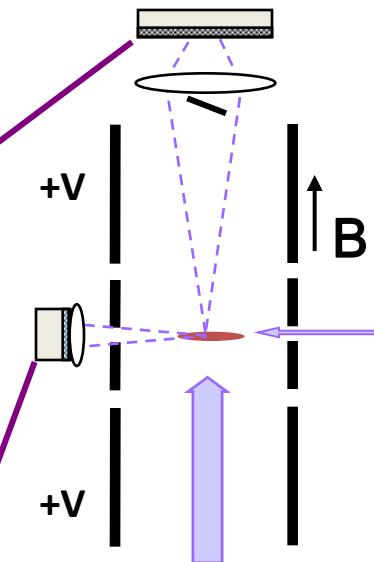
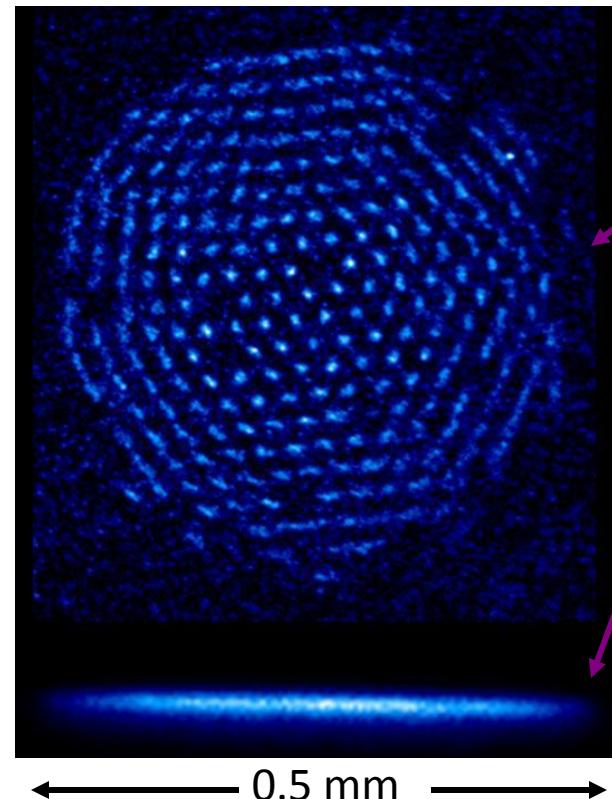


Quantum Information Experiments with Ion Crystals in Penning Traps

John Bollinger
NIST-Boulder
Ion storage group

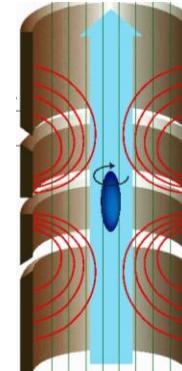
Michael Biercuk, Hermann Uys, Joseph Britton, Wayne Itano, and Nobuyasu Shiga

Long term collaborators: David Wineland,
Jim Bergquist, Dietrich Leibfried, Till
Rosenband, Joseph Tan, Pei Huang, Brana
Jelenkovic, Travis Mitchell, Brad King, Jason
Kriesel, Marie Jensen, Taro Hasegawa
Dan Dubin – UCSD(theory)



Outline:

- Penning traps
- Monday: Quint, Wada
Tuesday: Charlton
Friday: Schweikhard



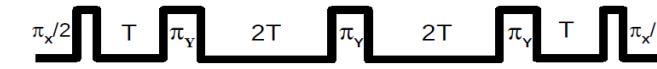
- High magnetic field qubit

$2s\ ^2S_{1/2}$

124 GHz

- Qubit coherence and dynamical decoupling techniques

CPMG Total Time: $2n\tau$

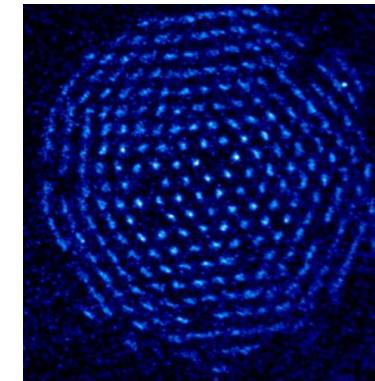


UDD



- Current effort: quantum simulation

$$H = -B_x \sum_i \sigma_i^x + \chi J_z^2$$

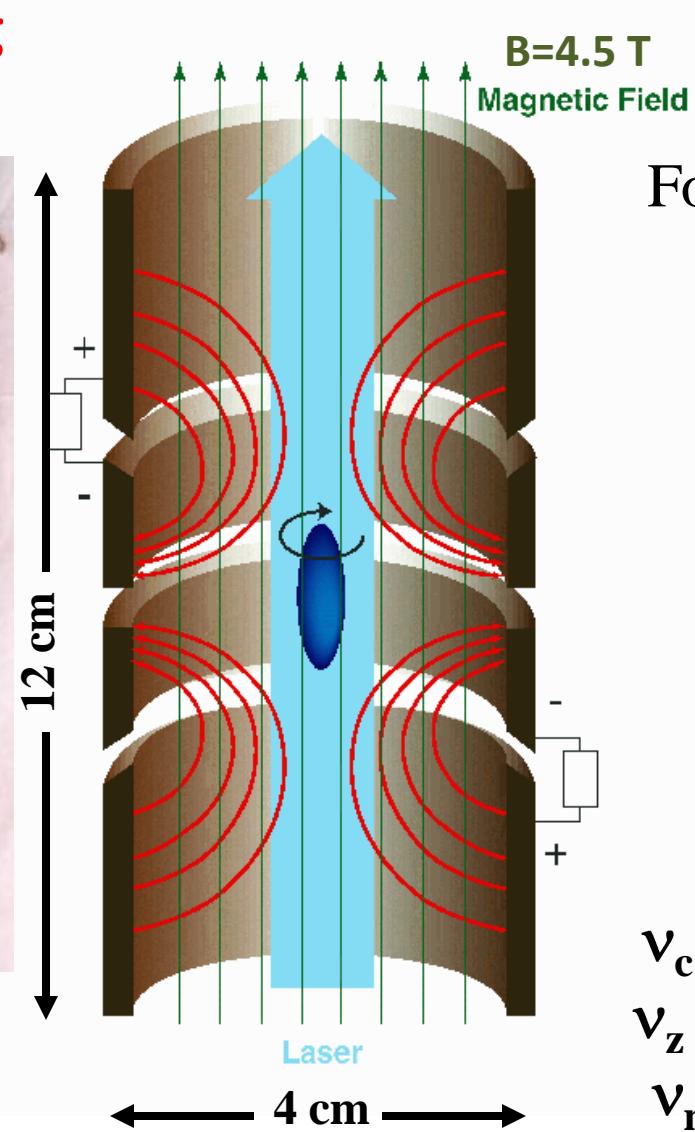


- Decoherence due to elastic Rayleigh scattering

$$\Gamma_{Rayleigh} = \Omega_R^2 \gamma \left(\sum_J a_{d \rightarrow d}^J - \sum_{J'} a_{u \rightarrow u}^{J'} \right)^2$$

Penning trap

NIST Penning trap



For $r, z \ll$ trap dimensions ,

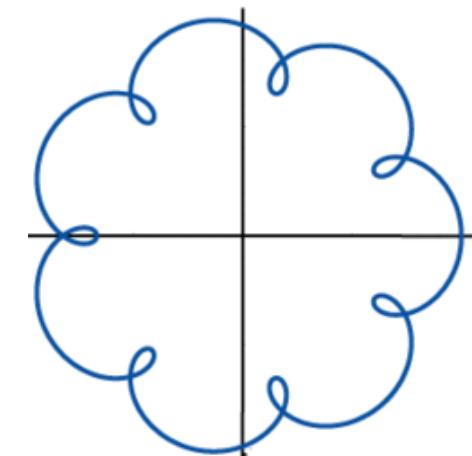
$$\varphi_{trap}(r, z) \approx \frac{1}{2} m \omega_z^2 \left(z^2 - \frac{r^2}{2} \right)$$

$$z(t) = z_o \sin(2\pi\nu_z t + \varphi_z)$$

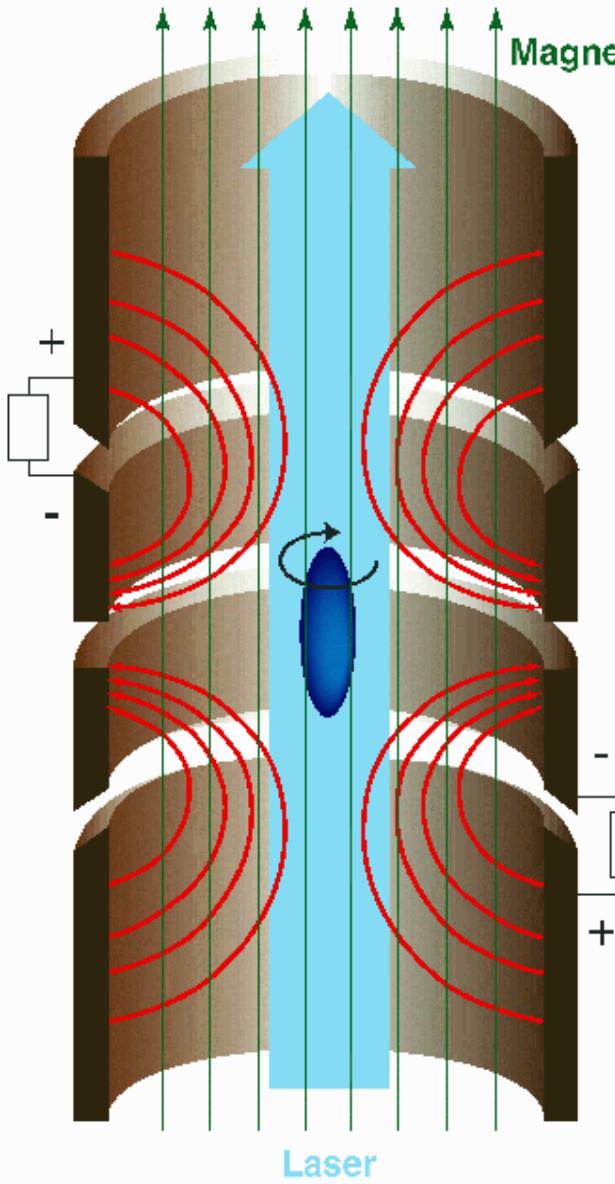
$$r(t) = r_c \sin(\pi(\nu_c - \nu_m)t + \phi_c)$$

$$r_m \sin(2\pi\nu_m t + \phi_m)$$

${}^9\text{Be}^+$
 $\nu_c \sim 7.6 \text{ MHz}$
 $\nu_z \sim 800 \text{ kHz}$,
 $\nu_m \sim 40 \text{ kHz}$



Penning trap: many particle confinement



axial confinement \leftrightarrow
conservation of energy

radial confinement \leftrightarrow
conservation of angular momentum

$$P_\theta = \sum_j \left(mv_{\theta j} + \frac{q}{c} A_\theta(r_j) \right) r_j$$

$$\approx \frac{qB}{2c} \sum_j r_j^2 \text{ for } A_\theta(r) = \frac{Br}{2} \text{ and } B \text{ large}$$

O'Neil, Dubin, UCSD

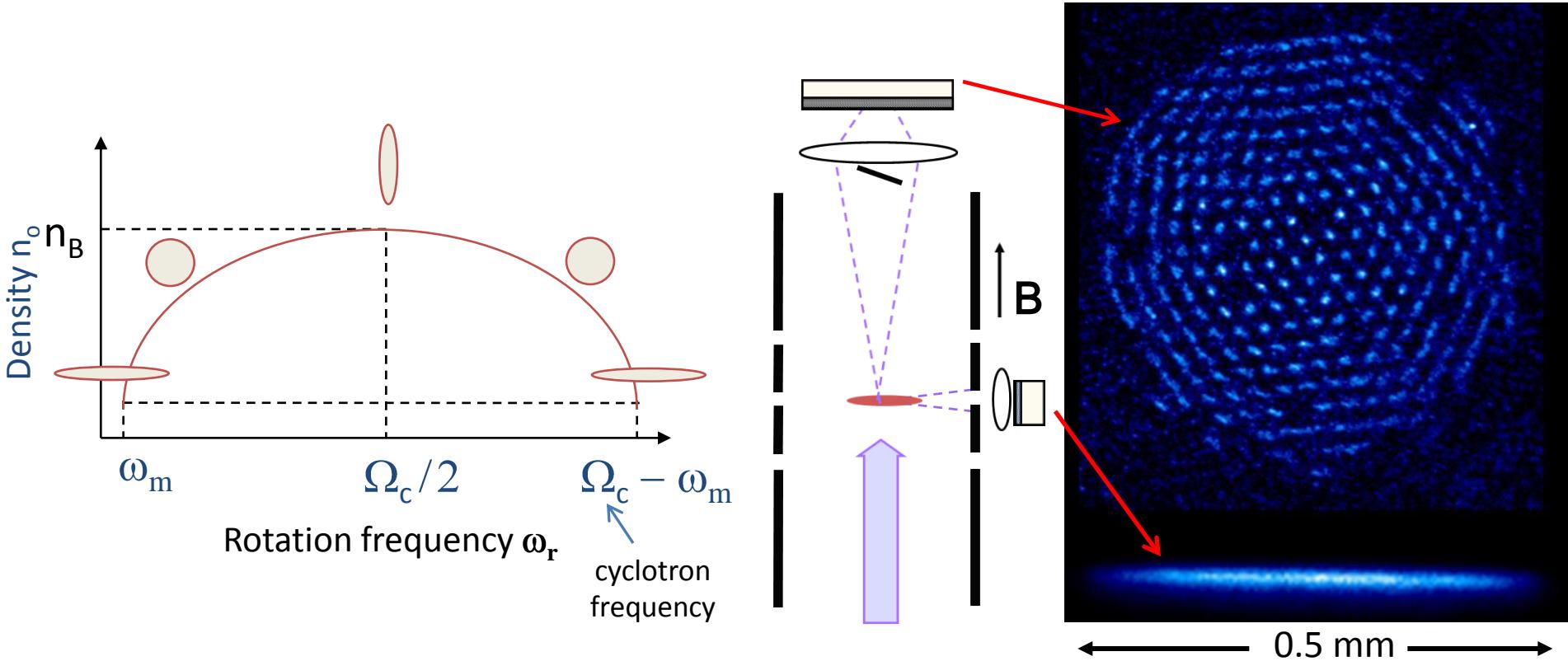
axial symmetry important – trap and B field aligned to 0.01°

radial confinement due to rotation –
ion plasma rotates $v_\theta = \omega_r r$ due to $E \times B$ fields

in rotating frame, $\omega_r r \hat{\theta} \times \hat{B} \hat{z}$ Lorentz force is directed radially inward

precisely control ω_r (and the radial binding force)
with a rotating electric field (rotating wall)

Planar ion arrays in Penning traps

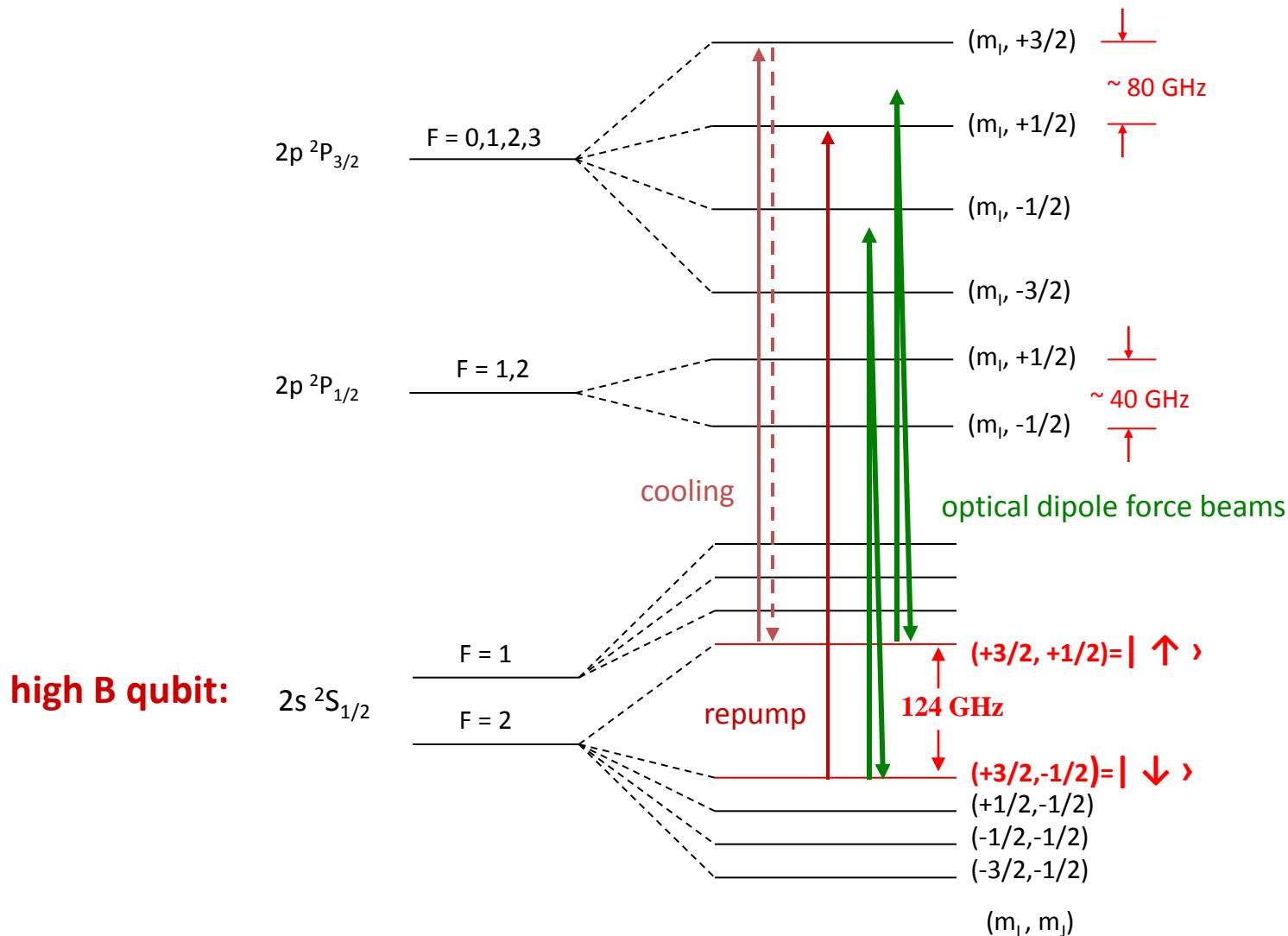


Features of Penning traps for quantum information/simulation experiments:

- static trapping fields enable large traps to be used; ions are far from electrode surfaces \Rightarrow low heating rates
- ion crystals form naturally from minimization of the Coulomb potential energy
- for a single plane, the minimum energy lattice is triangular \Rightarrow good for magnetically frustrated simulations
- ion crystals rotate ($\sim 50 \text{ kHz}$) but rotation precisely controlled \Rightarrow individual particle detection still possible

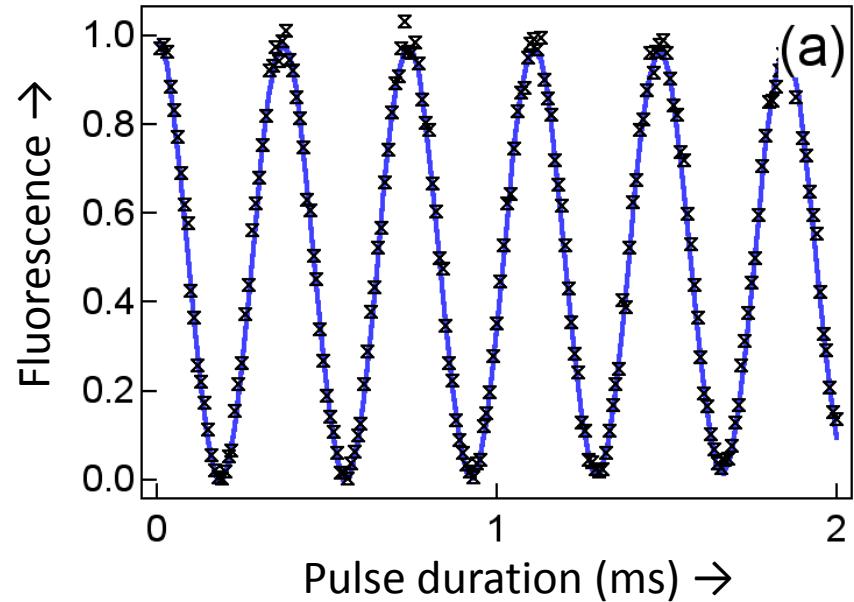
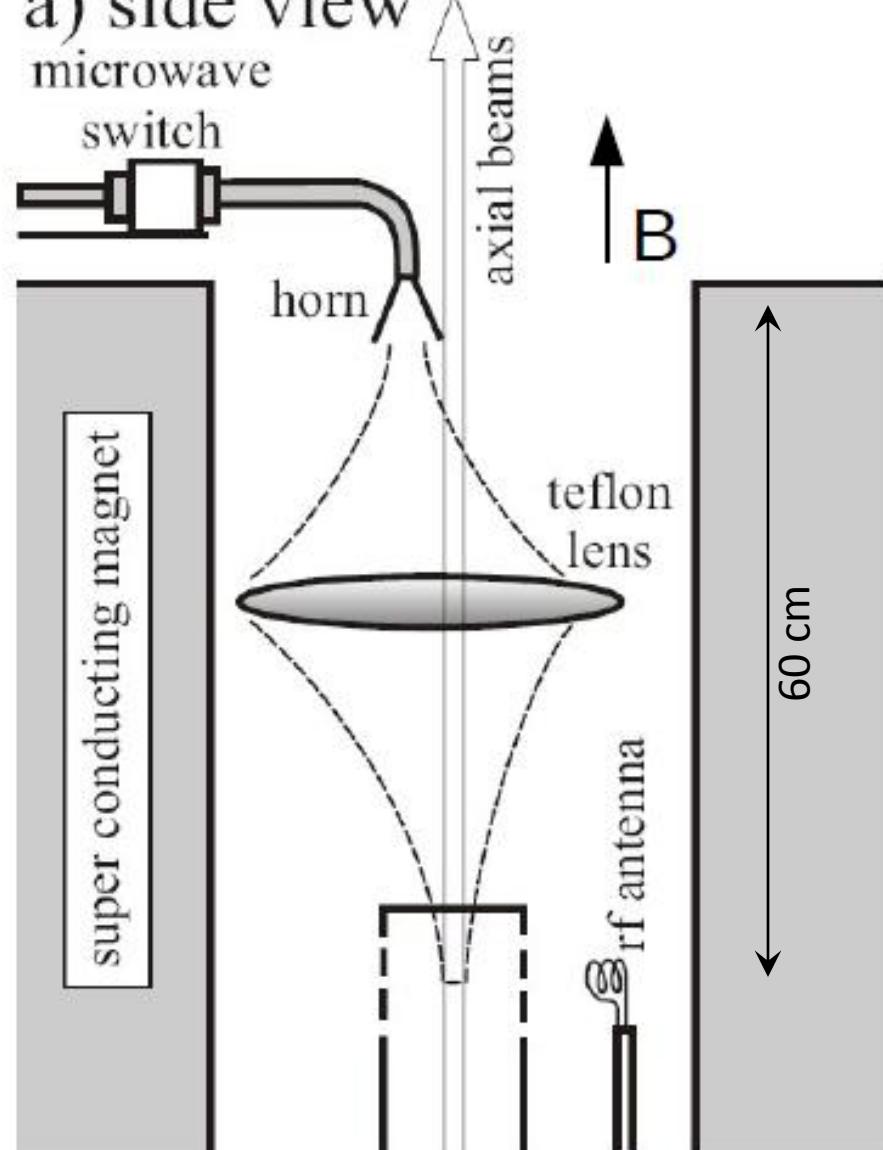
High magnetic field qubit

${}^9\text{Be}^+$, $B \sim 4.5 \text{ T}$, $\omega_0 / 2\pi \sim 124.1 \text{ GHz}$



Rabi flopping on 124 GHz electron spin flip

a) side view

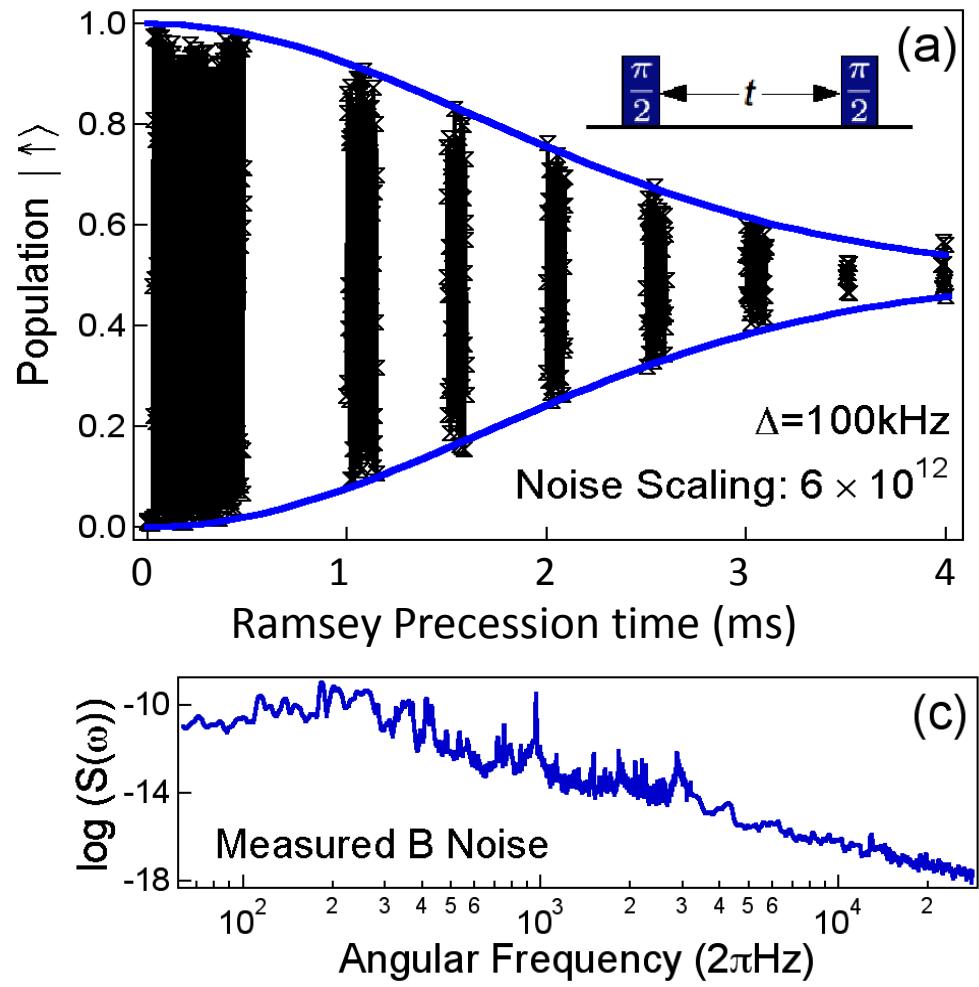
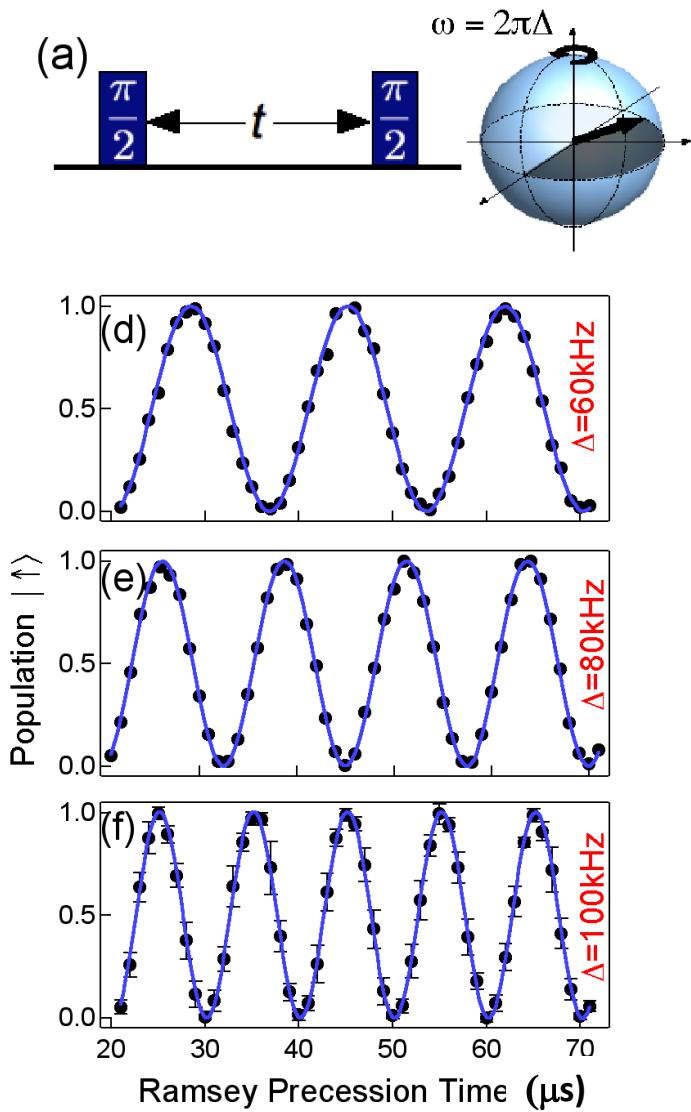


random benchmarking experiment:

Biercuk, *et al.*, Quantum Information and Control 9, 920 (2009).

π –pulse fidelity > 99.9%

Ramsey (T_2) coherence on 124 GHz electron spin flip



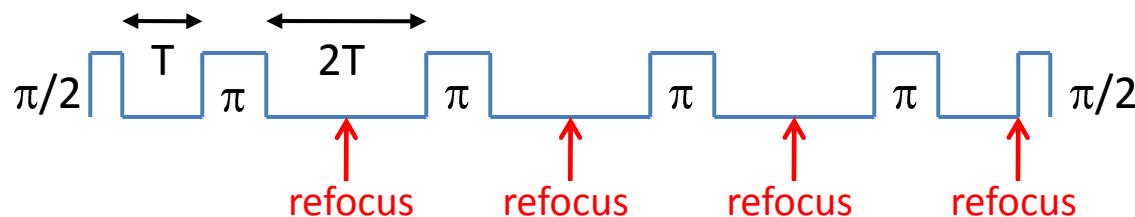
$$\frac{\delta B}{B} \sim 10^{-9}, \quad T_2 \sim 2\text{ ms}$$

coherence can be extended to 10's of ms with spin echo (dynamical decoupling)

Biercuk, Uys, VanDevender, Shiga, Itano, Bollinger, Nature 458, 996 (2009)

Dynamical decoupling sequences

- Carr-Purcell-Meiboom-Gill (CPMG) sequence – evenly spaced π -pulses

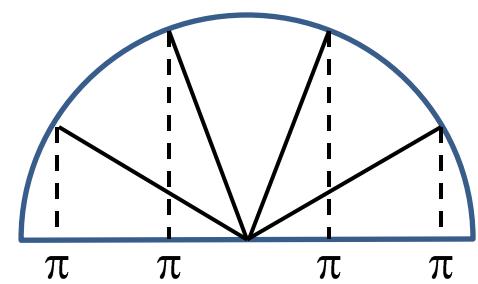


usually only need refocusing at sequence end \Rightarrow π -pulses need not be evenly spaced

- Uhrig sequence (UDD) - Uhrig, PRL 98, 100504 (2007)



UDD construction



$n \pi$ -pulse UDD sequence cancels lowest $n-1$ derivatives of the noise

$$B(t) = B(0) + B'(0)t + (1/2)B''(0)t^2 + (1/6)B'''(0)t^3 + \dots$$

- Theoretical determination of coherence: $F(\omega\tau)$ = noise filtration function of sequence

Uhrig

Cywinski, Das Sarma

Martinis

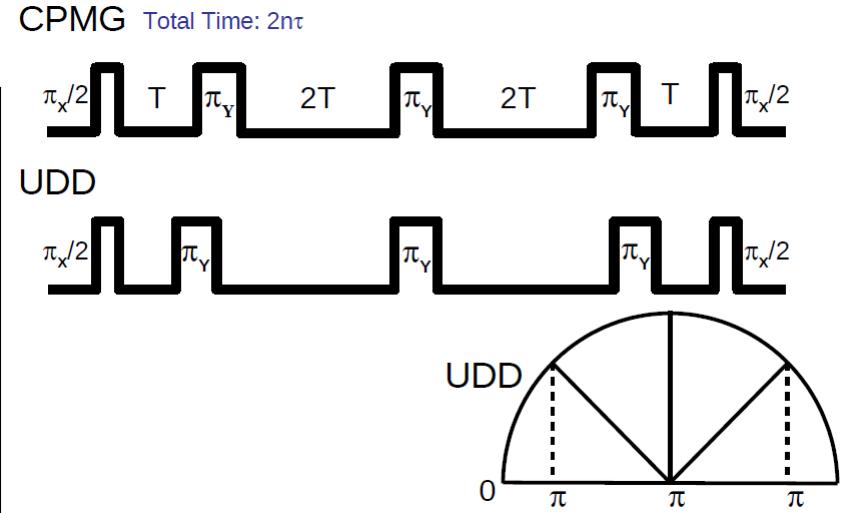
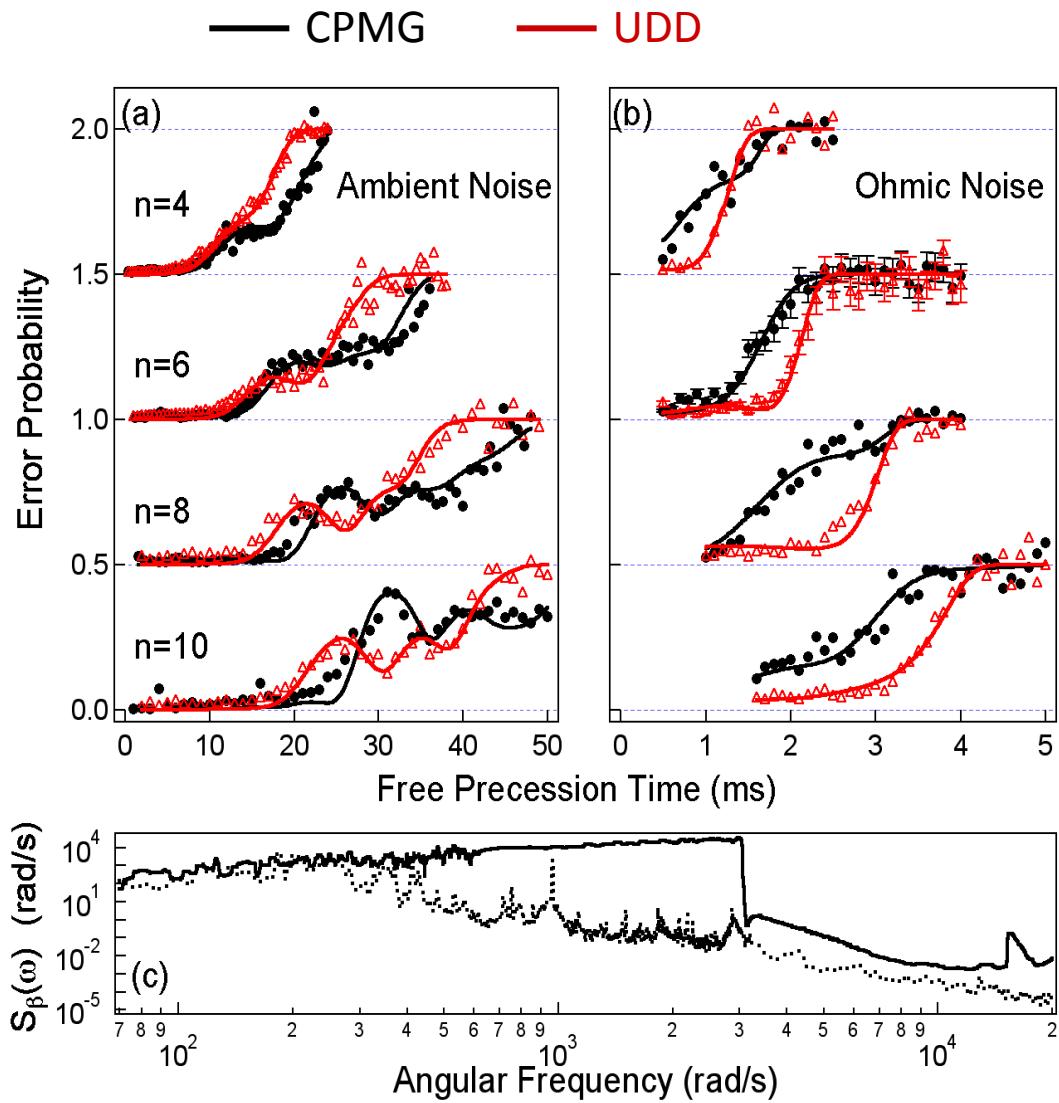
$S(\omega)$ = noise power spectrum

$$\text{coherence} = e^{-\chi(\tau)}, \quad \chi(\tau) = \frac{2}{\pi} \int_0^\infty \frac{S(\omega)}{\omega^2} F(\omega\tau) d\omega$$

Qubit coherence extended and accurately predicted by dynamical decoupling

Biercuk, et al., Nature 458, 996 (2009)

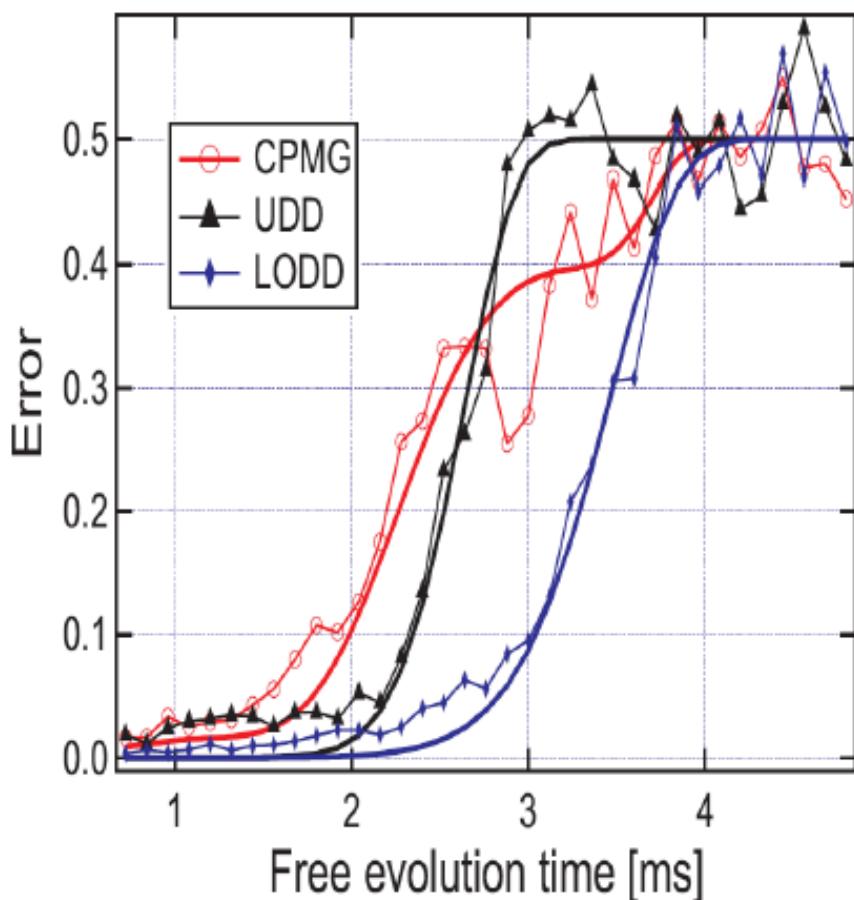
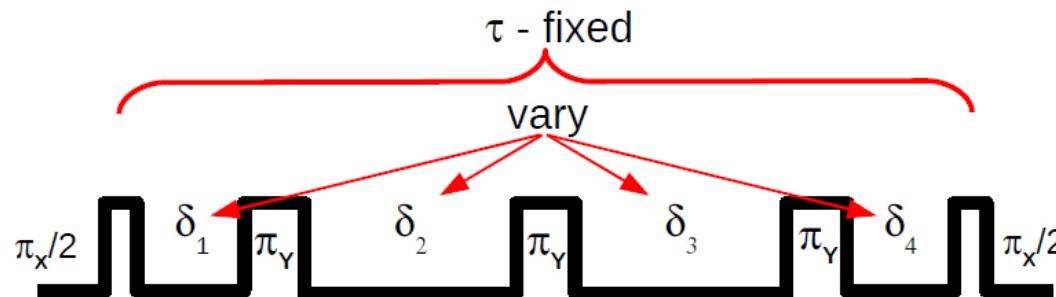
Uys, et al., PRL 103, 040501 (2009)



- Fits use analytical filter function and measured noise spectrum
- CPMG, UDD perform similarly for $1/f^2$ ambient noise
- UDD performs better for noise spectra with sharp cutoffs
- Ohmic noise with sharp cutoff injected

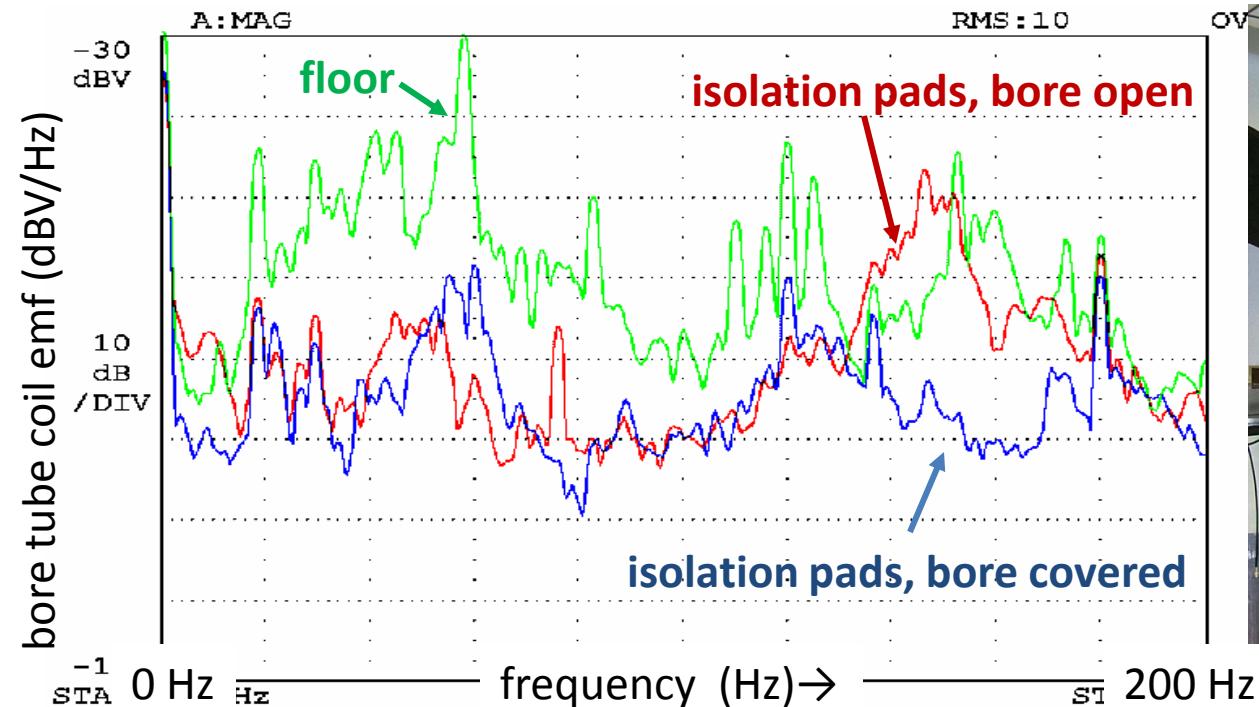
improved performance through feedback optimization

- vary inter-pulse delays for fixed precession time (Nelder-Mead simplex method)



- results shown for injected Ohmic noise
- improved performance with feedback optimization at each total precession time
- no prior knowledge of $S(\omega)$ required

Magnetic field noise reduced through vibration reduction



3 ms → 15 ms increase in spin echo coherence time (single arm time)

Present effort: quantum simulation (see Thursday tutorial by Schaetz)

- Porras, Cirac, PRL 92 (2004); Deng, Porras, Cirac, PRA 72 (2005)
- Schaetz group, Nature Physics 4 (2008); Monroe group, Nature 465 (2010)

uniform Ising model

$$H = -B_x \sum_i \sigma_i^x + \chi J_z^2$$

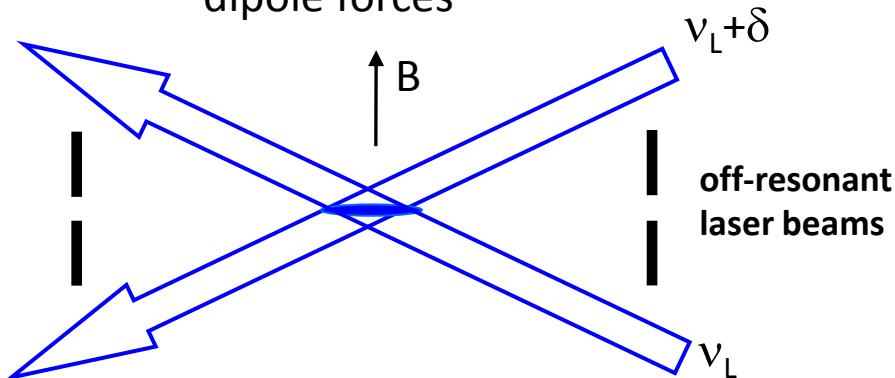
$$J_z^2 = \frac{1}{4} \left(\sum_i \sigma_z^i \right) \left(\sum_j \sigma_z^j \right) = \frac{N}{4} + \frac{1}{4} \sum_{i,j} \sigma_z^i \sigma_z^j$$

dipolar anti-ferromagnetic Ising interaction

$$H = -B_x \sum_i \sigma_i^x + J \sum_{i \neq j} \sigma_i^z \sigma_j^z \frac{d_o^3}{|\vec{r}_i - \vec{r}_j|^3}$$

- B_x derived from 124 GHz microwaves

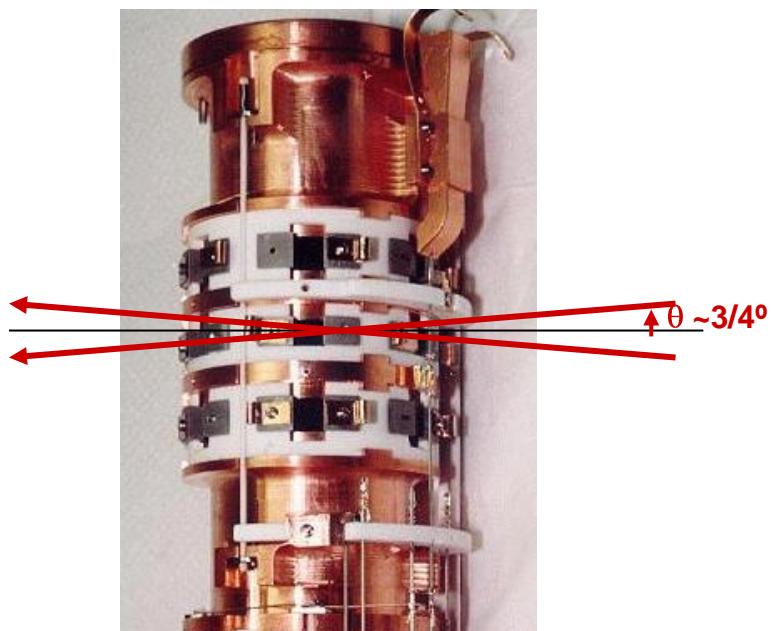
- χ, J derived from state-dependent optical dipole forces



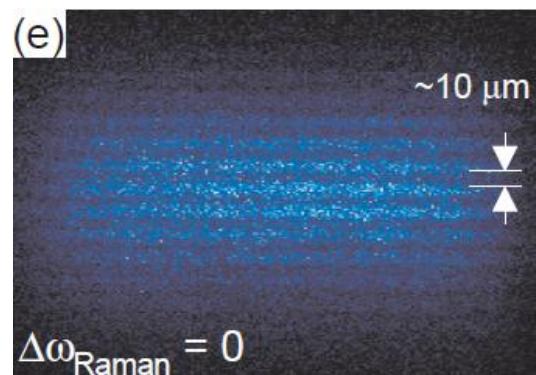
$$\delta \approx \omega_z \Rightarrow \chi J_z^2$$

uniform Ising interaction,
squeezing

$$\delta \gg \omega_z \Rightarrow \text{dipolar anti-ferromagnetic Ising interaction}$$



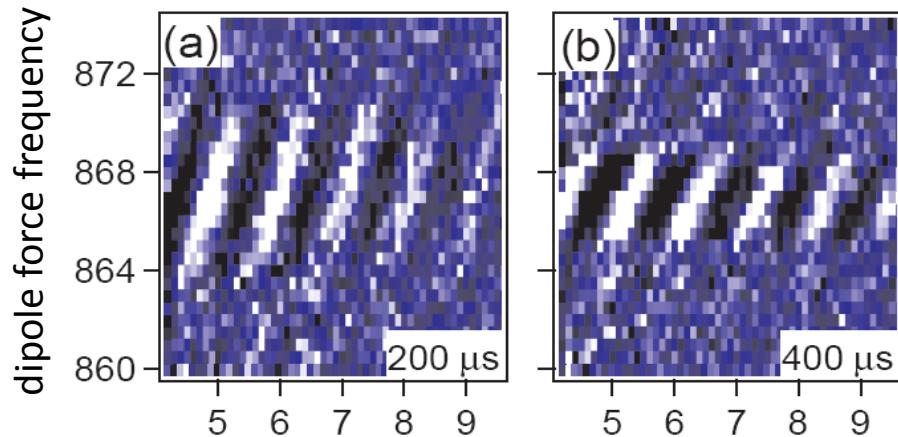
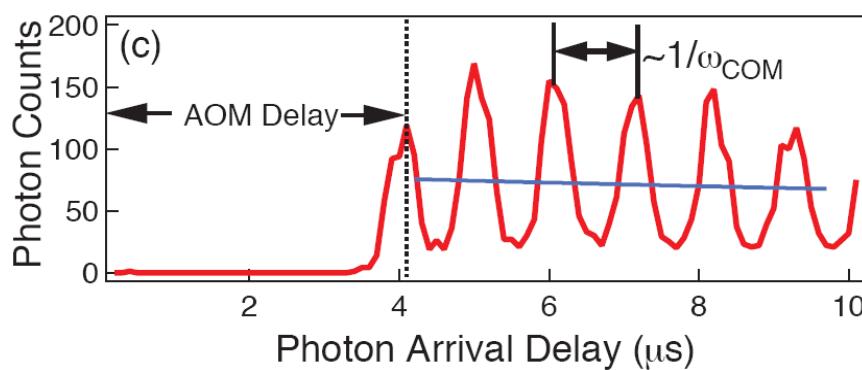
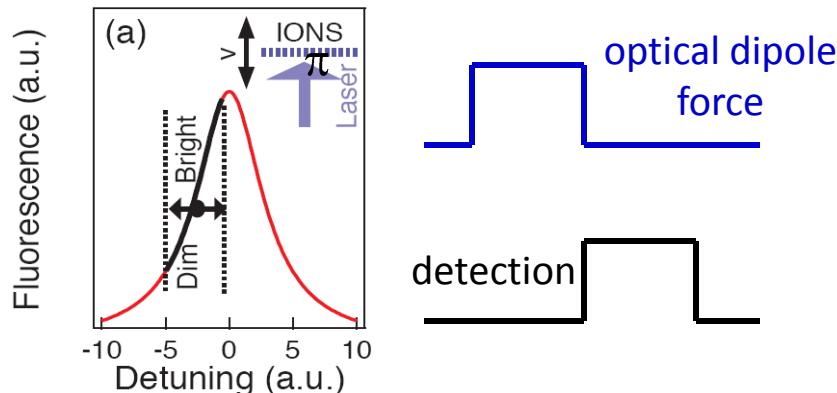
$$\delta = 0$$



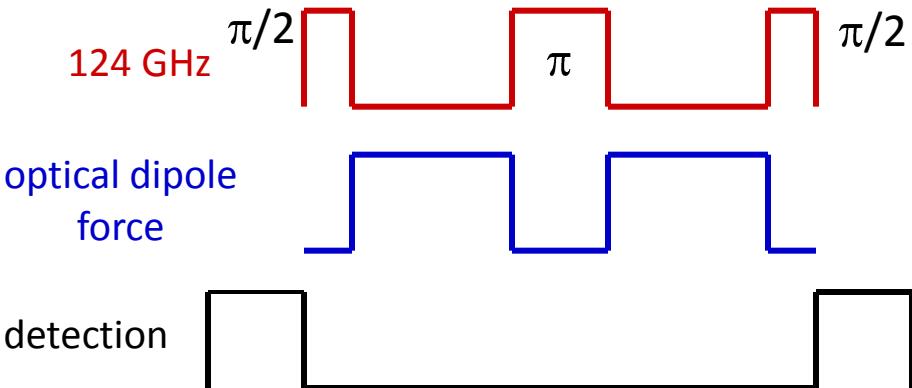
Optical dipole force excitation of ion planar arrays ($N > 100$)

triggered Doppler velocimetry

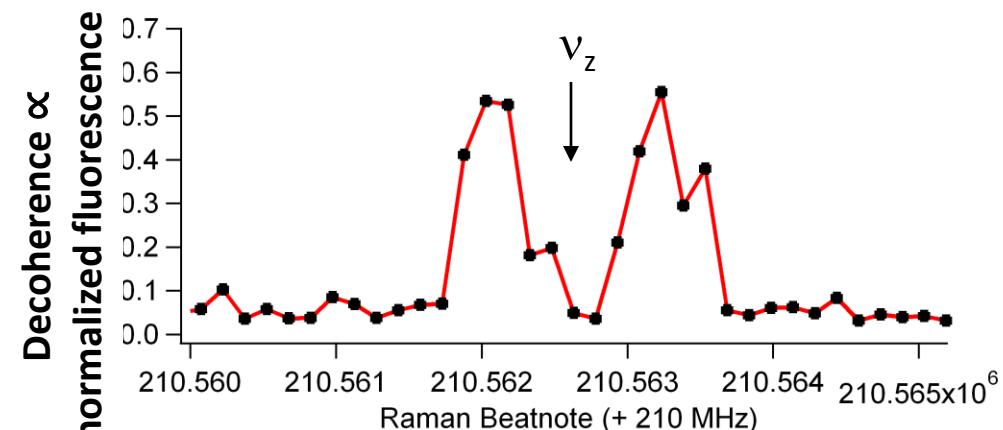
Biercuk et al., Nature Nanotechnology 5 (2010)



internal state dependence of optical dipole drive



decoherence due to spin/axial COM mode entanglement



Quantum simulation with planar ion arrays

uniform Ising model

$$H = -B_x \sum_i \sigma_i^x + \chi J_z^2$$

$B_x = 5 \text{ kHz}$; $> 50 \text{ kHz}$ with new μwave source

dipolar anti-ferromagnetic Ising interaction

$$H = -B_x \sum_i \sigma_i^x + J \sum_{i \neq j} \sigma_i^z \sigma_j^z \frac{d_o^3}{|\vec{r}_i - \vec{r}_j|^3}$$

first configuration:

$\theta = 0.72^\circ$, 10 mW/beam
 waist $\sim 500 \mu\text{m} \times 50 \mu\text{m}$
 $\Delta \sim (2\pi) \cdot 20 \text{ GHz}$, N=100

$$\left. \begin{array}{l} \chi \sim 2\pi \cdot 36 \text{ Hz} \\ J \sim 2\pi \cdot 7 \text{ Hz} \\ \Gamma/\chi \sim 1 \end{array} \right\}$$

Spontaneous emission limits coherent interaction effects; squeezing and entanglement still possible at short times $\chi t \sim 0.02(\pi/2)$, **but**

presently implementing:

$\theta \rightarrow 5^\circ$, 10 mW/beam
 waist $\sim 500 \mu\text{m} \times 50 \mu\text{m}$

$$\left. \begin{array}{l} \chi \sim 2\pi \cdot 1.7 \text{ kHz} \\ J \sim 2\pi \cdot 320 \text{ Hz} \\ \Gamma/\chi \sim 0.07 \end{array} \right\}$$

future configuration?

$\theta \rightarrow 35^\circ$, 10 mW/beam
 waist $\sim 500 \mu\text{m} \times 300 \mu\text{m}$
 sub-Doppler cooling required

$$\left. \begin{array}{l} \chi \sim 2\pi \cdot 2.1 \text{ kHz} \\ J \sim 2\pi \cdot 380 \text{ Hz} \\ \Gamma/\chi \sim 0.01 \end{array} \right\}$$

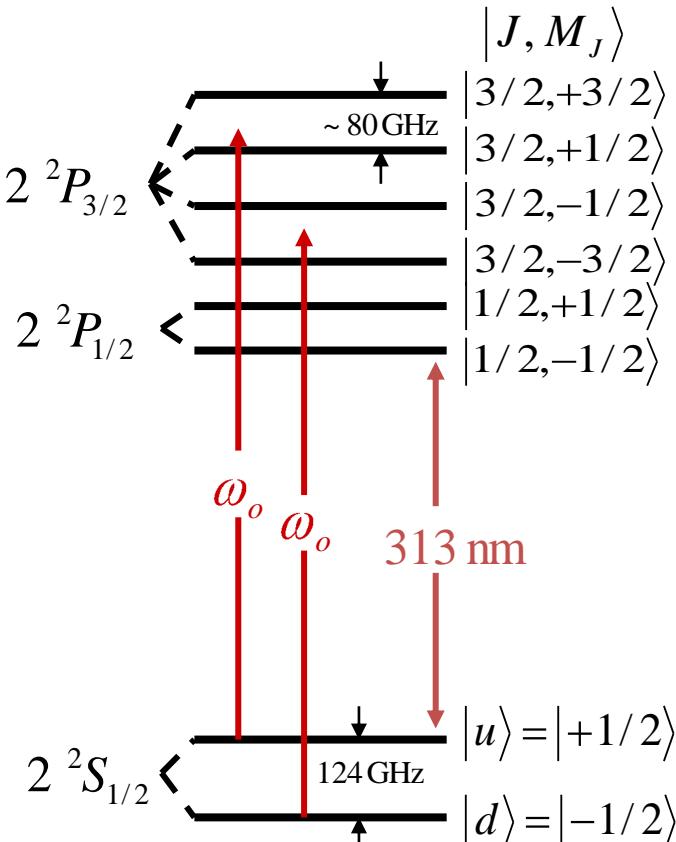
with high power fiber laser based system
 $10 \text{ mW/beam} \rightarrow 100 \text{ mW/beam}$



$$\begin{aligned} \chi &\rightarrow 100 \cdot \chi \\ J &\rightarrow 100 \cdot J \end{aligned}$$

Decoherence due to elastic Rayleigh scattering

Uys, et al., [arXiv:1007.2661](https://arxiv.org/abs/1007.2661)



qubit superposition state described by density matrix

$$\rho \equiv \begin{pmatrix} \rho_{uu} & \rho_{ud} \\ \rho_{du} & \rho_{dd} \end{pmatrix}$$

non-resonant light (ω_o) scattering causes decoherence,

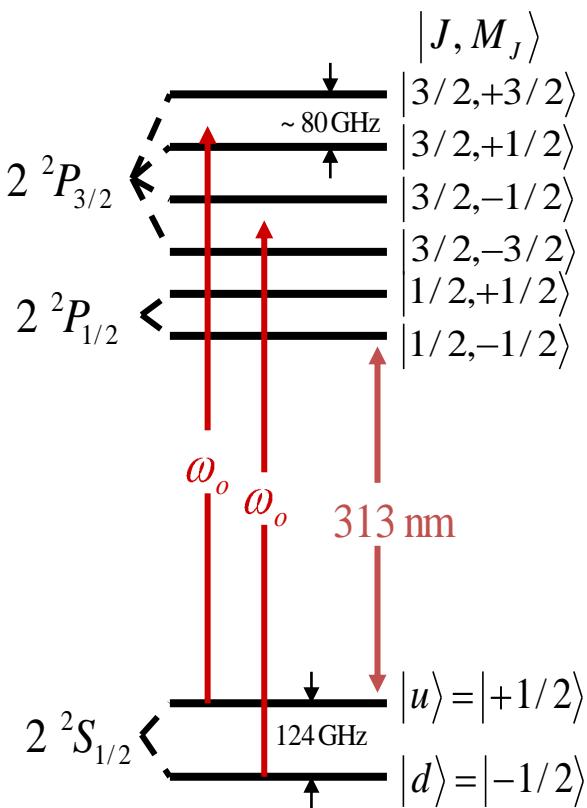
$$\frac{d\rho_{ud}}{dt} = -\frac{\Gamma}{2} \rho_{ud}$$
$$\Gamma = ???$$

Be^+ energy levels, $B=4.5 \text{ T}$

Raman scattering vs Rayleigh scattering

Raman scattering: $|u\rangle \rightarrow |d\rangle; |d\rangle \rightarrow |u\rangle$

final qubit state entangled with the polarization or frequency of the scattered photon
 ⇒ decoherence after single scattering event



Raman scattering rate given by Kramers-Heisenberg formula

$$\Gamma_{ij} = \Omega_R^2 \gamma \left(\sum_J a_{i \rightarrow j}^J \right)^2, \quad a_{i \rightarrow j}^J = \frac{\langle j | \vec{d} \cdot \hat{\varepsilon}_{\lambda+(i-j)}^* | J, \lambda+i \rangle \langle J, \lambda+i | \vec{d} \cdot \hat{\varepsilon}_\lambda | i \rangle}{\delta_{i;J,\lambda+i}}$$

$$\frac{d\rho_{ud}}{dt} = -\frac{\Gamma_{Raman}}{2} \rho_{ud}, \quad \Gamma_{Raman} = \Gamma_{ud} + \Gamma_{du}$$

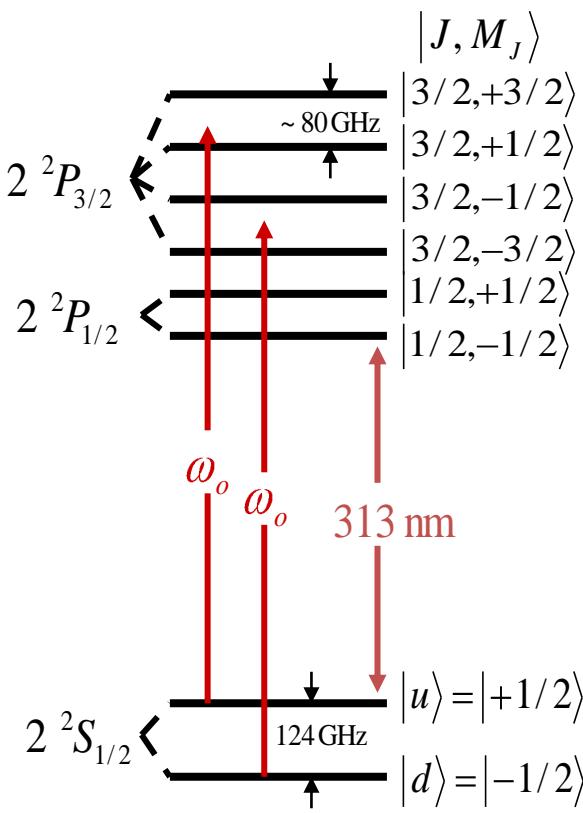
Ozeri *et al.*, Phys. Rev. Lett. **95**, 030403 (2005) –
 precise test of the above prescription for calculating
 decoherence due to Raman scattering

Raman scattering vs Rayleigh scattering

elastic Rayleigh scattering: $|u\rangle \rightarrow |u\rangle; |d\rangle \rightarrow |d\rangle \quad \Gamma_{Rayleigh} = ???$

literature indicates that elastic Rayleigh scattering should not produce decoherence when the elastic scatter rates are equal

- “when of equal rate from both qubit levels, off-resonance Rayleigh scattering of photons did not affect the coherence of a hyperfine superposition”, PRA 75, 042329 (2007)
- “In our system, Rayleigh scattering occurs at the same rate for the two clock states, does not reveal the atomic state, and so does not harm the coherence”, PRL 104, 073602 (2010)



estimate based on difference in elastic scatter rates,
from Ozeri *et al.*, PRA 75, 042329 (2007)

$$\Gamma_{Rayleigh,diff} \sim \frac{(\Gamma_{uu} - \Gamma_{dd})^2}{(\Gamma_{uu} + \Gamma_{dd})/2}$$

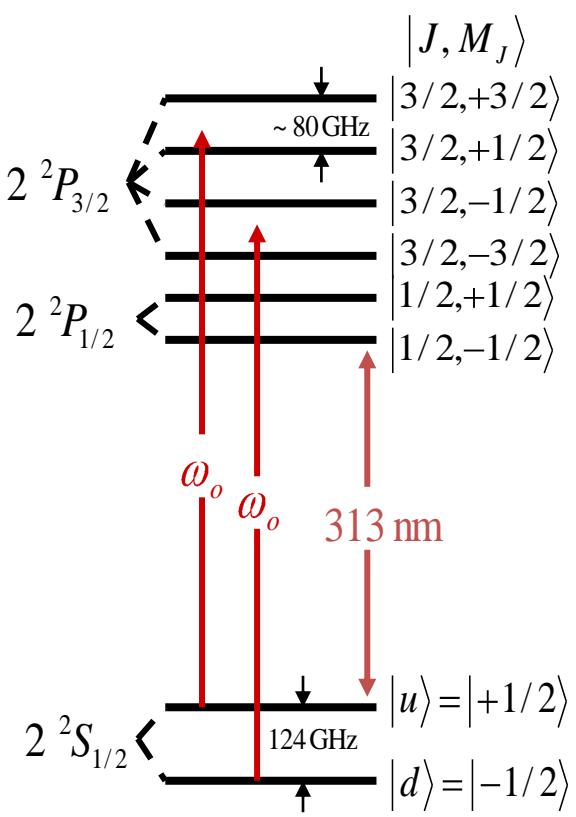
Master equation treatment of decoherence due to light scattering

- consistent treatment of decoherence due to both Raman and Rayleigh scattering
- credit to: Hermann Uys

$$\frac{\partial \rho}{\partial t} = \begin{pmatrix} -\Gamma_{ud}\rho_{uu} + \Gamma_{du}\rho_{dd} & -\frac{1}{2}(\Gamma_{Raman} + \Gamma_{Rayleigh})\rho_{ud} \\ -\frac{1}{2}(\Gamma_{Raman} + \Gamma_{Rayleigh})\rho_{du} & -\Gamma_{du}\rho_{dd} + \Gamma_{ud}\rho_{uu} \end{pmatrix}$$

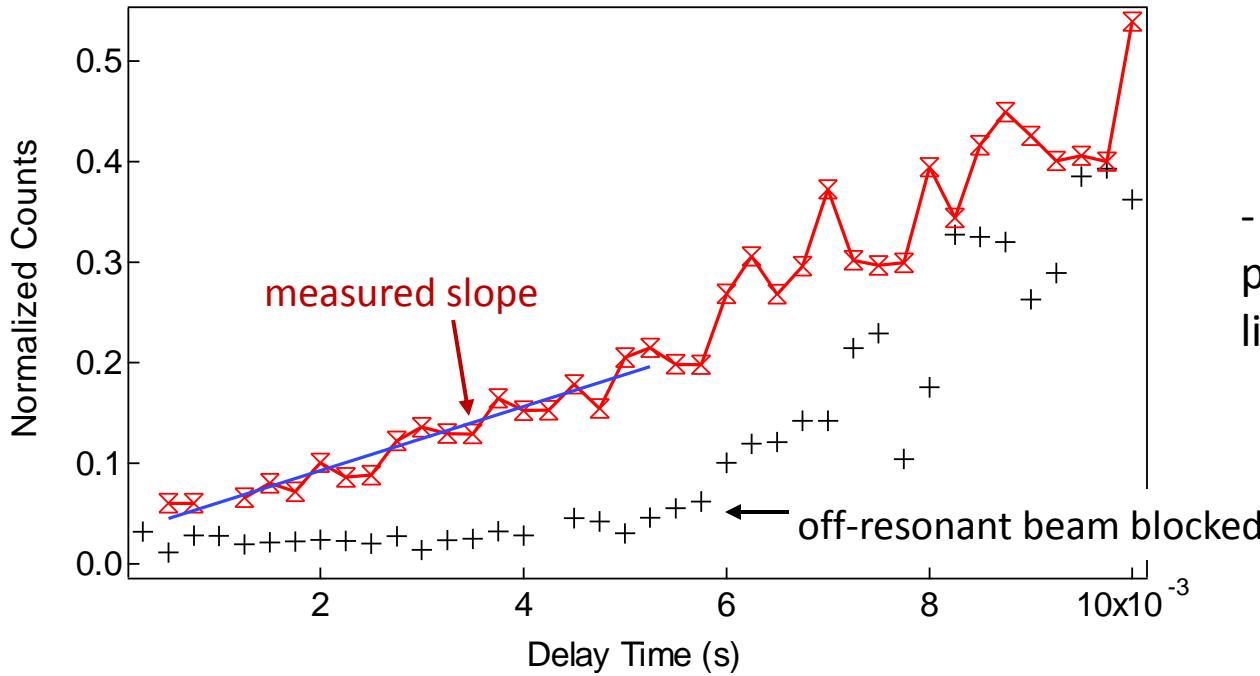
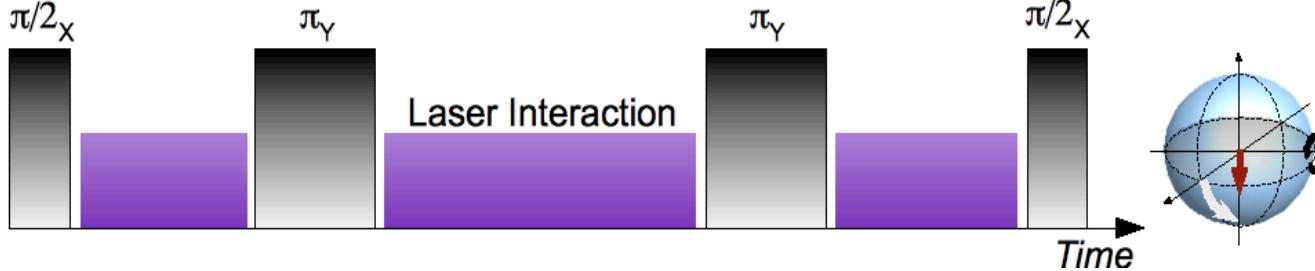
$$\Gamma_{Rayleigh} = \Omega_R^2 \gamma \left(\sum_J a_{d \rightarrow d}^J - \sum_{J'} a_{u \rightarrow u}^{J'} \right)^2$$

decoherence due to difference in the elastic scattering *amplitudes* !



- large decoherence expected if scattering amplitudes have opposite sign
- sign of scattering amplitude determined by the detuning
- physically the state of the qubit can be determined from the phase of the scattered photon

Decoherence measured from decrease in the Bloch vector

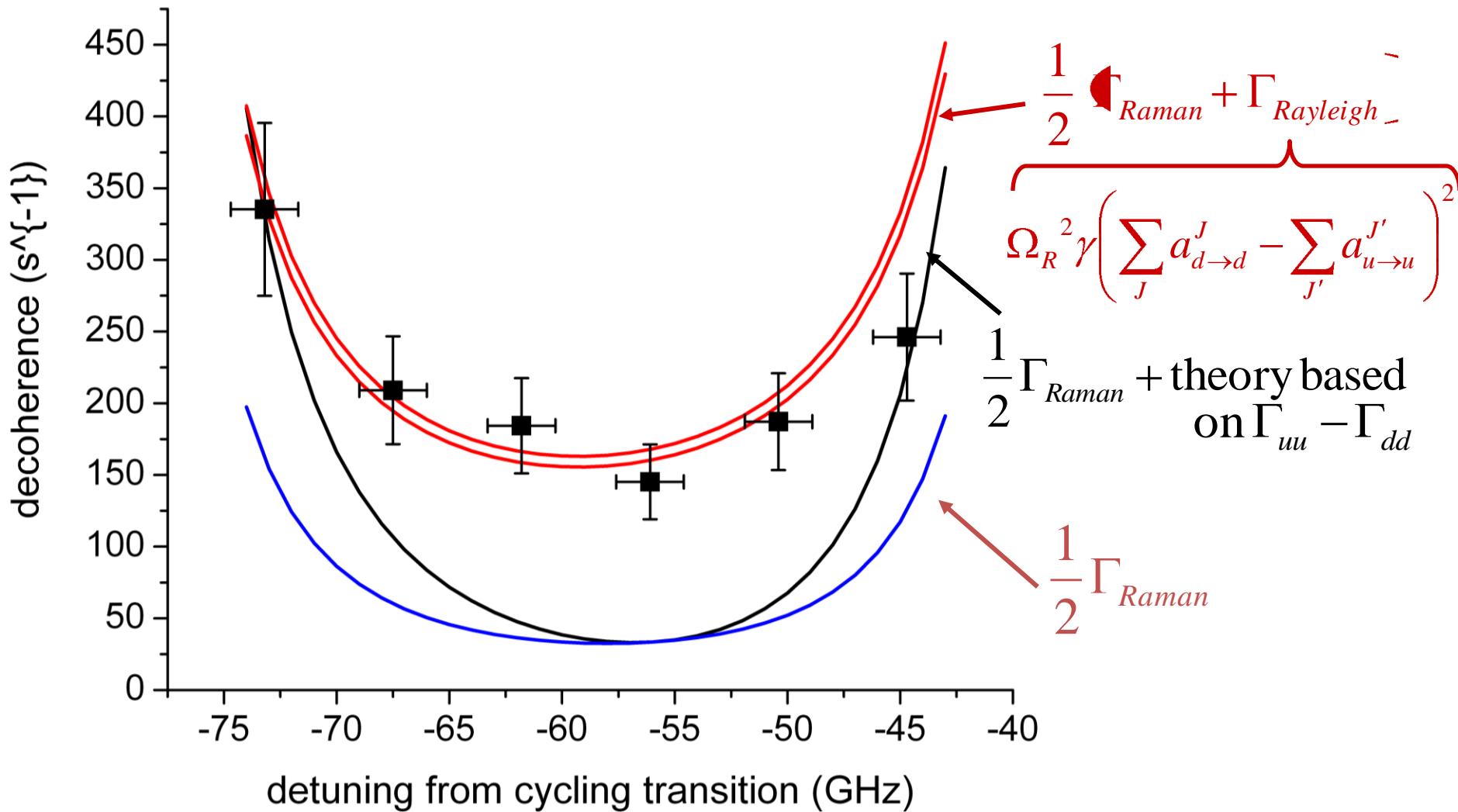


- off-resonant laser beam polarization adjusted to null light shift

$$\text{Normalized counts} = \frac{1}{2} \left[1 - e^{-\frac{1}{2}(\Gamma_{\text{Raman}} + \Gamma_{\text{Rayleigh}})\tau} \right] \approx \frac{1}{2} \frac{\Gamma_{\text{Raman}} + \Gamma_{\text{Rayleigh}}}{2} \tau, \tau = \text{total laser on time}$$

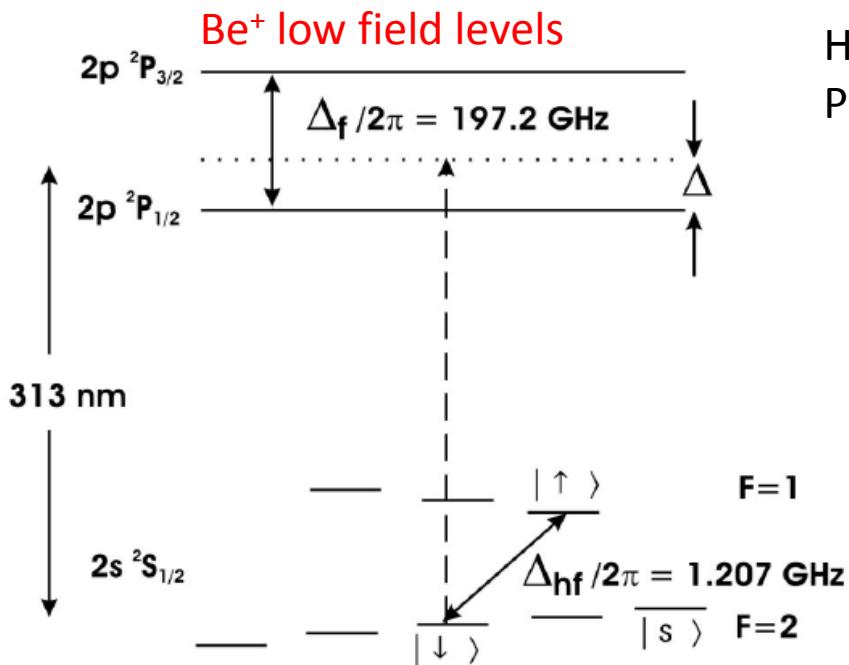
$$\text{decoherence } e = \frac{\Gamma_{\text{Raman}} + \Gamma_{\text{Rayleigh}}}{2} \approx 2 \times (\text{measured slope})$$

Good agreement between theory and experiment



- Laser electric field calibrated from measured light shift and Raman rates

Contribution of Γ_{Rayleigh} to low field trapped ion experiments



Hyperfine Coherence in the Presence of Spontaneous Photon Scattering, Ozeri , et al., PRL 95 (2005)

- no decoherence due to Rayleigh scattering
- conditions: 1. clock states
2. $\Delta \gg \Delta_{hf}$
- $\Gamma_{\text{Raman}} \sim 1/\Delta^4 \rightarrow$ gate errors due to spontaneous emission reduced with large detunings Δ

For clock states and $\Delta \gg \Delta_{hf}$:

$$\Gamma_{\text{Rayleigh}} = \Omega_R^2 \gamma \left(\sum_J a_{d \rightarrow d}^J - \sum_{J'} a_{u \rightarrow u}^{J'} \right)^2 \sim \frac{1}{\Delta^2} \left(\frac{\Delta_{hf}}{\Delta} \right)^2 \sim \frac{1}{\Delta^4}$$

For qubits that are not clock states:

$$\Gamma_{\text{Rayleigh}} \sim \frac{1}{\Delta^2}$$

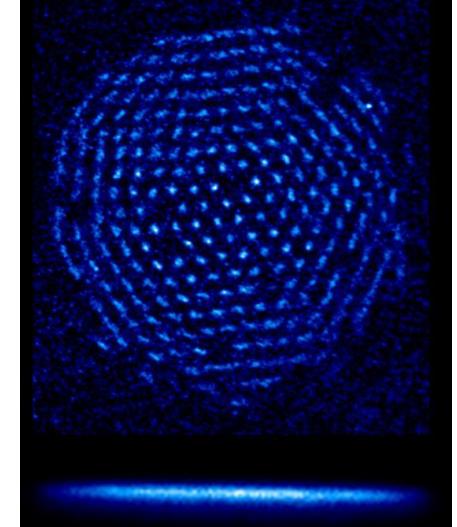
two-ion gate strength:

$$\chi \sim \frac{\eta^2}{\delta} \frac{V^2}{\Delta^2} \sim \frac{1}{\Delta^2}$$

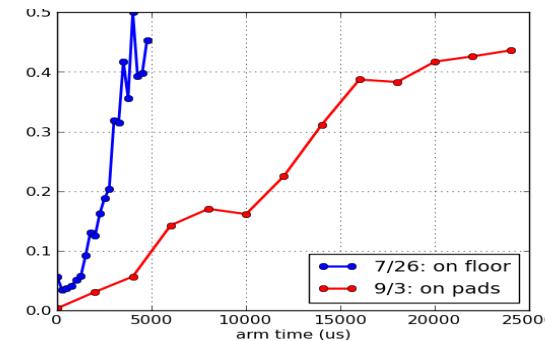
No apparent advantage of large Δ if qubits are not clock states

Summary:

- ion crystals in Penning traps look like a good platform for simulating quantum magnetic models with an intractable number of ions



- qubit coherence limited by 20 -200 Hz magnetic field fluctuations;
 $3\text{ ms} \rightarrow 15\text{ ms}$ coherence time through vibration reduction



- current effort: Ising model simulation using state dependent optical dipole forces to push the ions in the axial direction

