Lead-lag relationship between spot and futures stock indexes: Intraday data and Regime Switching Models.

Introduction

A key question in finance is the lead-lag relationship between spot and futures markets. In the last decades, a significant strand of both theoretical and empirical research has focused on the study of the dynamic relationship between futures and spot prices. However, this question remains ambiguous because the empirical evidence diverges across articles.

According to the efficient market theory of financial economics, the price of an asset reflects all relevant information that is available about the intrinsic value of the asset. Numerous articles have focused on the study of deviations from the Cost of Carry model and have investigated the relationship between futures and spot prices (Kawaller, Koch and Koch, 1987; Ng, 1987; Stoll and Whaley, 1990 and Chan, 1992 among others). Spot and futures prices are different from each other as a result of the difference in the Cost of Carry. However, in efficient markets there will be a simultaneously perfect relationship between spot index and index future contract price changes. Therefore, all available information would simultaneously be reflected both in spot prices and future prices and there should be no lead-lag relationship between prices in two markets. Notwithstanding, due to market imperfections, one market may reflect information faster than the other one and, as a result of that, a lead-lag relationship exists.

A relevant issue which has to be considered when studying empirically the lead-lag relationship between two markets or assets, and thus, the presence of arbitrage opportunities, is the frequency of the analyzed data. Numerous empirical studies suggest that the lead-lag relationship is an important stylized fact at high frequency data; nevertheless, it vanishes when the frequency of observations decreases (Huth and Abergel, 2014). Harris et al. (1995) show evidence that frequency is crucial to test pricing dynamics between markets which are cointegrated due to two reasons: 1) If time interval is too wide it might provoke that the error correction took place inside an interval which had not been taken into account, so, higher frequency trading strategies cannot be detected considering daily prices, and 2) Cointegration models allow establishing long term relationships between temporal series which may diverged in very short periods but they readjust to the cointegration pattern. For this reason, the best way to approach this investigation is considering high frequency data.

Besides, an increasing empirical strand of the literature suggests that the dynamic relationship between futures and spot prices may be characterized by a nonlinear relationship and failure to account for this might lead to biased results (Brooks, 1996; Hieh, 1991). Regime switching models (RSM hereafter) have the ability to adequately characterize and capture unusual movements that appear in the relationship between spot and futures markets. These models can capture these changes of behavior, and the fact that the new dynamics of prices and fundamentals persist for several periods (Sarno and Valente, 2000). Ang and Timmermann (2011) determine three reasons why RSM have become popular in financial modeling: 1) the idea of regime shifts is natural and intuitive. When this methodology is implemented in financial series, regimes determined by econometric methods are often identified with different periods in regulation, policy, and other secular changes (Hamilton, 1989); 2) RSM parsimoniously capture stylized behavior of many financial return series, and 3) They can capture nonlinear dynamics of asset

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1They find strong evidence against the hypothesis of linear dynamics and in favor of the capability of regime-switching-vector-equilibrium models (non linear models) to capture properly the time-series properties of the data.
returns in a framework based on linear specifications, or conditionally normal or log-normal distributions, within a regime, which allows its analytical study.

In this paper, high-frequency data on a five-minute interval basis of transaction prices from January 2014 to September 2015 from different markets, namely CAC40, DAX30, FTSE100 and EUROSTOXX, for both the stock index and index futures are used.

A well-known stylized fact about the intraday statistical characteristics of many financial markets is the strong intraday periodicity in the average absolute returns which leads to an observable repetitive pattern in the autocorrelations of the absolute returns each day, suggesting a gradually decaying U-shape pattern, (see among others Wood et al, 1985 and Tse, 1999). Due to the strong intraday periodicity detected in the average absolute returns, consistent with earlier evidence found by Andersen and Bollerslev, 1997, standard ARCH models which usually imply a monotone geometric decay in the autocorrelogram of the absolute returns are not appropriate.

The Fourier Flexible Form (FFF) implemented in this study is particularly convenient to deal with this issue and involves polynomial methods which approximate the intraday periodic component by using the linear polynomials regression and Fourier methods which consider sinus and cosines to approximate the periodic intraday component.

The main goal and major contribution of this investigation consist of analyzing the lead-lag relationship between different stock markets and their respective future contracts. To do so, two aspects will be taken into account, given that, as indicated above, they are considered essential in such studies: high frequency data and regime switching (high versus low volatility). The study intends to determine if the degree and direction of price adjustment process between spot and futures prices differs when high and low volatility regimes are considered. Although high frequency data has been used in many studies, to the best of authors’ knowledge there is no study that analyzes the contribution of spot and futures market to the price discovery process on a high frequency interval basis while considering Markovian regime shifts.

Review of literature

The lead–lag relation between price movements of stock index futures and the underlying cash market describe how fast one market reflects innovations relative to the other market, and the linkage between both markets. When one market respond faster to new information, and the other market react later a lead lag relationship is observed (Chan, 1992). Numerous empirical studies conclude than when this equilibrium is perturbed it is spot price which make the adjustment towards equilibrium. There are other studies which conclude that both markets contribute to price discovery but, generally, futures price movements lead the stock market. A significant body of the literature has attempted to determine whether price discovery takes place primarily in the spot or futures markets.

Engle and Granger (1987) demonstrated that cointegrated series have an error correction term representation (also named speed of adjustment coefficient) which allows correcting in one period the disequilibrium detected in the previous one. Not only does the error correction term indicate the

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2 An appropriate intraday dynamic analysis requires computing and extracting the intraday periodic component of return volatility (Andersen y Bollerslev, 1997) and standardizing returns before estimating models RSM.

3 One interpretation of the error correction term is that it reflects the effect of arbitrage. If the futures price is too low compare to the index value, arbitragers will sell the stocks underlying the index and buy the futures contract. On the contrary, if the futures price is too high they will sell the futures contract and buy the stocks underlying the index.
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percentage of disequilibrium from one period that is corrected in the next period, but it also show the relative magnitude of adjustments in both markets towards equilibrium. The rationale behind the concept of cointegration is that two variables can deviate in the short run from each other, but market forces will bring them back together and therefore there exists a long-run equilibrium relationship between these two variables. If the error correction term is not considered, then the model is misspecified. Usually, spot and futures prices are cointegrated with an order of one and the linear vector error correction model has been used to investigate the error correction process between spot and futures prices.

Kawaller, Koch and Koch (1987) examine the intraday price relationship between S&P 500 Index and S&P 500 Futures using minute-to-minute data for all trading days during 1984 and 1985 and conclude that futures market lead the stock market over twenty to forty-five minutes, however movements in the spot market rarely affect futures more than one minute.

Ng (1987) used S&P 500 Index and S&P 500 Futures daily data for about 5 years and concluded that futures prices lead than lag spot prices by one day, although the magnitude of the lead coefficients was rather weak. It was no detected lead for spot prices.

Stoll and Whaley (1990) examine the time series properties of 5-minute intraday returns for about 5 years of stock index and stock index futures contract and conclude that on one hand S&P500 Index and MM index futures returns tend to lead stock market returns over five minutes, on average, but sometimes 10 minutes or more. On the other hand lagged stock index returns have a moderate predictive impact on futures returns, so the effect is bidirectional, but future market has more predictive capability.

Chan, Chan and Karolyi (1991) studied, simultaneously, the intraday relationship between returns and returns volatility (utilizing the GARCH models) in the S&P 500 stock index and stock index futures market from 1984 to 1989. Each day, trading hours are partitioned into five-minute intervals. Their evidence is consistent with the hypothesis that both markets contribute to price discovery. Therefore, price innovations which emerge in either the stock or futures market can predict the volatility in the other market.

Chan (1992) studies the lead–lag relationship between intraday futures and cash index prices for two sample periods, August 1984–June 1985 and January 1987–September 1987. The article analyzes data on MMI and an index comprising of 20 actively traded stocks. The intraday time series are partitioned into five-minute intervals. The article finds strong evidence that there is an asymmetric lead–lag relationship between the two markets with the strong evidence that futures index leads the cash index and weak evidence that the cash index leads the futures.

Wahab and Lashgari (1993) extended the study of the lead-lag relationship by applying a cointegration approach to investigate the robustness of previous studies including an alternative model parameterization: the error correction model. They use daily closing spot and futures prices for both the S&P 500 Index and the Financial Time index from January 4, 1988 to May 30, 19. They find evidence that feedback relationship exists between cash and futures markets which is consistent with the important price discovery role served by both stock and index futures markets and confirm the hypothesis of Chan et al. (1991), which established that both markets contribute to price discovery.

Tse (1999) examines the intraday price discovery process and volatility spillovers between the DJIA futures and index using minute by minute data for the six-month period of November 1997 to April
1998. The article concludes that the informational contribution attributable to the futures market is 88.3% implying that DJIA futures dominates the cash market in price discovery. It is the spot price which makes the greater adjustment in order to reestablish the equilibrium. Or, to put it another way, the futures price leads the cash price in price discovery.

Sarno and Valente (2000) examine the dynamic relationship between spot and futures prices in stock index futures markets using weekly data for the S&P 500 and the FTSE 100 indices from January 1, 1998 to December 26, 1997 and using nonlinear Markov-switching vector equilibrium correction models that allow for three regimes in the mean of the equilibrium correction model as well as in the variance-covariance matrix. They find that strong evidence against the hypothesis of linear dynamics and in favor of the capability of regime-switching-vector-equilibrium models to capture well the time-series properties of the data.

Pardo and Climent (2000) study the temporal relationship between IBEX 35 Index and IBEX 35 Futures Contract using minute by minute data for the entire year 1996 and applying a cointegration parameterization. They conclude that both markets contribute to price discovery, but there exists strong evidence that the predictive capability of the future market is greater.

Blanco (2003) investigates the temporal relationship between IBEX 35 Index and IBEX 35 Futures Contract using five minute interval data Futures from January 11, 1995 to October 27, 1995. The article concludes that there exists bidirectional causality between spot and future prices, although the predictive capability of the future market is greater.

Li (2009) examines the dynamics of the relationship between spot and futures markets of three mature markets (S&P500, FTSE100, DAX) and two emerging markets (BOVESPA, BSI) from the period April 3, 1995 to December 12, 2005 using daily data. The author uses a traditional VECM and a Markov-switching vector error correction model (in which the parameter of the deviation of spot-futures prices changes according to the phase of the volatility regime) and compares the results. When a conventional VECM is used to examine the spot-futures price discovery process the conclusions among markets are inconsistent. However, when a MS-VECM is considered the findings show that: 1) The spot-futures disequilibrium adjustment process depends primarily on the futures market during a high variance state and on the spot market in the low variance state, in other words, the futures price leads the spot price in price discovery during stable periods, conversely, during volatile periods, the price discovery takes place on the spot market. This finding is robust for all markets. 2) The scale of price adjustment occurring in the futures market during a high variance state is greater than that occurring in the spot markets during a low variance state. 3) The correlation between the spot and futures markets for the high variance state is lower than that for the low variance state in all cases. The study also denotes that the process of the price adjustment between spot and futures markets occurs very rapidly in mature markets, meanwhile, for emerging markets the disequilibrium between spot and futures prices takes longer to reduce. This phenomenon explains why the deviation of spot-futures prices in the two emerging markets, particularly in the high variance state, is considerably higher in absolute magnitude than that in the three mature markets.

Theissen (2012) examines the intraday price discovery process of two data sets: 1) DAX index values from the spot equity market and DAX index futures data from the first quarter of 1999 at a frequency of 15 s., and 2) DAX EX and DAX index futures data from the last quarter of 2010 at a frequency of 1 minute. The data sets comprise a complete record of all transaction prices, bid and ask quotes. The author estimates a threshold error correction model to allow for arbitrage opportunities to have impact on the return dynamics introducing two dummy variables to identify those arbitrage opportunities that
require selling in the spot market and selling in the futures market. Bidirectional Granger causality is found and the evidence shows that the dependence of the spot market on the futures market is much stronger than the reverse dependence. Therefore, the futures market leads in the process of price discovery, and furthermore, the presence of arbitrage opportunities has a strong impact on the dynamics of the price discovery process. Therefore, the results suggest that the leading role of the futures market in the price discovery process is particularly pronounced when arbitrage opportunities exist.

**Methodology and data**

This study uses high-frequency observations on a five-minute interval basis of transaction prices from different markets, namely CAC40, DAX30, FTSE100 and EUROSTOXX, for both the stock index and index futures. The sample period extends from January 2014 to September 2015 and only data for the period of simultaneous operation of both markets are used in our analysis.

Before studying the price discovery process of market returns we first filter out and standardize data for the sake of removing the intraday periodic component.

The vast majority of the aforementioned studies do not consider intraday dynamic returns. As has been underlined by Andersen and Bollerslev (1997) an appropriate intraday dynamic analysis requires computing and extracting the intraday periodic component of return volatility. Filtering out the intraday periodicity of volatility is essential for high frequency data analysis. The Flexible Fourier Form, originally proposed by Gallant (1981, 1982) is the method used in this study to address this issue. Examples of articles which implement this methodology are, among others, Andersen and Bollerslev (1997), Andersen, Bollerslev, and Cai (2000) and Andersen (2000).

![Figure 1](image.jpg)

**Figure 1.** The figure depicts the correlogram for the absolute 5 minute CAC 40 stock index raw returns (blue line), filtered returns (green line) and standardized returns (red line), out to a length 5 trading days. The sample period extends from January 2014 to September 2015, for a total length of 45,217 observations.
Figure 1 depicts the correlation structure for the absolute 5-minute raw, filtered and standardized returns for CAC 40 performance stock index. Noticeably, the periodic dependencies have been considerably reduced for the filtered and standardized absolute returns compared to the raw absolute returns. It can also be appreciated the accelerated initial rate of decay in the autocorrelations followed by a remarkable slow rate of decay thenceforward. A similar pattern is observed in the other markets analyzed in this study.

Therefore, the contribution of this paper is two-fold: the authors reexamine the price discovery process using a nonlinear equilibrium correction model based on an extension of Markovian regime shifts in time series proposed by Hamilton (1988, 1989) which allows the error correction term to follow a nonlinear adjustment process in the mean equation. Additionally, high frequency data, previously standardized to filter out the intraday periodicity of volatility, are used in this study to improve our understanding of the dynamic properties of price discovery process.

**Anticipated outcomes**

This study hypothesizes that convergence process towards the spot-futures equilibrium and its speed of adjustment depends on the volatility regimes.

**References**


