High-Fidelity Composite Pulse Techniques for Quantum Computation and Simulation

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QUANTUM OPTICS & QUANTUM INFO @ SOFIA UNI

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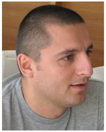

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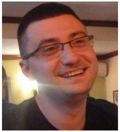

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ACTIVITIES: quantum optics & control, quantum computation, quantum simulation, classical & nonlinear optics, femtosecond physics

EU: STREP iQIT   AvH: CLGrant

Outline

- Two-state systems
  - Broadband, narrowband, passband, fractional-π pulses
  - Local addressing (subwavelength localization)
  - Composite adiabatic passage
- Multistate systems
  - Composite STIRAP
  - Suppression of unwanted transitions
- Conditional gates and entanglement
  - Dicke and NooN states
  - Toffoli gate and C^n-NOT gates
  - C-phase gate
- Lineshapes
  - smooth pulses of finite duration
  - power narrowing

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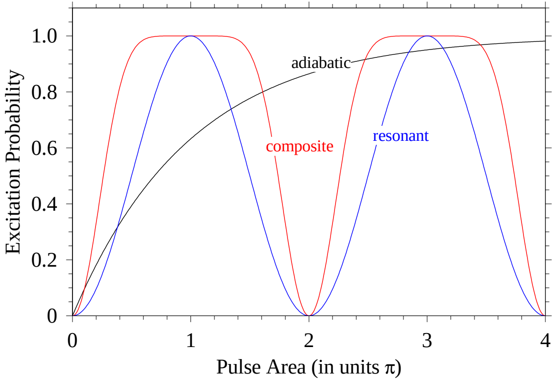
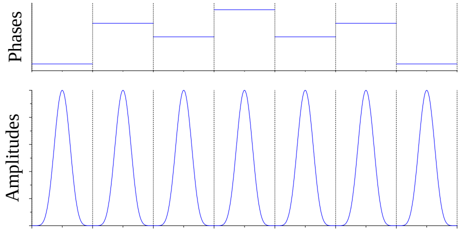
Coherent excitation



resonant
small area (fastest)
accurate
sensitive

adiabatic
large area (slow)
inaccurate
robust

composite
moderate area (fast)
robust, accurate
flexible



A sequence of phased pulses with an electric field $E(t) = \sum_{n=1}^{N} e^{i\phi_n} E_n(t)$ [Rabi frequencies $\Omega(t) = \sum_{n=1}^{N} e^{i\phi_n} \Omega_n(t)$] generates the propagator

$$\mathbf{U}(\phi_1, \phi_2, \ldots, \phi_N) = \mathbf{U}(\phi_N)\mathbf{U}(\phi_{N-1}) \cdots \mathbf{U}(\phi_2)\mathbf{U}(\phi_1)$$

The phases ϕ_n are used as control parameters which are determined from the condition to produce a transition probability profile of a desired shape.

composite pulses ≡ discrete optimal control theory

Composite Pulses

NMR

Levitt & Freeman, JMR **33**, 473 (1979)
Freeman, Kempsell & Levitt, JJMR **38**, 453 (1980)
Levitt, Prog. NMR Spectrosc. **18**, 61 (1986)
Freeman, *Spin Choreography* (Spektrum, Oxford, 1997)

Polarization Optics

West & Makas, JOSA **39**, 791 (1949)
Destriau & Prouteau, J. Phys. Radium **10**, 53 (1949)
Pancharatnam, Proc. Indian Acad. Sci. **A41**, 130 (1955); **A41**, 137 (1955)
Harris, Ammann & Chang, JOSA **54**, 1267 (1964)
McIntyre & Harris, JOSA **58**, 1575 (1968)

Quantum Optics & Quantum Info

Blatt's group: Nature **422**, 408 (2003); Nature Phys. **4**, 839 (2008); PRL **102**, 040501 (2009)
Wunderlich's group: PRL **98**, 023003 (2007); PRA **77**, 052334 (2008); PRL **110**, 200501 (2013)

Composite pulses: SU(2) method

coherently driven two-state quantum system

$$\mathrm{i}\hbar\partial_t \mathbf{c}(t) = \mathbf{H}(t)\mathbf{c}(t) \qquad \mathbf{H}(t) = (\hbar/2)\Omega(t)\,\mathrm{e}^{-\mathrm{i}D(t)}|\psi_1\rangle\langle\psi_2| + \text{h.c.}$$

$D(t) = \int_0^t \Delta(t')\mathrm{d}t'$; detuning $\Delta = \omega_0 - \omega$; Rabi frequency $\Omega(t)$

evolution described by the SU(2) propagator $\mathbf{U}$: $\mathbf{c}(t) = \mathbf{U}(t, t_i)\mathbf{c}(t_i)$

$$\mathbf{U} = \begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$$ Cayley-Klein parameters a and b

transition probability $p = |b|^2 = 1 - |a|^2$

$\Delta = 0 \implies a = \cos(A/2),\ b = -\mathrm{i}\sin(A/2)$ $\qquad A = \int_{t_\mathrm{i}}^{t_\mathrm{f}} \Omega(t)\mathrm{d}t$ pulse area

constant phase shift ϕ in the field: $\Omega(t) \to \Omega(t)\,\mathrm{e}^{\mathrm{i}\phi}$

$$\mathbf{U}(\phi) = \begin{bmatrix} a & b\,\mathrm{e}^{\mathrm{i}\phi} \\ -b^*\,\mathrm{e}^{-\mathrm{i}\phi} & a^* \end{bmatrix}$$

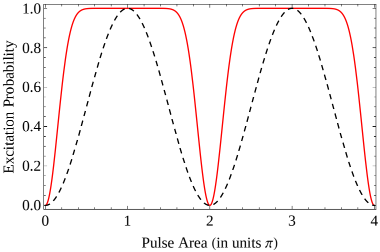
composite sequence of N phased pulses $\implies$ overall propagator

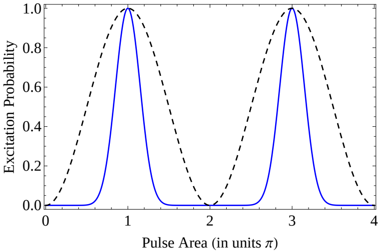
$$\mathbf{U}^{(N)} = \mathbf{U}(\phi_N)\mathbf{U}(\phi_{N-1})\cdots\mathbf{U}(\phi_2)\mathbf{U}(\phi_1)$$

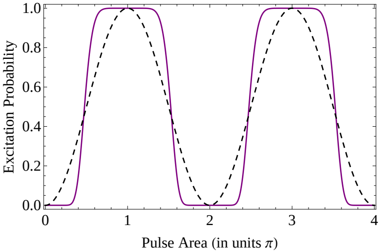
Torosov & NVV, PRA **83**, 053420 (2011)

BB, NB and PB composite pulses

broadband (BB), narrowband (NB), passband (PB)







BB: flat top around $A = \pi$

$$[\partial_A^k U_{11}^{(N)}]_{A=\pi} = 0 \quad (k = 0, 1, 2, \ldots)$$

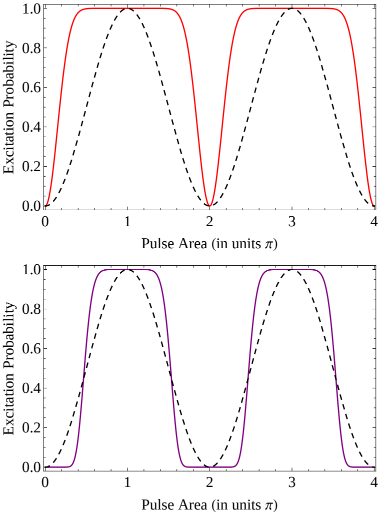
NB: flat bottom around $A = 0$ and 2π

$$[\partial_A^k U_{12}^{(N)}]_{A=0} = 0 \quad (k = 0, 1, 2, \ldots)$$

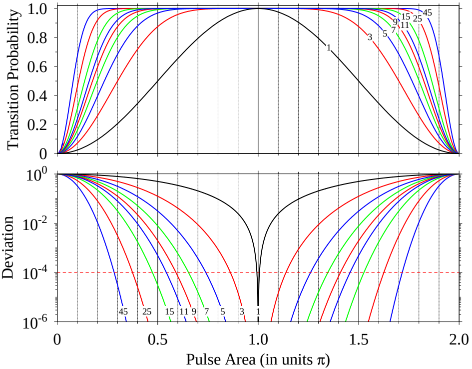
PB: flat top around $A = \pi$ and flat bottom around $A = 0$

BB, NB and PB composite pulses

broadband (BB), narrowband (NB), passband (PB), half-π



Broadband composite pulses



range wherein error $< 10^{-4}$

0.006π for 1 pulse

0.26π for 5 pulses

0.62π for 25 pulses

0.82π for 125 pulses

a sequence of $N = 2n+1$ identical pulses with anagram phases

$$(\phi_1, \phi_2, \cdots, \phi_n, \phi_{n+1}, \phi_n, \cdots, \phi_2, \phi_1)$$

$$\phi_k^{(N)} = \left(n + 1 - \left\lfloor \frac{k+1}{2} \right\rfloor\right) \left\lfloor \frac{k}{2} \right\rfloor \frac{2\pi}{N}$$

$$\left(0, \tfrac{2}{3}, 0\right)\pi$$
$$\left(0, \tfrac{4}{5}, \tfrac{2}{5}, \tfrac{4}{5}, 0\right)\pi$$
$$\left(0, \tfrac{6}{7}, \tfrac{4}{7}, \tfrac{8}{7}, \tfrac{4}{7}, \tfrac{6}{7}, 0\right)\pi$$
$$\left(0, \tfrac{8}{9}, \tfrac{6}{9}, \tfrac{12}{9}, \tfrac{8}{9}, \tfrac{12}{9}, \tfrac{6}{9}, \tfrac{8}{9}, 0\right)\pi$$
$$\cdots$$

$$p = 1 - \cos^{2N}(A/2)$$

transition probability

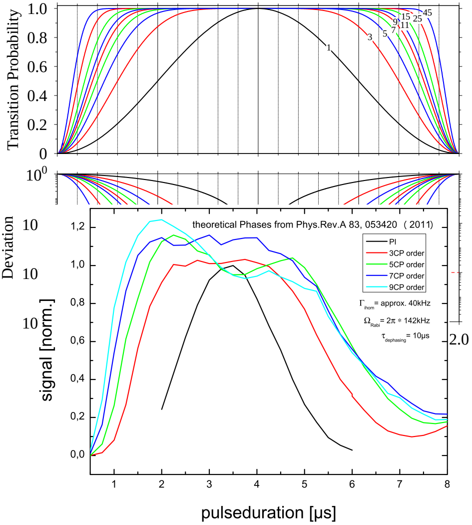
$p \to 1$ for large N if $A \neq 2m\pi$

optimize against variations in pulse area to arbitrary order

$A = \pi(1+\epsilon) \Longrightarrow p \sim 1 - (\pi\epsilon/2)^{2N}$

Torosov & NVV, PRA **83**, 053420 (2011)

Broadband composite pulses



AG Halfmann (Darmstadt); $Pr^{3+}: Y_2 \, Si \, O_5$

a sequence of $N = 2n + 1$ identical pulses with anagram phases
$(\phi_1, \phi_2, \cdots, \phi_n, \phi_{n+1}, \phi_n, \cdots, \phi_2, \phi_1)$

$$\phi_k^{(N)} = \left(n + 1 - \left\lfloor \frac{k+1}{2} \right\rfloor\right) \left\lfloor \frac{k}{2} \right\rfloor \frac{2\pi}{N}$$

$$(0, \tfrac{2}{3}, 0)\ \pi$$
$$(0, \tfrac{4}{5}, \tfrac{2}{5}, \tfrac{4}{5}, 0)\ \pi$$
$$(0, \tfrac{6}{7}, \tfrac{4}{7}, \tfrac{8}{7}, \tfrac{4}{7}, \tfrac{6}{7}, 0)\ \pi$$
$$(0, \tfrac{8}{9}, \tfrac{6}{9}, \tfrac{12}{9}, \tfrac{8}{9}, \tfrac{12}{9}, \tfrac{6}{9}, \tfrac{8}{9}, 0)\ \pi$$
$$\cdots$$

$$p = 1 - \cos^{2N}(A/2)$$

transition probability
$p \to 1$ for large N if $A \neq 2m\pi$
optimize against variations in pulse area to arbitrary order
$A = \pi(1 + \epsilon) \implies p \sim 1 - (\pi\epsilon/2)^{2N}$

Torosov & NVV, PRA **83**, 053420 (2011)

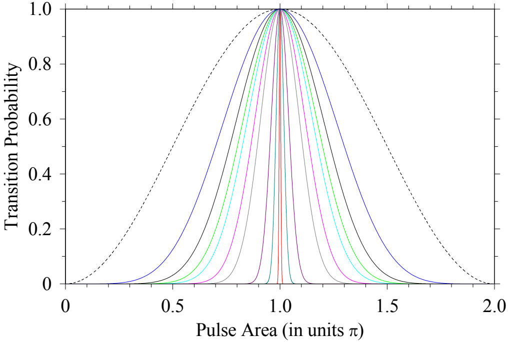
Narrowband composite pulses

a sequence of $N = 2n + 1$ identical pulses
with sign-alternating phases
$(\phi_1, \phi_2, -\phi_2, \phi_3, -\phi_3, \cdots, \phi_{n+1}, -\phi_{n+1})$

$$\phi_k^{(N)} = (-)^k \left\lfloor \frac{k}{2} \right\rfloor \frac{2\pi}{N}$$

$(k = 1, 2, \ldots, N)$

$$\left(0, \tfrac{2}{3}, -\tfrac{2}{3}\right)\pi$$
$$\left(0, \tfrac{2}{5}, -\tfrac{2}{5}, \tfrac{4}{5}, -\tfrac{4}{5}\right)\pi$$
$$\left(0, \tfrac{2}{7}, -\tfrac{2}{7}, \tfrac{4}{7}, -\tfrac{4}{7}, \tfrac{6}{7}, -\tfrac{6}{7}\right)\pi$$
$$\left(0, \tfrac{2}{9}, -\tfrac{2}{9}, \tfrac{4}{9}, -\tfrac{4}{9}, \tfrac{6}{9}, -\tfrac{6}{9}, \tfrac{8}{9}, -\tfrac{8}{9}\right)\pi$$
$$\cdots$$



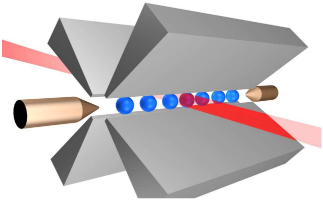
$$p = \sin^{2N}(A/2) = p^N$$

transition probability

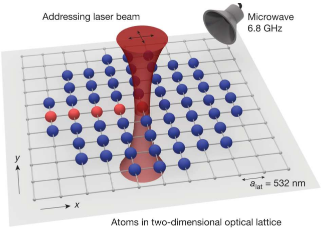
NVV, PRA **84**, 065404 (2011)

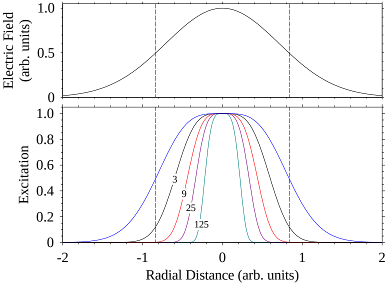
Local addressing

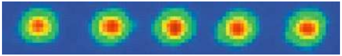
Blatt, PRL **102**, 040501 (2009): Toffoli gate



Bloch, Nature **471**, 319 (2011)







Excitation can be localized in any desired spatial range around the center.

⇓

We can "beat the diffraction limit"!!!

The physical reason is the destructive interference of the ingredient pulses in the composite sequence.

SS Ivanov & NVV, Opt. Lett. **36**, 1275 (2011)

nested NB $(\nu_1, \nu_2, \ldots, \nu_{2m+1})$
and BB $(\beta_1, \beta_2, \ldots, \beta_{2n+1})$

$$N_3 = (\nu_1, \nu_2, \nu_3) = (0, \tfrac{2}{3}, -\tfrac{2}{3})\,\pi$$
$$B_3 = (\beta_1, \beta_2, \beta_3) = (0, \tfrac{2}{3}, 0)\,\pi$$

$$\nu_k^{(N)} = (-)^k \left\lfloor \tfrac{k}{2} \right\rfloor \frac{2\pi}{2m+1}$$

$$P = \sin^{2N}(A/2) = p^N$$

$$\phi_k^{(N)} = \left(n + 1 - \left\lfloor \tfrac{k+1}{2} \right\rfloor\right) \left\lfloor \tfrac{k}{2} \right\rfloor \frac{2\pi}{N}$$

$$P = 1 - \cos^{2N}(A/2) = 1 - (1-p)^N$$

nested NB into BB

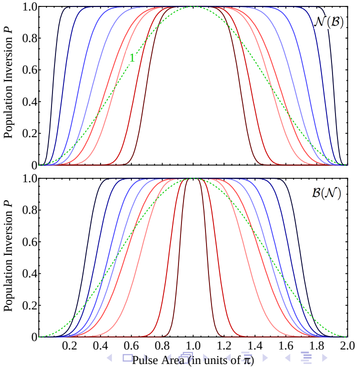
$$B_3(N_3) = (\beta_1 + \nu_1, \beta_1 + \nu_2, \beta_1 + \nu_3, \beta_2 + \nu_3, \beta_2 + \nu_2, \beta_2 + \nu_1, \beta_3 + \nu_1, \beta_3 + \nu_2, \beta_3 + \nu_3)$$
$$B_3(N_3) = (0, \tfrac{2}{3}, \tfrac{4}{3}, 0, \tfrac{4}{3}, \tfrac{2}{3}, 0, \tfrac{2}{3}, \tfrac{4}{3})\pi$$

nested BB into NB

$$N_3(B_3) = (\beta_1 + \nu_1, \beta_2 + \nu_1, \beta_3 + \nu_1, \beta_1 + \nu_2, \beta_2 + \nu_2, \beta_3 + \nu_2, \beta_1 + \nu_3, \beta_2 + \nu_3, \beta_3 + \nu_3)$$
$$N_3(B_3) = (0, \tfrac{2}{3}, 0, \tfrac{2}{3}, \tfrac{4}{3}, \tfrac{2}{3}, \tfrac{4}{3}, 0, \tfrac{4}{3})\pi$$

$$P_{N(B)} = [1 - (1-p)^{N_b}]^{N_n}$$

$$P_{B(N)} = 1 - (1 - p^{N_n})^{N_b}$$



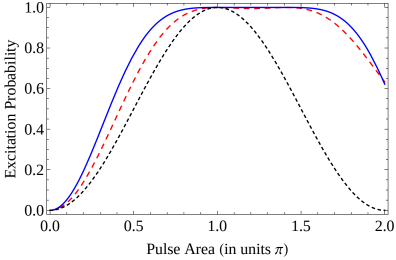
E. Kyoseva & NVV, PRA, Dec 2013

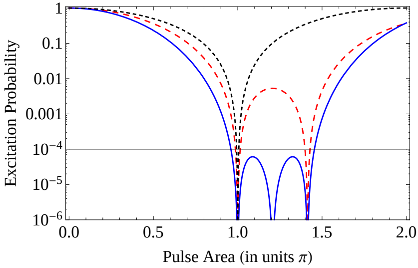
Multi-point composite pulses

$P = 1$ at areas $\pi, \pi\sqrt{2}$

$[(A/\sqrt{8})_0, A_{\pi/2}, (A/\sqrt{8})_0]$

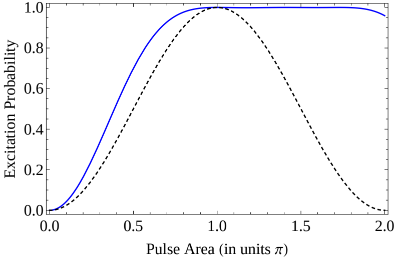
$[(0.83A)_0, (0.83A)_{0.65\pi}, (0.83A)_0]$

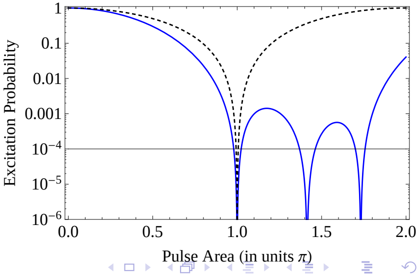




$P = 1$ at areas $\pi, \pi\sqrt{2}, \pi\sqrt{3}$

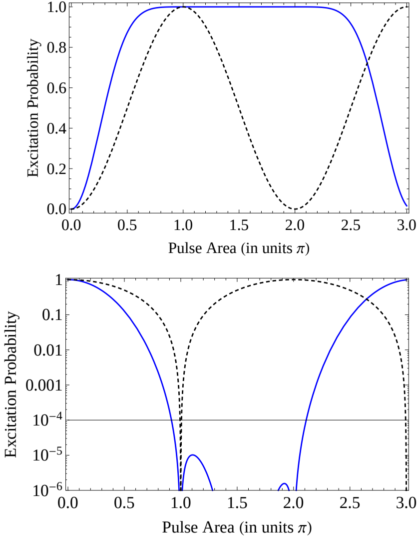
$[(0.72A)_0, (0.72A)_{0.63\pi}, (0.72A)_0]$



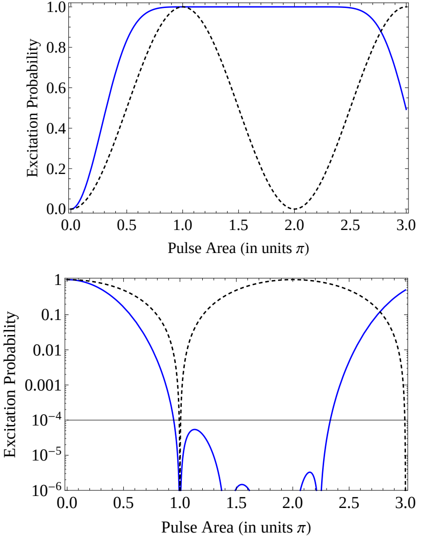


Multi-point composite pulses

$P = 1$ at areas $\pi, \pi\sqrt{2}, \pi\sqrt{3}, \pi\sqrt{4}$



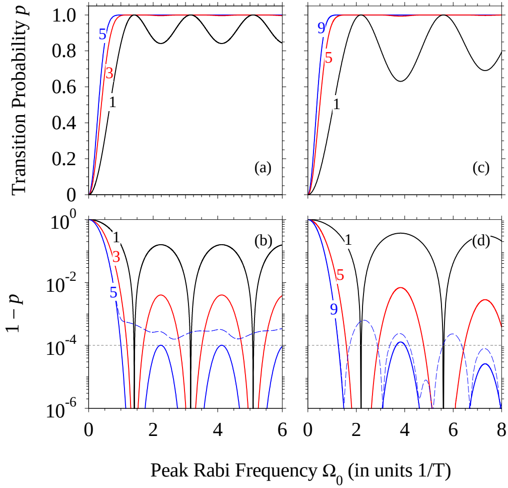
$P = 1$ at areas $\pi, \pi\sqrt{2}, \pi\sqrt{3}, \pi\sqrt{4}, \pi\sqrt{5}$



may be useful for sideband cooling

Composite adiabatic passage (level crossing)

Torosov, Guerin & NVV, PRL **106**, 233001 (2011)



$$\phi_k^{(N)} = \left(n + 1 - \left\lfloor \frac{k+1}{2} \right\rfloor\right) \left\lfloor \frac{k}{2} \right\rfloor \frac{2\pi}{N}$$

these "magic phases" optimize AP against variations in the pulse area and the chirp rate to arbitrary order

$$p = 1 - a^{2N}$$

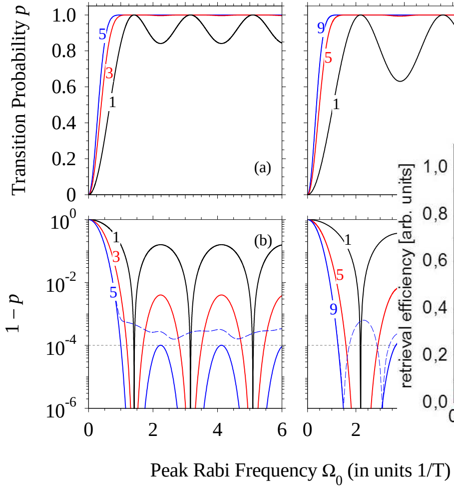
transition probability

$\Longrightarrow p \to 1$ for large N $(a \neq \pm 1)$

$(0, 2, 0)\, \pi/3$
$(0, 4, 2, 4, 0)\, \pi/5$
$(0, 6, 4, 8, 4, 6, 0)\, \pi/7$
$(0, 8, 6, 12, 8, 12, 6, 8, 0)\, \pi/9$
$\cdots$

Composite adiabatic passage (level crossing)

Torosov, Guerin & NVV, PRL **106**, 233001 (2011)



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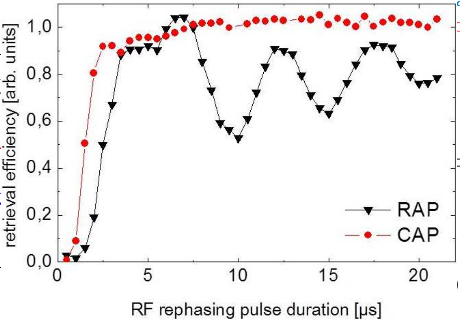
these "magic phases" optimize AP against variations in the pulse area ... order



experiment in $Pr^{3+}: Y_2 Si O_5$

Schraft, Halfmann, Genov & NVV, PRA **88**, 063406 (2013)

Universal composite pulses

$$\mathbf{U}(\phi) = \begin{bmatrix} a & b\,e^{i\phi} \\ -b^*\,e^{-i\phi} & a^* \end{bmatrix}$$

$$\mathbf{U}^{(N)} = \mathbf{U}(\phi_N)\mathbf{U}(\phi_{N-1})\cdots\mathbf{U}(\phi_2)\mathbf{U}(\phi_1)$$

eliminate the coefficients of the lowest powers of a in $U^{(N)}$

$\Longrightarrow$ a set of equations for the phases

the phases do not depend on the interaction parameters!

$$p = 1 - |a|^m \to 1$$

for sufficiently large $m(N)$ for any field!

5 pulses:

$(0, 5, 2, 5, 0)\pi/6$

$(0, -1, 2, -1, 0)\pi/6$

13 pulses:

$(0, 9, 42, 11, 8, 37, 2, 37, 8, 11, 42, 9, 0)\pi/24$

$(0, 15, 6, 13, 40, 35, 46, 35, 40, 13, 6, 15, 0)\pi/24$

Genov, Schraft, NVV & Halfmann, in preparation

Universal composite pulses

$$\mathbf{U}(\phi) = \begin{bmatrix} a & b\,e^{i\phi} \\ -b^*\,e^{-i\phi} & a^* \end{bmatrix}$$

$$\mathbf{U}^{(N)} = \mathbf{U}(\phi_N)\mathbf{U}(\phi_{N-1})\cdots\mathbf{U}(\phi_2)\mathbf{U}(\phi_1)$$

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5 pulses:

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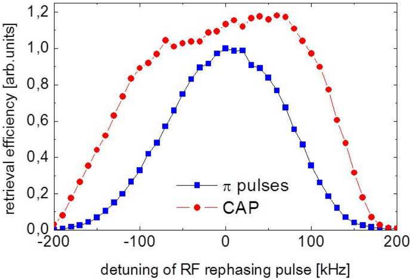
$(0, -1, 2, -1, 0)\pi/6$

13 pulses:

$(0, 9, 42, 11, 8, 37, 2, 37, 8, 11, 42, 9, 0)\pi/24$

$(0, 15, 6, 13, 40, 35, 46, 35, 40, 13, 6, 15, 0)\pi/24$

Genov, Schraft, NVV & Halfmann, in preparation



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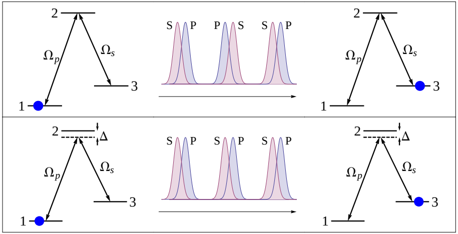
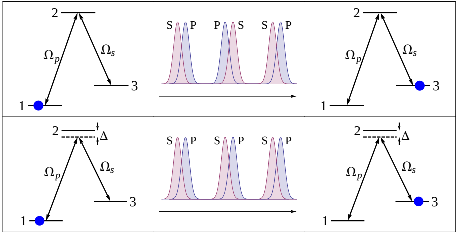
Composite STIRAP

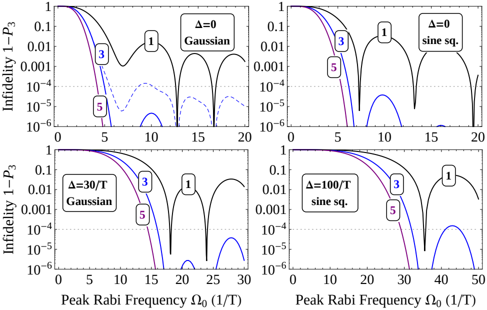


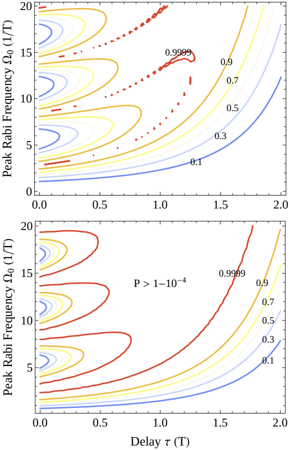
Composite STIRAP



$(0, 1; 3, 3; 1, 0)\pi/3$

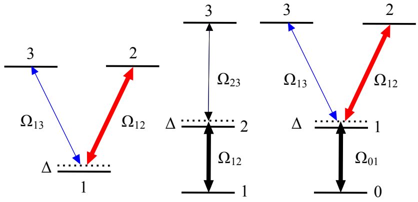
$(0, 4; 5, 8; 3, 3; 8, 5; 4, 0)\pi/5$

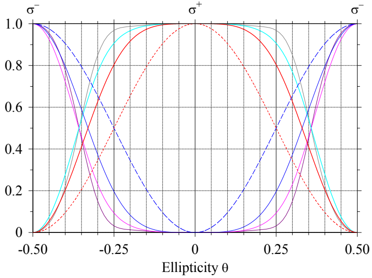


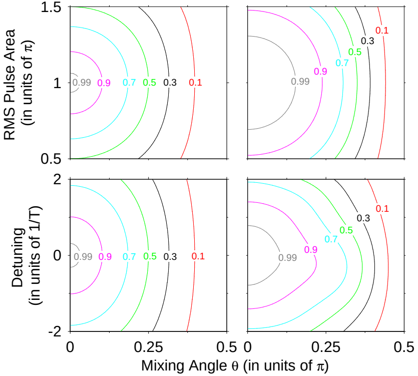


Torosov& NVV, PRA **87**, 043418 (2013)

Suppression of Unwanted Transitions







GT Genov & NVV, PRL **110**, 133002 (2013)

Outline

- Two-state systems
  - Broadband, narrowband, passband, fractional-π pulses
  - Local addressing (subwavelength localization)
  - Composite adiabatic passage
- Multistate systems
  - Composite STIRAP
  - Suppression of unwanted transitions
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  - Dicke and NooN states
  - Toffoli gate and C^n-NOT gates
  - C-phase gate
- Lineshapes
  - smooth pulses of finite duration
  - power narrowing

Entangled Dicke and NooN states

New Journal of Physics
The open access journal for physics

Creation of arbitrary Dicke and NOON states of trapped-ion qubits by global addressing with composite pulses

Svetoslav S Ivanov[1,2,3], **Nikolay V Vitanov**[2] **and Natalia V Korolkova**[1]

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New Journal of Physics **15** (2013) 023039 (11pp)

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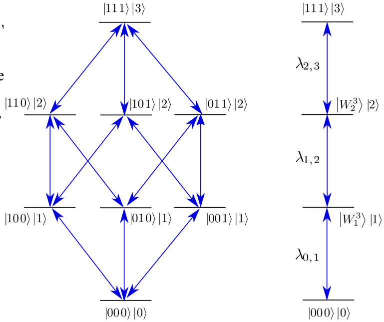
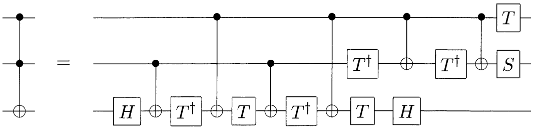


Toffoli gate (C^2-NOT)

$\vert cct\rangle \to \vert cct\rangle$
$\vert 000\rangle \to \vert 000\rangle$
$\vert 001\rangle \to \vert 001\rangle$
$\vert 010\rangle \to \vert 010\rangle$
$\vert 011\rangle \to \vert 011\rangle$
$\vert 100\rangle \to \vert 100\rangle$
$\vert 101\rangle \to \vert 101\rangle$
$\vert 110\rangle \to \vert 111\rangle$
$\vert 111\rangle \to \vert 110\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

- 6 CNOT gates by the circuit model



- 5 CNOT gates with an ancilla state   Lanyon *et al.*, Nature Phys. **5**, 134 (2009)

Beyond the circuit model

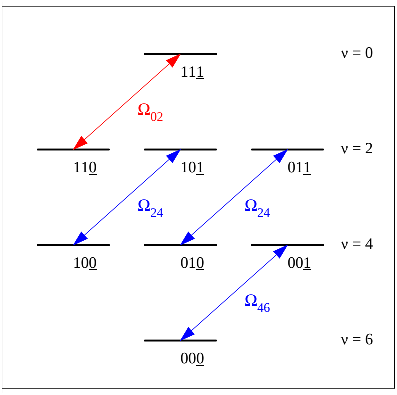
Blatt's group: trapped ions, 15 laser pulses, 71% fidelity, LD regime

T. Monz *et al.*, Phys. Rev. Lett. **102**, 040501 (2009)

Toffoli gate (C^2-NOT)

$|110\rangle \to |111\rangle \quad |111\rangle \to |110\rangle$

SS Ivanov & NVV, Phys. Rev. A **84**, 022319 (2011)



for the second-sideband transition

$$\Omega_{\nu,\nu+2} = \Omega e^{-\eta^2/2}\eta^2 \chi_{\nu,\nu+2}$$

$$\chi_{\nu,\nu+2} = \frac{L_\nu^2(\eta^2)}{\sqrt{(\nu+1)(\nu+2)}}$$

$$\chi_{02} = \frac{1}{\sqrt{2}}$$

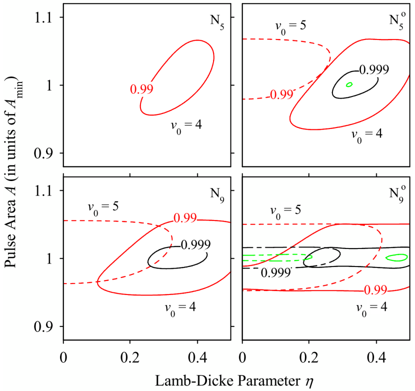
$$\chi_{24} = \frac{12-8\eta^2+\eta^4}{4\sqrt{3}}$$

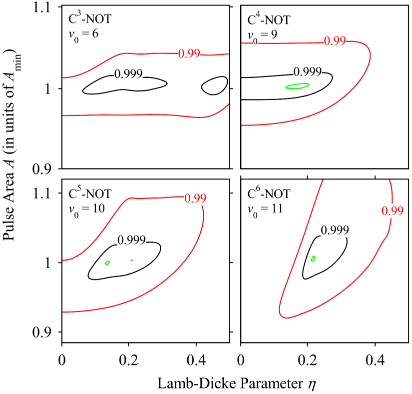
$$\chi_{46} = \frac{360-480\eta^2+180\eta^4-24\eta^6+\eta^8}{24\sqrt{30}}$$

Toffoli gate: Ω_{02} is a π-pulse
Ω_{24} and Ω_{46} are even-π-pulses

C. Monroe et al., PRA **55**, R2489 (1997); B. DeMarco et al., PRL **89**, 267901 (2002)

Toffoli gate (C^2-NOT) and C^n-NOT gates





SS Ivanov & NVV, Phys. Rev. A **84**, 022319 (2011)

C-phase gate with overlapping pulses

bichromatic field (first red and blue sidebands)

$H(t) = \sum_{k=1}^{N} g_k(t)\sigma(\phi_k^+)(a^\dagger e^{-i\phi_k^-} + a e^{i\phi_k^-})$

control-phase gate

$U = D(\alpha)G(J_{k,l})$ $\qquad$ $U = \exp\left(i\frac{\pi}{4}\sigma_1^x\sigma_2^x\right)$

$\alpha(t,t_i) = -i\sum_{k=1}^{N} A_k(t,t_i)e^{-i\phi_k}\sigma_k^x$ $\qquad$ $A_k(t,t_i) = \int_{t_i}^{t} dt_1 g_k(t_1)$

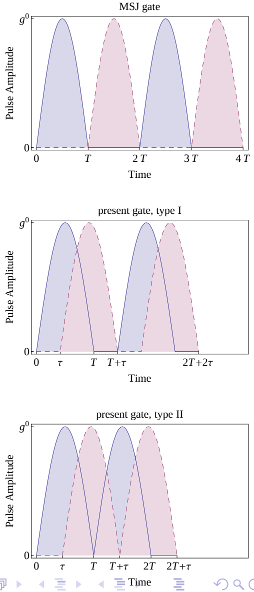
$G(J_{k,l}) = \exp\left[i\sum_{k<l}^{N} J_{k,l}\sigma_k^x\sigma_l^x\right]$

$J_{k,l}(t,t_i) = \sin(\phi_k - \phi_l)\int_{t_i}^{t} dt_1 \int_{t_i}^{t_1} dt_2 [g_k(t_2)g_l(t_1) - g_k(t_1)g_l(t_2)]$

Sorensen-Molmer: off-resonant, sequential pulses

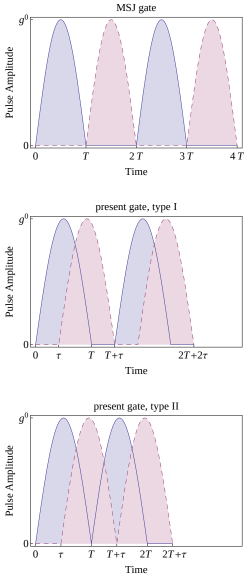
Milburn-Schneider-James: resonant, sequential pulses

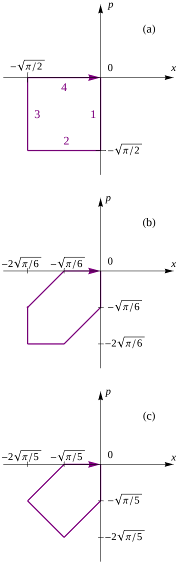
Simeonov *et al*: resonant, overlapping pulses
equal areas; phases $0, \frac{\pi}{2}, \pi, -\frac{\pi}{2}$



Sorensen & Molmer, PRA **62**, 022311 (2000)
Milburn, Schneider & James, FP **48**, 801 (2000)
Simeonov, Ivanov & NVV, PRA (2014)

C-phase gate with overlapping pulses





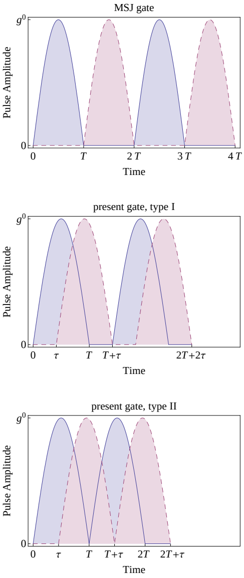
pulse shape	**MSJ** T	**present, type I** τ/T	T	speed-up
rect	0.627	0.500	0.724	15.5%
sin	0.984	0.413	1.094	27.3%
$\sin^2$	1.253	0.362	1.370	34.4%
$\sin^3$	1.477	0.327	1.597	39.3%
$\sin^4$	1.671	0.303	1.794	43.0%
$\sin^5$	1.846	0.284	1.971	45.9%
$\sin^6$	2.005	0.268	2.131	48.4%

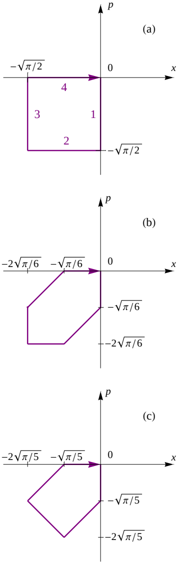
pulse shape	**MSJ** T	**present, type II** τ/T	T	speed-up
rect	0.627	0.500	0.793	26.5%
sin	0.984	0.454	1.091	47.1%
$\sin^2$	1.253	0.424	1.327	55.8%
$\sin^3$	1.477	0.393	1.539	60.4%
$\sin^4$	1.671	0.365	1.731	63.3%
$\sin^5$	1.846	0.341	1.905	65.5%
$\sin^6$	2.005	0.321	2.065	67.3%

gate	duration
$4T$	for MSJ
$2T+2\tau$	for type I
$2T+\tau$	for type II

L Simeonov, P Ivanov & NVV, PRA 2014

C-phase gate with overlapping pulses





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$4T$	for M
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L Simeonov, P Ivanov & NVV, PRA 2014

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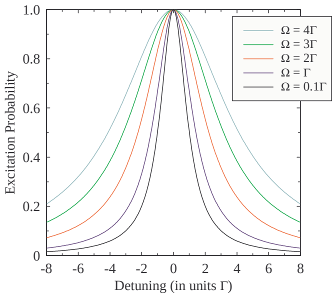
Lineshapes in Coherent Excitation



$$P \propto \frac{1}{\Delta^2 + \Gamma^2/4 + \Omega^2/4}$$

CW laser excitation:
power broadening is a paradigm!

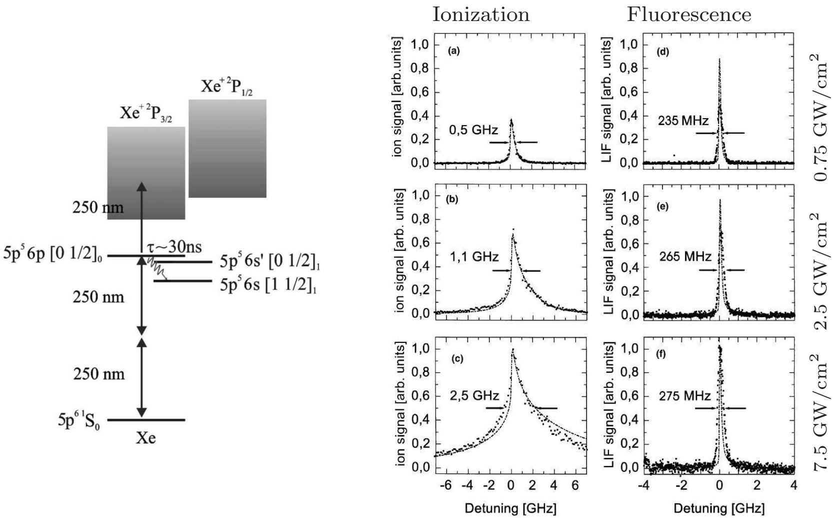
Pulsed excitation:
extent of power broadening depends on how the signal is collected!

If collected **during** excitation:
→ typical power broadening

If collected **after** excitation:
strong dependence on the **pulse shape** due to coherent adiabatic population return!

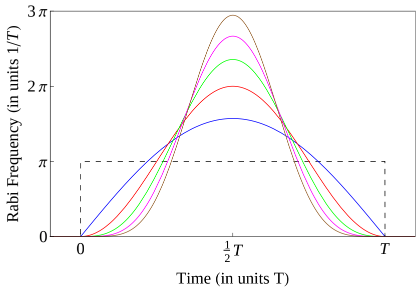
Shape	Duration	Smooth	Linewidth	Broadening
rectangular	finite	no	$\Delta \propto \Omega$	linear
$\sin^n$	**finite**	partly	$\Delta \propto \Omega^{1/(n+1)}$	root
gaussian	infinite	yes	$\Delta \propto \ln \Omega$	logarithmic
sech	infinite	yes	$\Delta \propto$ const	no
lorentzian^n	infinite	yes	$\Delta \propto \Omega^{-1/(2n-1)}$	NARROWING!

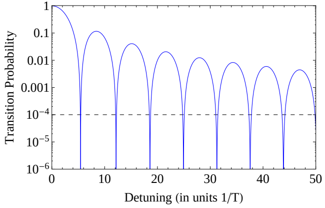
Power *non*-broadening in pulsed excitation

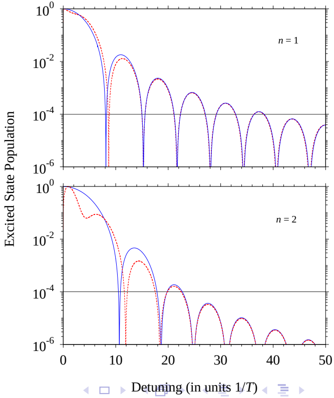


Halfmann, Rickes, NVV & Bergmann, Opt. Commun. **220**, 353 (2003)

Smooth pulses of finite duration







$$f(t) = \sin^n(\pi t/T) \quad (0 \leqq t \leqq T)$$

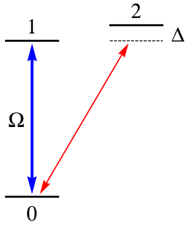
non-analytic!

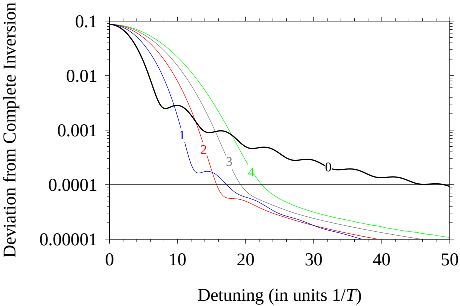
$$P = \frac{(n!\pi^n\Omega)^2}{(n!\pi^n\Omega)^2 + (\Delta^{n+1}T^n)^2}\sin^2\eta_{n+1}$$

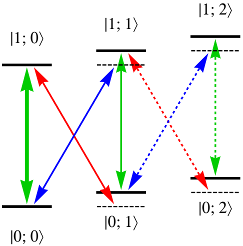
$$\Delta_{\frac{1}{2}} = \left(\frac{n!\pi^n\Omega}{T^n}\right)^{\frac{1}{n+1}}$$

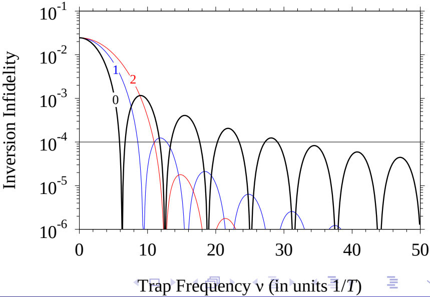
Boradjiev & NVV, PRA **88**, 013402 (2013)

Smooth pulses of finite duration

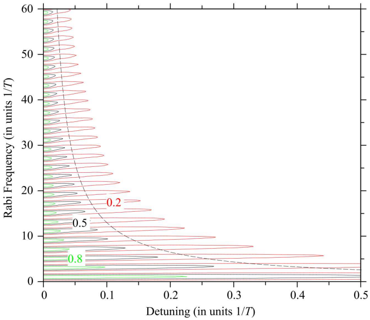








Boradjiev & NVV, PRA **88**, 013402 (2013)



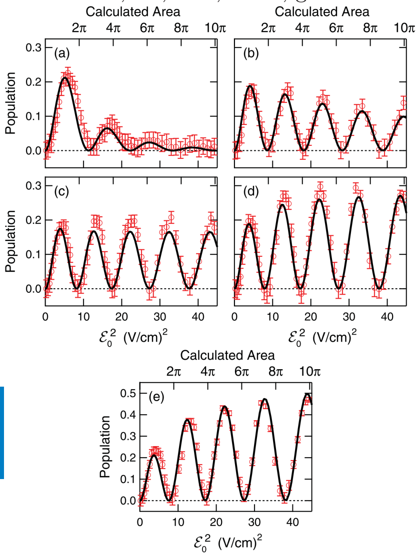
$$f(t) = \frac{1}{[1+(t/T)^2]^n}$$

$$\Delta_{\frac{1}{2}} T = \left[\frac{(2n+1)^{n+\frac{1}{2}}(2n-1)^{2n-\frac{1}{2}}}{(4n)^n \Omega T} \right]^{\frac{1}{2n-1}}$$

from adiabatic condition

Boradjiev & NVV, Opt. Commun. **288**, 91 (2013)

lz, lz^2, sech, sech^2, gs



Raman-driven fine-structure doublet in Na Rydberg atoms by microwave pulses

Conover, PRA **84**, 063416 (2011)

CAMEL 10

Control of Atoms, Molecules and Ensembles by Light

Nessebar, Black Sea coast, 23-27 June 2014

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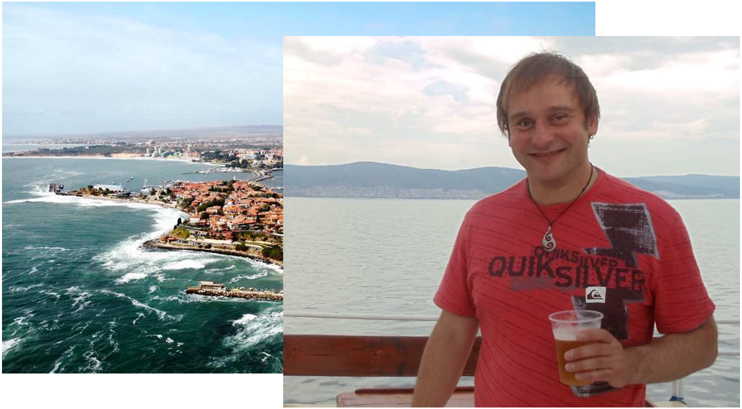

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