Statistical and empirical properties of Factor Model Quantile Simulation (FMQS)

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Simulation of Stock Returns through FMQS

Quantile Regression on Factor Model and Simulation of Market Returns

Factor Model

\[ R_i = \alpha_i + \beta_i R_M + \gamma_i R_M^2 + \varepsilon_i \]

Model for Market Return

\[ dY_t = \mu dt + \sqrt{V_t} dW^Y_t + dJ^Y_t \]

Distribution of each Stock Return

Potential Applications

- Risk Management
- Importance Sampling
- Portfolio Optimization
- ...
Outline of FMQS Methodology (1/6)

Adjustment of the historical Data

- **Time Series Data**
- **Quantile Regression**
- **Market Simulation**
- **Quantile Calculation**
- **Interpolation**
- **Inversion Sampling**

- **Investment Universe**: All 30 stocks currently in DJIA
- **Market Return**: Estimated through index DJIA*
- **Time Horizon**: 4 Mai 1999 to 25 February 2016
- **Adjustments**: Visa stock prices prior 18 March 2008 were reconstructed by PCA
Quantile Regression estimates the Parameters of the Factor Model

**Quadratic CAPM**

\[ R_i = \alpha_i + \beta_i R_M + \gamma_i R_M^2 + \varepsilon_i \]

**Quantile Regression**

\[ Q^q(R_i | X) = \alpha_i^q + \beta_i^q R_M + \gamma_i^q R_M^2 + \varepsilon_i^q \]

**Results**

Estimation of \( \alpha_i^q, \beta_i^q, \gamma_i^q \) \( \forall i, q \)
Outline of FMQS Methodology (3/6)

Market Return Simulation through a Stochastic Volatility Jump Diffusion Model

**Time Series Data**

**Quantile Regression**

**Market Simulation**

**Quantile Calculation**

**Interpolation**

**Inversion Sampling**

\[
\begin{align*}
\text{Stock Process:} & \quad dY_t = \mu dt + \sqrt{V_t}dW_t^y + dJ_t^y \\
\text{Volatility Process:} & \quad dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t}dW_t^v + dJ_t^v \\
\text{Jump Process:} & \quad dJ_t^y = \xi_t^y dN_t
\end{align*}
\]

**Results**

Simulation of $R_M$
Outline of FMQS Methodology (4/6)

Quantiles can be calculated using the previous Results

Quantile Calculation

Time Series Data

Quadratic CAPM

Quantiles $\alpha^q_i$, $\beta^q_i$, $\gamma^q_i$ \forall i, q

Market Simulation

$R_M$

$R_i = \alpha_i + \beta_i R_M + \gamma_i R_M^2 + \varepsilon_i$

Inversion Sampling

Interpolation

Quantile Regression

Graph showing the relationship between quantiles and returns.
Outline of FMQS Methodology (5/6)

Through Interpolation we generate the Distribution from the Quantiles

- Time Series Data
- Quantile Regression
- Market Simulation
- Quantile Calculation
- Interpolation
- Inversion Sampling

**Monotonicity**
- Interpolation maintains monotonicity

**Preservation of shape**
- Interpolation is continuously differentiable
Inversion Sampling can be used to sample from the calculated Distribution

Let $U$ be an uniform random variable on $(0, 1)$ and let $F^{-1}$ be the inverse of some distribution $F$. Then $Y := F^{-1}(U)$ follows the distribution $F$. 

Time Series Data
Quantile Regression
Market Simulation
Quantile Calculation
Interpolation
Inversion Sampling
Empirical Results

Parameter estimation for Quantile Regression and SVJD Model

**Quantile Regression**

\[ R_i = \alpha_i + \beta_i R_M + \gamma_i R_M^2 + \varepsilon_i \]

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**SVJD Model**

\[
\begin{align*}
\text{d}Y_t &= \mu \text{d}t + \sqrt{V_t} \text{d}W_t^y + \text{d}J_t^y \\
\text{d}V_t &= \kappa (\theta - V_t) \text{d}t + \sigma \sqrt{V_t} \text{d}W_t^v + \text{d}J_t^v \\
\text{d}J_t^y &= \xi_t^y \text{d}N_t 
\end{align*}
\]

<table>
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References

Summary of the most relevant References


Thank you for your attention
Backup slides
Analytical results

Assuming a normal distributed market return

**Distribution**

\[ F(r) = \frac{1}{2} \left( \text{Erfc} \left( \frac{\beta - \sqrt{\beta^2 + 4\alpha\gamma - 4\alpha\gamma + 2\gamma\mu}}{2\sqrt{2}\gamma\sigma} \right) - \text{Erfc} \left( \frac{\beta + \sqrt{\beta^2 + 4\alpha\gamma - 4\alpha\gamma + 2\gamma\mu}}{2\sqrt{2}\gamma\sigma} \right) \right) I_{\{r \geq \frac{\beta^2}{4\gamma}\}} \]

**Density**

\[ f(r) = \frac{\exp \left( -\frac{\beta^2 + 2\beta\gamma\mu + 2\gamma(r - \alpha + \gamma^2)}{2\gamma^2\sigma^2} \right) \exp \left( \frac{\beta - \sqrt{\beta^2 + 4(r - \alpha)\gamma + 2\gamma\mu}^2}{8\gamma^2\sigma^2} \right) + \exp \left( \frac{\beta + \sqrt{\beta^2 + 4(r - \alpha)\gamma + 2\gamma\mu}^2}{8\gamma^2\sigma^2} \right)}{\sqrt{2\pi} \sqrt{\beta^2 + 4(r - \alpha)\gamma\sigma}} I_{\{r \geq \frac{\beta^2}{4\gamma}\}} \]

**Example: Standard normal market return**

![Graphs showing distribution and density functions for different values of alpha, beta, and gamma.](image)
Moments

Correlation

\[
\text{Cor}(R_1, R_2) = \frac{(\beta_1 + 2\gamma_1 \mu)(\beta_2 + 2\gamma_2 \mu)\sigma^2 + 2\gamma_1\gamma_2 \sigma^4}{\sqrt{\sigma^4((\beta_1 + 2\gamma_1 \mu)^2 + 2\gamma_1^2 \sigma^2)(\beta_2 + 2\gamma_2 \mu)^2 + 2\gamma_2^2 \sigma^2})}
\]

\[
\text{mean} = \alpha + \beta \mu + \gamma(\mu^2 + \sigma^2)
\]

\[
\text{variance} = (\beta + 2\gamma \mu)^2 \sigma^2 + 2\gamma^2 \sigma^4
\]