

Other systems





PhD position available

Thanks