Lieb-Robinson bounds & nonequilibrium dynamics in trapped ions

Phys. Rev. Lett. ***111****, 230404 (2013)*

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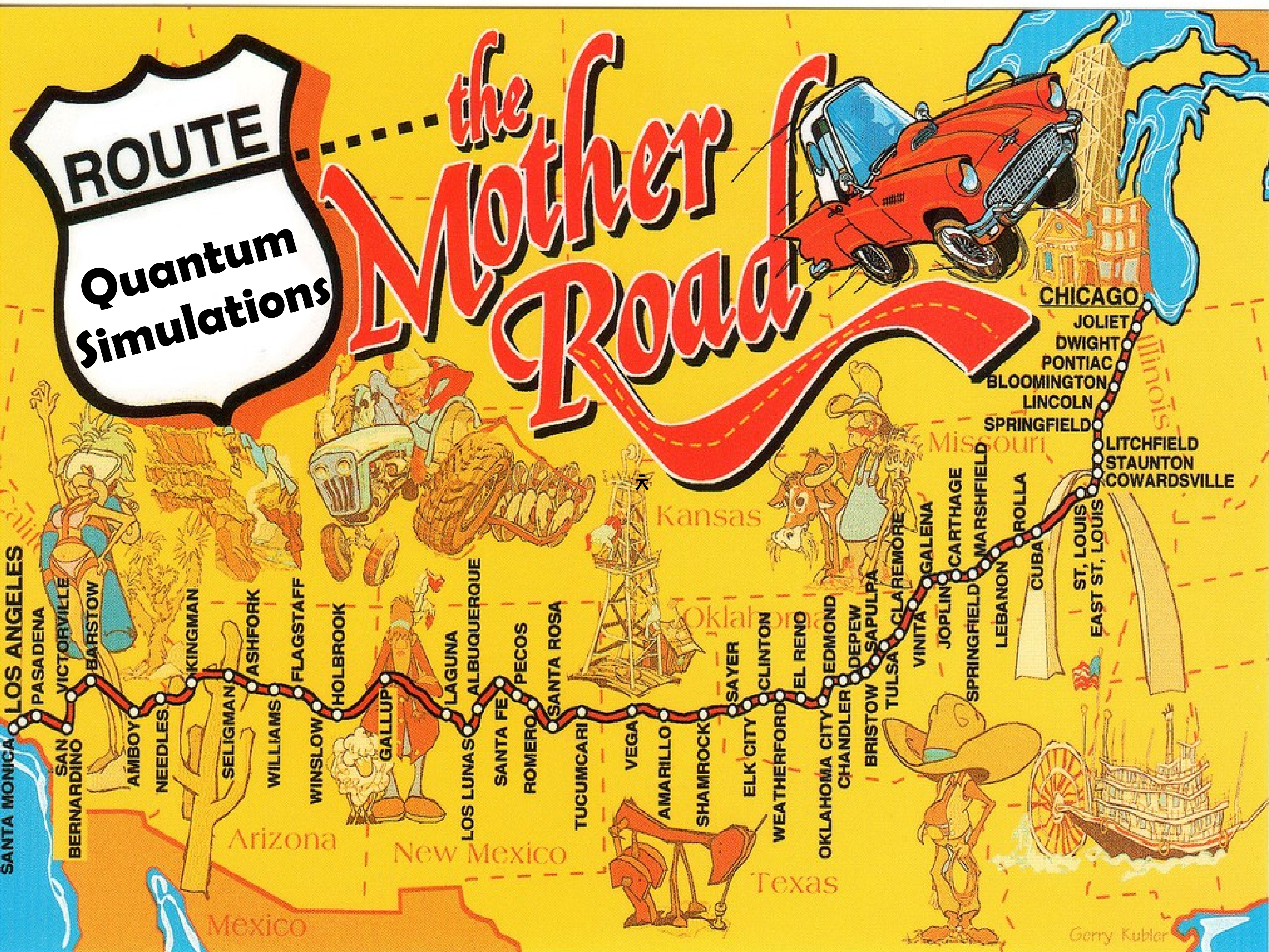
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Gerry Kubler

To-do

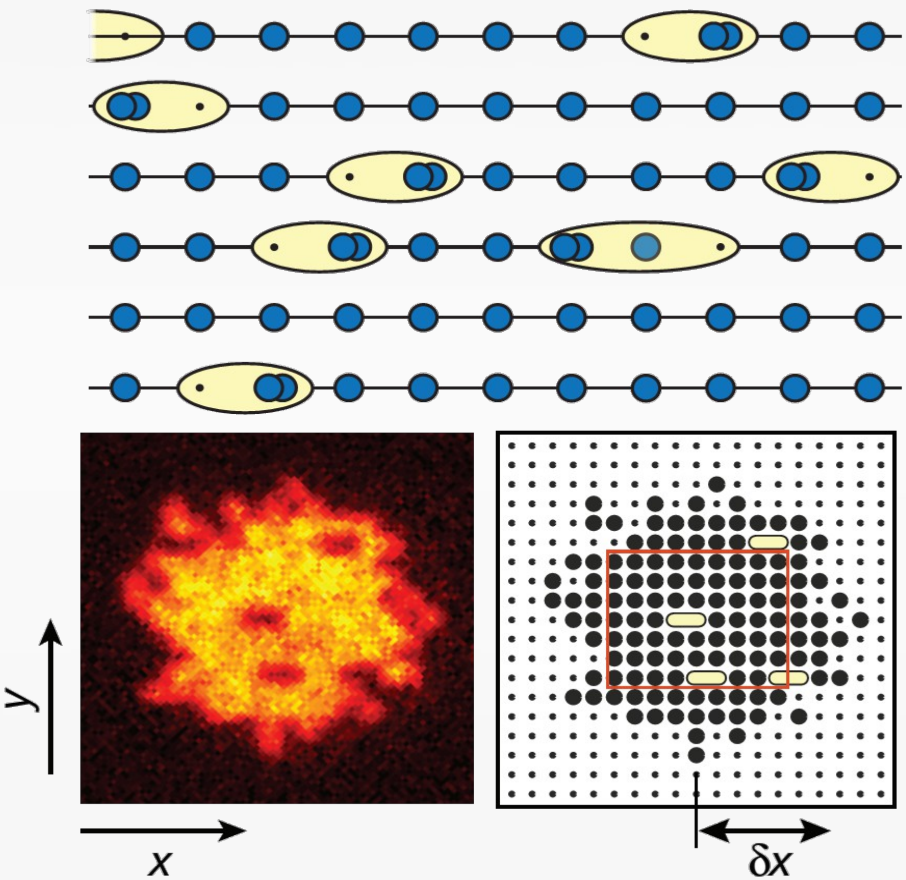
- ☑ Experimental tools
- ☑ Theoretical tools
- ☑ Fundamental questions
- ☑ Concrete applications

To-do

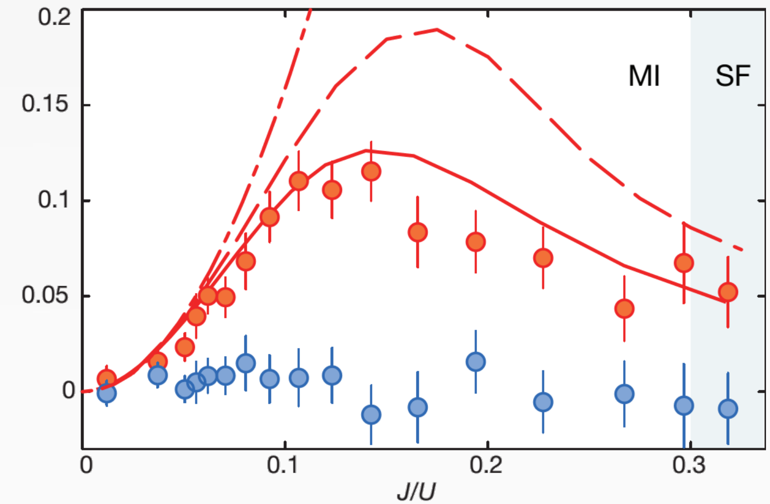
- [x] Experimental tools
- [x] Theoretical tools
- [x] Fundamental questi
- [x] Concrete applica

Learning by example

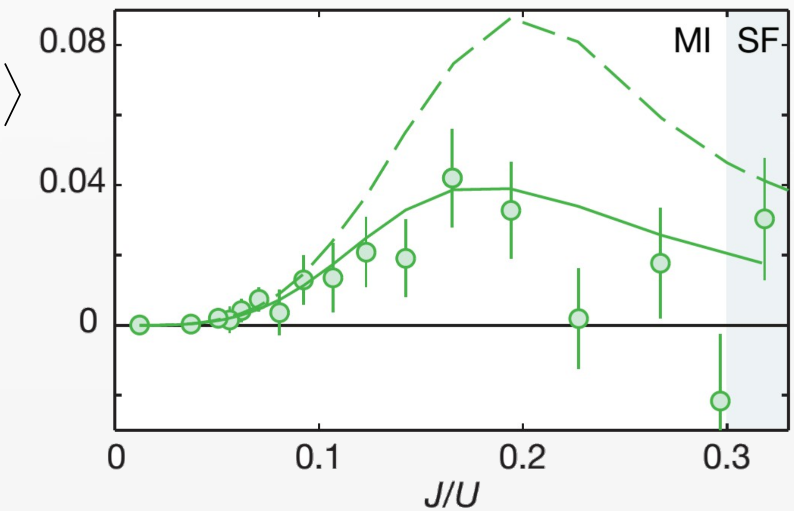
Static correlations & order



$$\langle S_i^z S_{i+1}^z \rangle$$

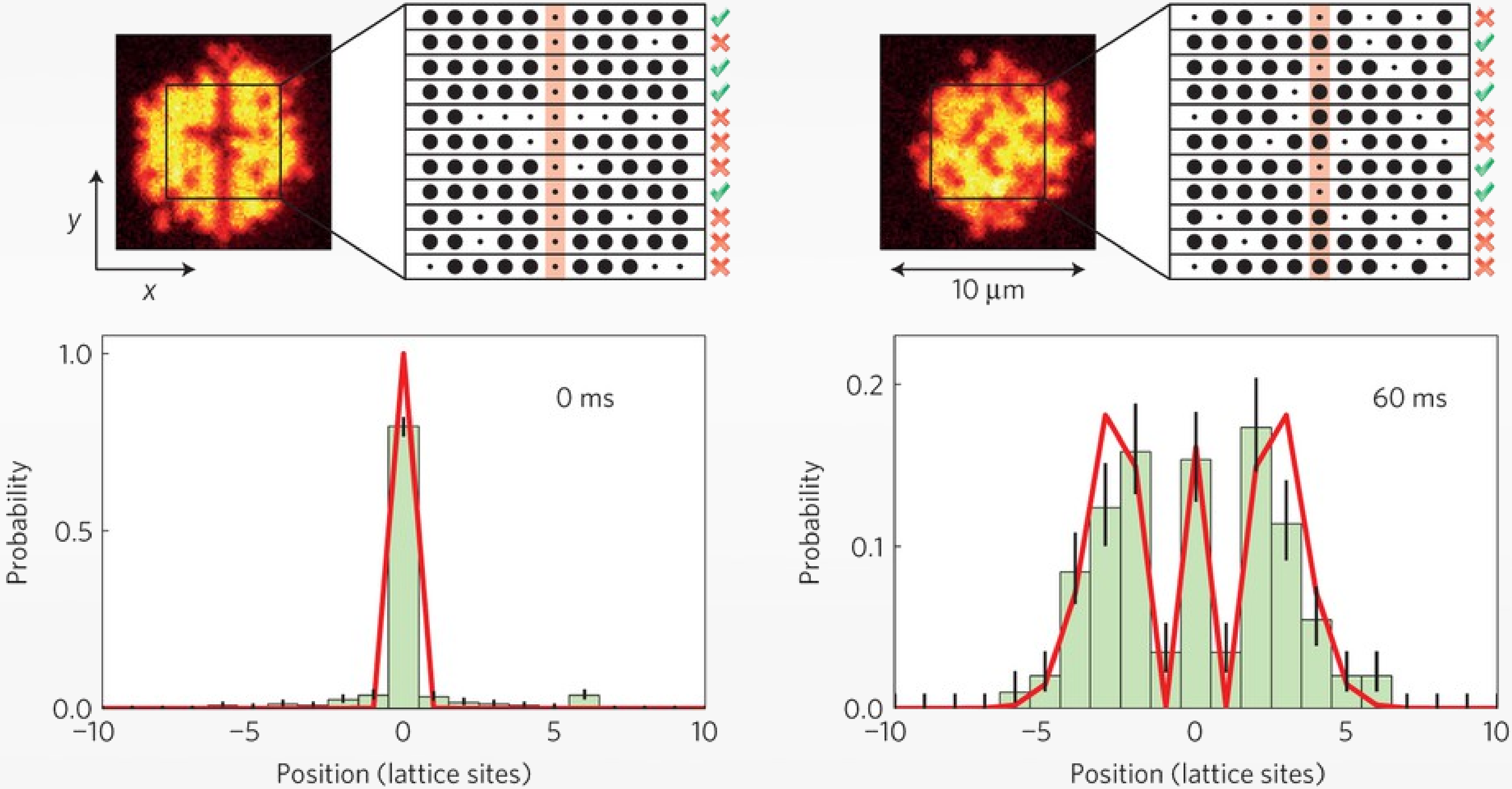


$$\langle S_1^z S_2^z S_3^z \rangle$$



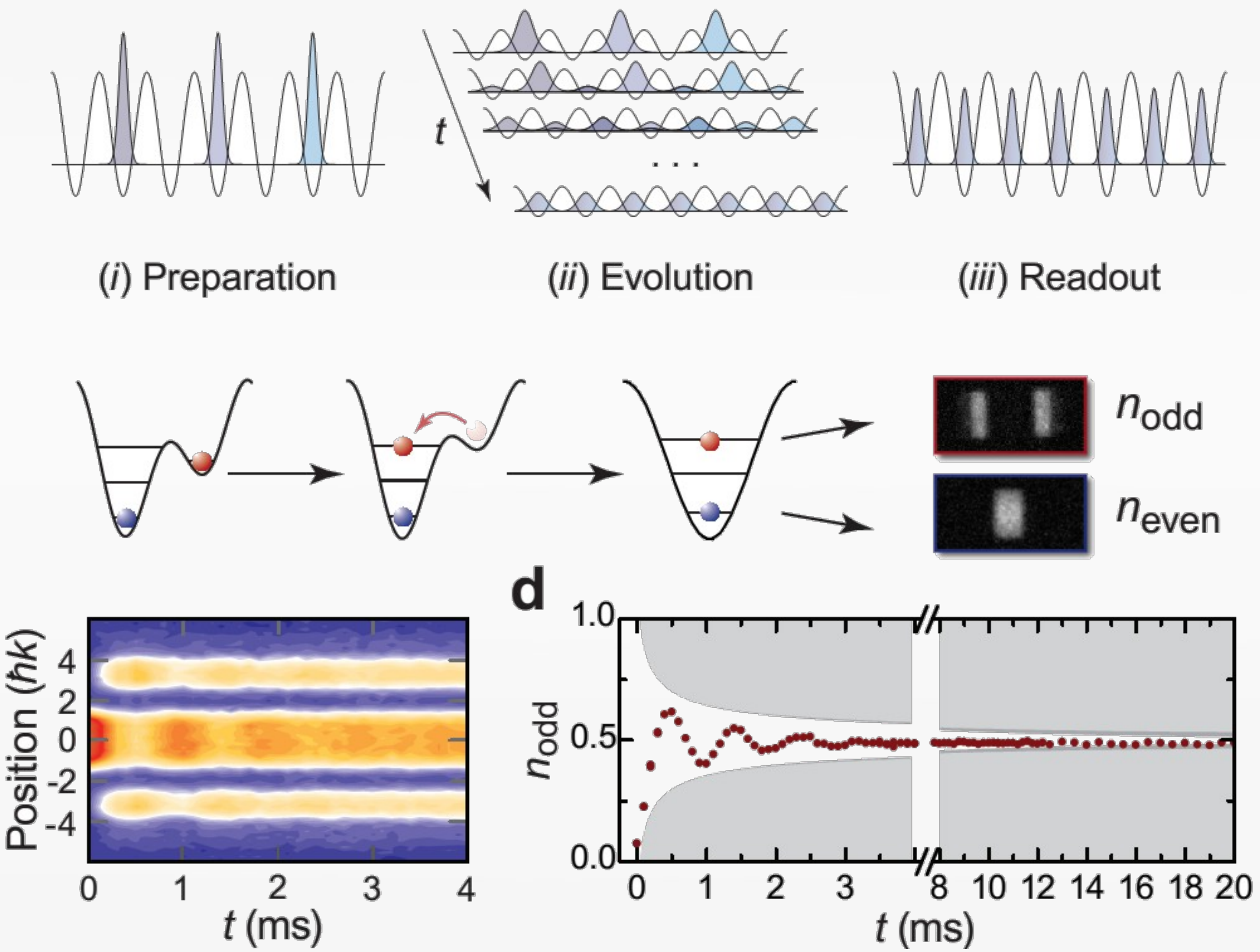
M. Endres et al., Science ***334****, 200-203 (2011)*

Quasiparticle dynamics



T. Fukuhara, Nature Physics ***9****, 235–241 (2013)*

Equilibration



S. Trotzky et al, Nature Physics ***8****, 325–330 (2012)*

CMP Problems

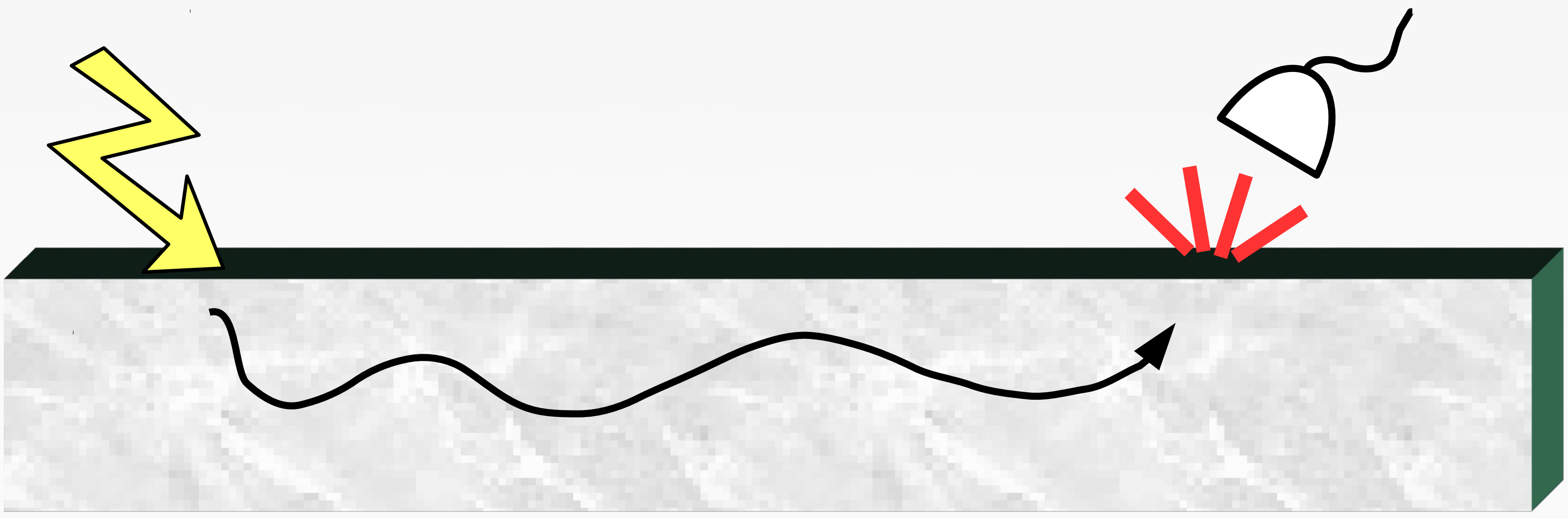
Equilibrium

- Correlations
- Entanglement
- Topological order
- Phase transitions

Nonequilibrium

- Energy gaps
- Excitations / quasiparticles
- Equilibration / thermalization
- Correlation propagation
- Causality

Linear response

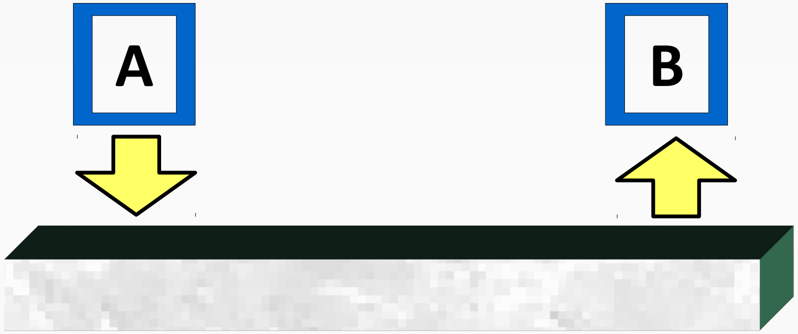


$$|\psi_0\rangle \rightarrow \left(1 - i\frac{\varepsilon}{\hbar}A(0)\right)|\psi_0\rangle + O(\varepsilon^2)$$

$$\langle B(t)\rangle \rightarrow \langle B(t)\rangle + \frac{\varepsilon}{\hbar}\langle [B(t), A(0)]\rangle + O(\varepsilon^2)$$

Entanglement

Mutual influence

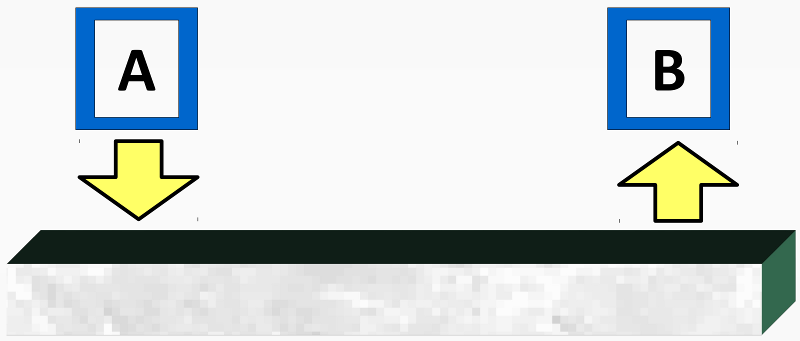


Vacuum fluctuations

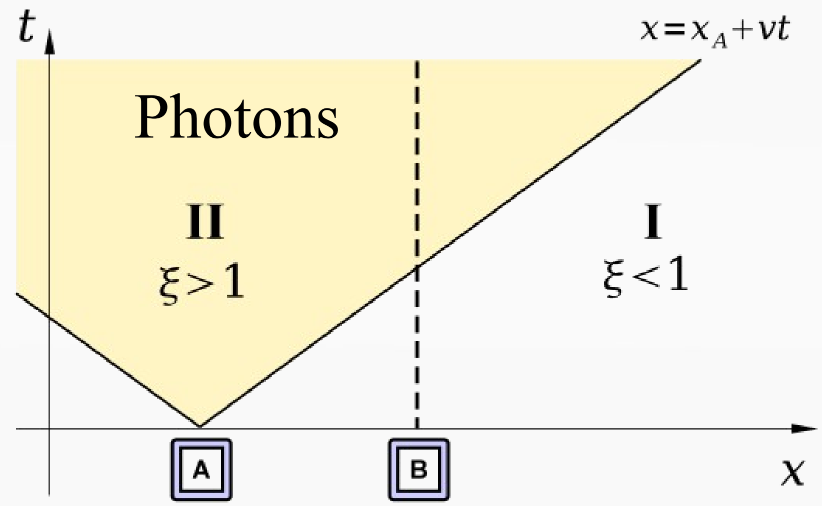
A Retzker, JI Cirac, B Reznik,
Phys. Rev. Lett. ***94****, 050504 (2005)*

Entanglement

Mutual influence



Causality



$$\|[A(x,t), B(0,0)]\| = 0$$
$$|x| > v|t|$$

C. Sabin, M. del Rey, JJGR, J. Leon
Phys. Rev. Lett 107, 150402 (2011)

Effective causality

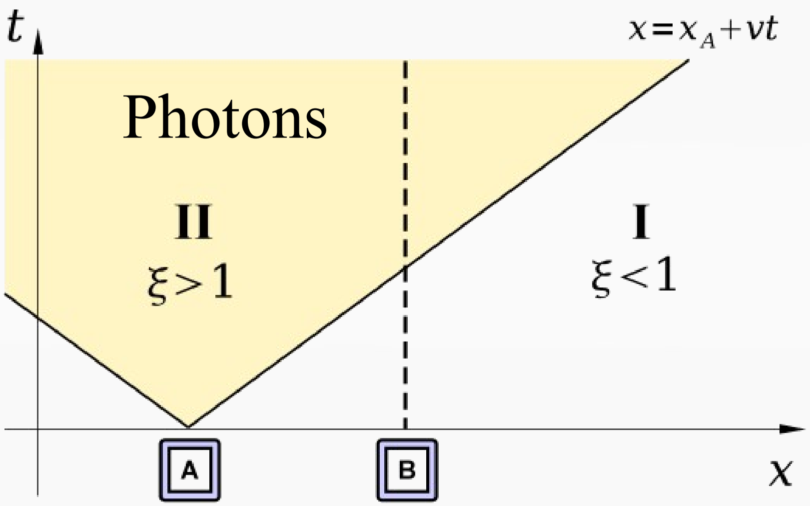
Approximate

Non-relativistic,many-body system

$$H = \sum_{\langle i,j \rangle} \sigma_i \sigma_j$$

$$\|[A(x,t), B(0,0)]\| \leq c \exp(\mu(v) t)$$
$$\forall |x| > v|t|$$

Causality

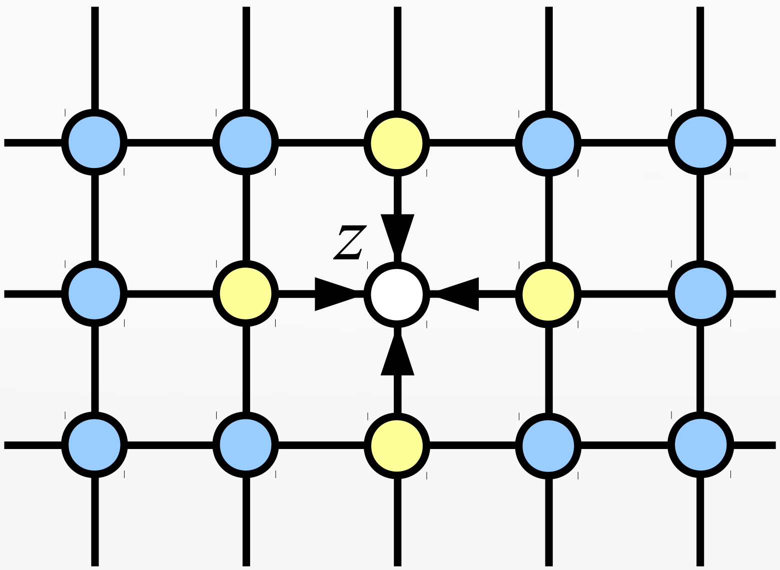


$$\|[A(x,t), B(0,0)]\| = 0$$
$$|x| > v|t|$$

Spin models

Hopping of particles:

$$H = \sum_{\langle i\,,\,j\rangle} J_{ij}\left(\sigma_i^{+}\sigma_j - + \sigma_i^{-}\sigma_j^{+}\right)$$



Hopping from the **z** nearest neighbors is slower than

$$z \times max_{\langle i\,,\,j\rangle}|J|$$

Thus the group velocity is smaller than that.

M. W. Hastings, "Locality in Quantum Systems", Les Houches Notes
E. H. Lieb, D. W. Robinson, Commun. Math. Phys. 28, 251-257 (1972)

Basic idea

Starting point

$$\frac{\mathrm{d}}{\mathrm{dt}}[B_X(t), A_Y] = [[B_X(t), H], A_Y]$$

with a local model

$$H = \sum_{\langle x, y \rangle} H_{xy}$$

M. W. Hastings, "Locality in Quantum Systems", Les Houches Notes
E. H. Lieb, D. W. Robinson, Commun. Math. Phys. 28, 251-257 (1972)

Basic idea

Starting point

$$\frac{\mathrm{d}}{\mathrm{dt}}[B_X(t), A_Y] = [[B_X(t), H], A_Y]$$

Quantity to bound

$$C(t, X, Y) = \|[B_X(t), A_Y(t)]\|$$

Recurrence relation

$$C(t, X, Y) \leq \sum_{\langle Z, X \rangle} \|H_{XZ}\| \|B_X\| \int C(t, Z, Y)\,\mathrm{dt}$$

M. W. Hastings, "Locality in Quantum Systems", Les Houches Notes
E. H. Lieb, D. W. Robinson, Commun. Math. Phys. 28, 251-257 (1972)

Basic idea

Starting point

$$\frac{\mathrm{d}}{\mathrm{dt}}\left[B_X(t), A_Y\right]$$

Quantity to bound

$$C(t, X, Y$$

Bounded Hamiltonians

Recurrence relation

$$C(t, X, Y) \leq \sum_{\langle Z, X\rangle} \|H_{XZ}\| \|B_X\| \int C(t, Z, Y)\,\mathrm{dt}$$

M. W. Hastings, "Locality in Quantum Systems", Les Houches Notes
E. H. Lieb, D. W. Robinson, Commun. Math. Phys. 28, 251-257 (1972)

Beyond here

What has been done

- Harmonic oscillators
  - Bounded nonlinearity
  - Generalized oscillators
- Dissipative models
- Bounds on selected observables

What for

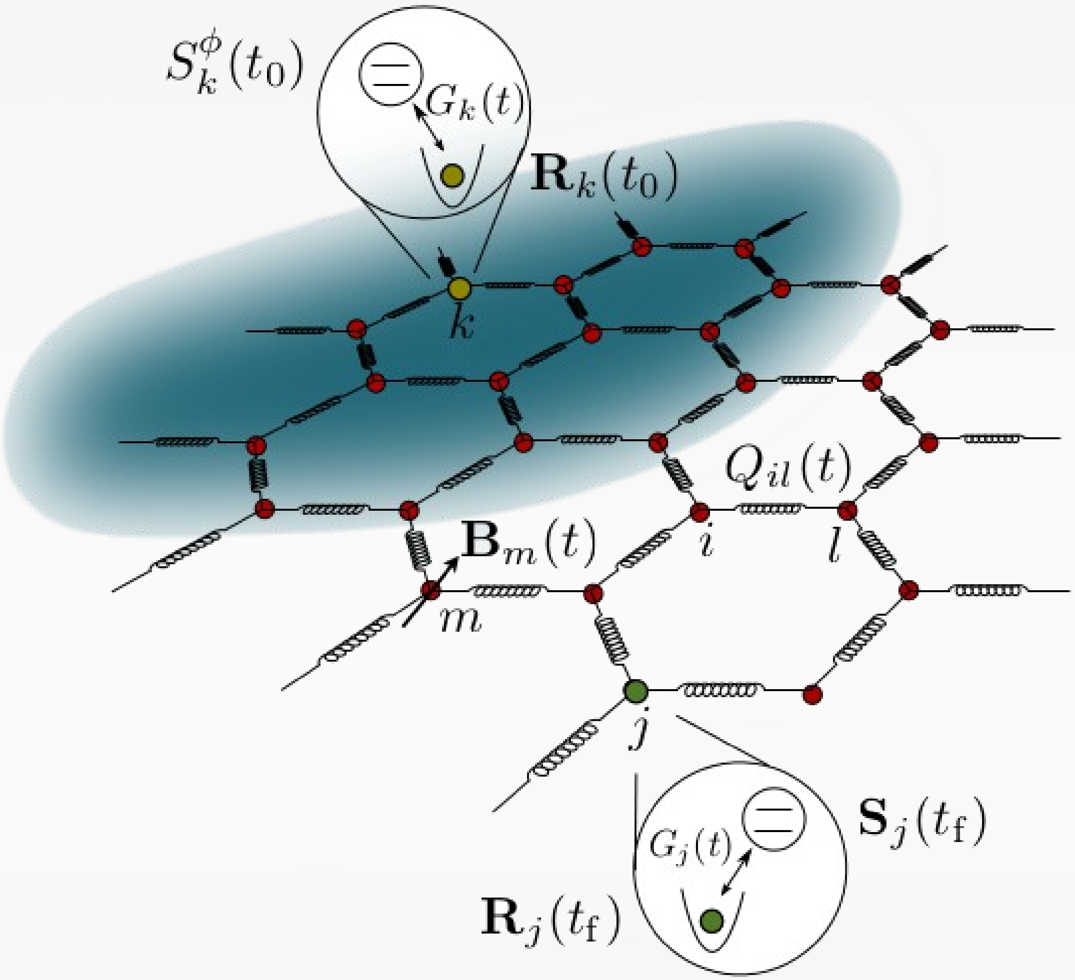
- Condensed Matter
- Proof of exponential clustering of correlations
  - Area law
  - Correlation length
- Proof of MPS simulability
- Generalized adiabatic theorem.
- Proof of robustnes of topological order

Lieb-Robinson bounds for spin-boson lattice models and trapped ions

J. Jünemann, A. Cadarso, D. Pérez-García, A. Bermúdez, JJGR

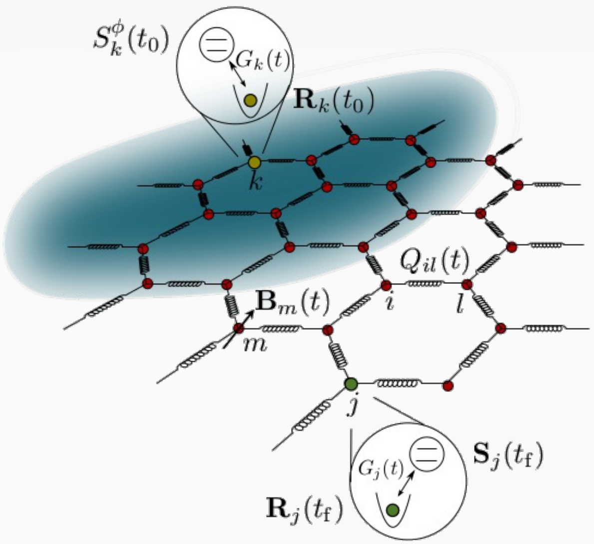
Phys. Rev. Lett. ***111****, 230404 (2013)*
arXiv:1307.1992

Spin-boson model



$$H = \sum_m \frac{\Delta_m}{2}\sigma_m^z + \sum_k \omega_k a_k^+ a_k + \sum_m g_m(t)\sigma_m^z x_m$$

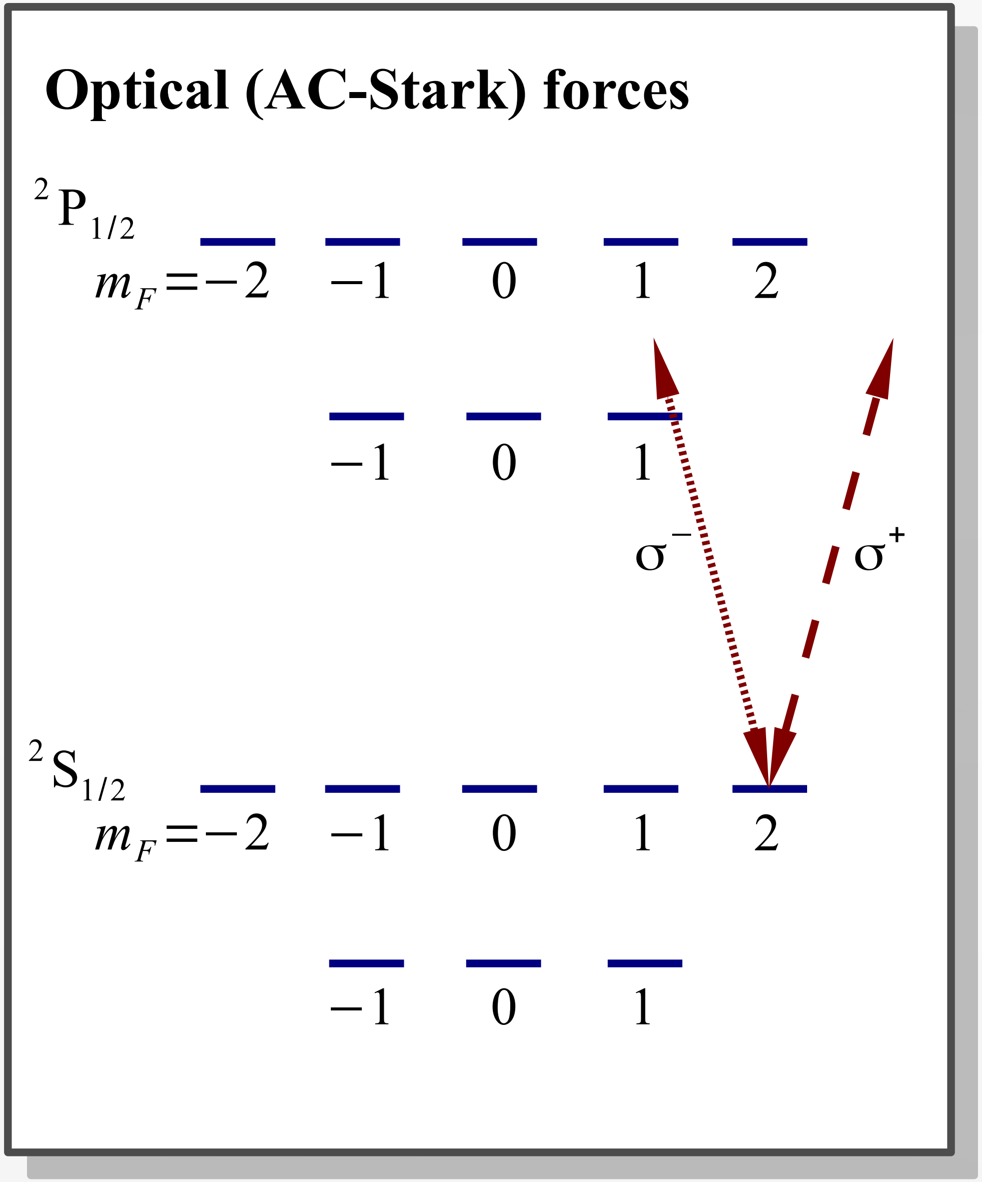
Spin-boson model

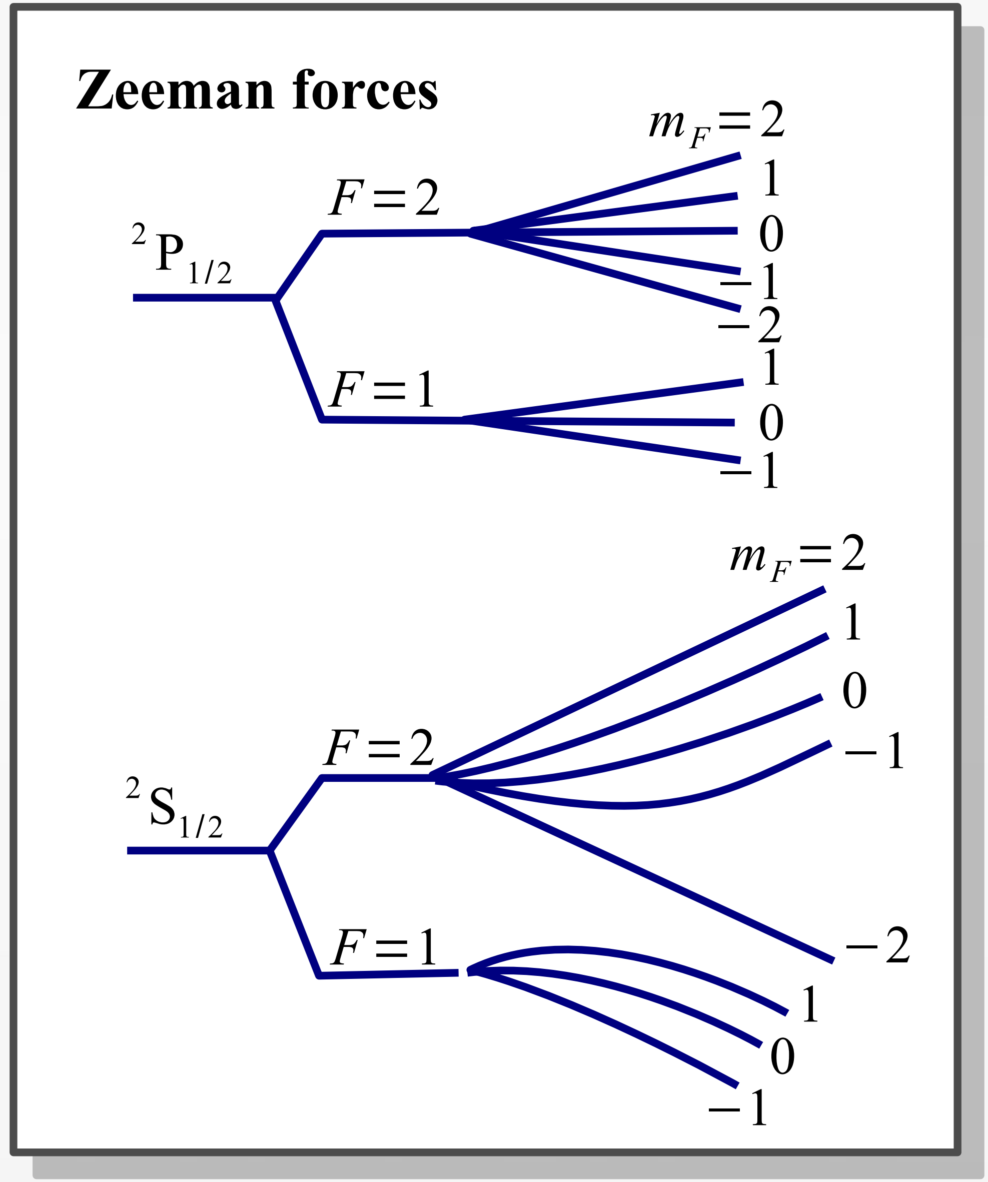


$$H=\sum_m \frac{\Delta_m}{2}\sigma_m^z+\sum_k \omega_k a_k^+ a_k+\sum_m g_m(t)\sigma_m^z x_m$$

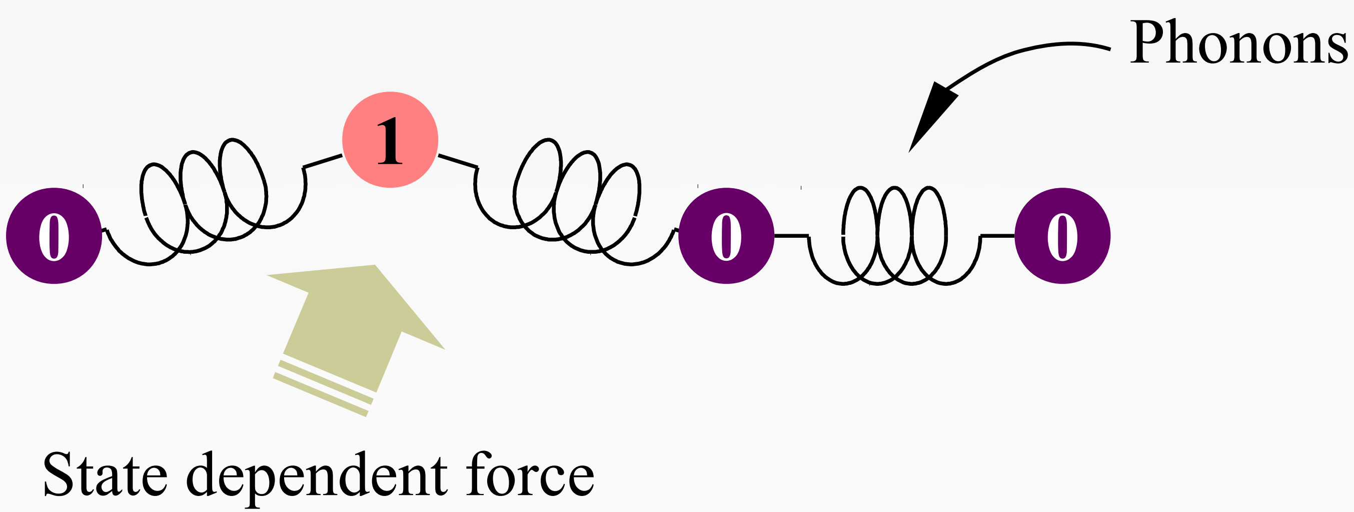
- Two models with LR bounds
- Infinite-dimensional
- Highly nonlinear
- Non-integrable
- Multiple experimental applications
  - Trapped ions, c-QED, nanophotonics, etc.

State dependent forces





Spin-boson model



$$H = \sum_k \omega_k a_k^+ a_k + \sum_m \frac{\Delta_m}{2} \sigma_m^z + \sum_m F_m(t) \sigma_m^z x_m$$

Balance between Coulomb force and trap creates a crystal

Light pushes the atoms depending on the state

Effective interactions

Accummulated phase can be computed

$$\varphi = \sum_{i,j} \sigma_i^z \sigma_j^z \times \int_0^T \int_0^{t_1} F_i(t_1) G_{ij}(t_1 - t_2) F_j(t_2) \, \mathrm{dt}_1 \, \mathrm{dt}_2$$

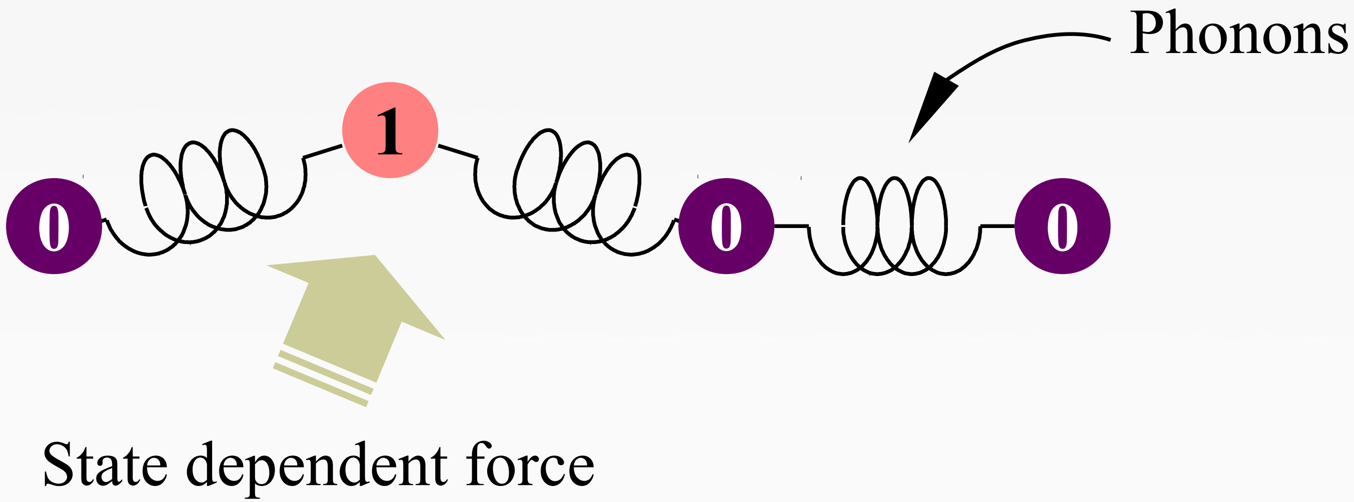
$$G_{ij}(t) = \sum_{ij} \frac{M_{ik} M_{jk}}{m \omega_k \hbar} \sin(\omega_k t)$$

This may be recasted as an spin-spin interaction

$$H_{eff} \sim \sum_{i,j} \sigma_i^z J_{ij} \sigma_j^z$$

which can be used for quantum simulation, QIPC, entanglement

Quantum simulation

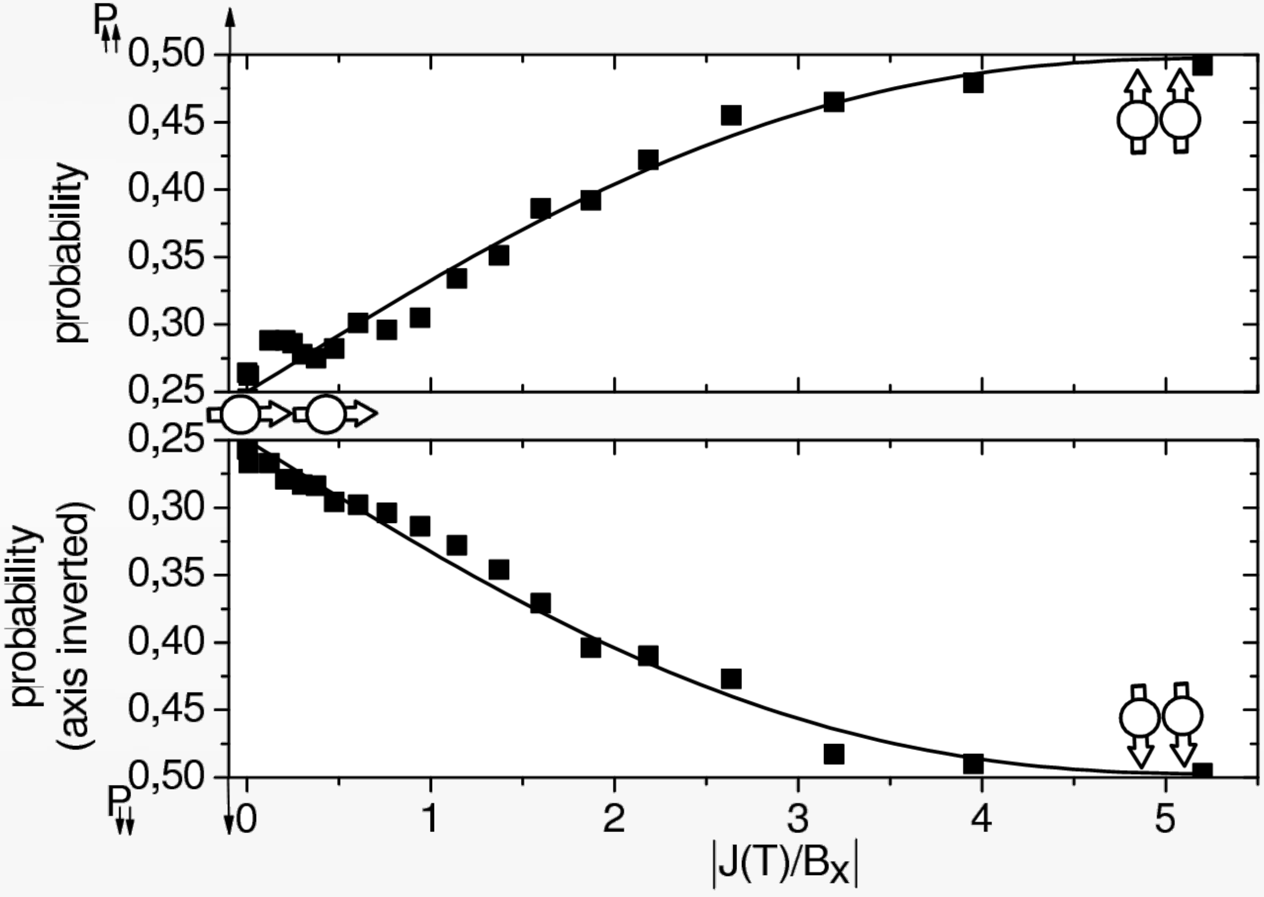


$$H = \sum_k \omega_k a_k^+ a_k + \sum_m \frac{\Delta_m}{2} \sigma_m^z + \sum_m F_m(t) \sigma_m^z x_m$$

$$F, \dot{F} \ll \omega, \Delta; \langle n_i \rangle \simeq 0$$

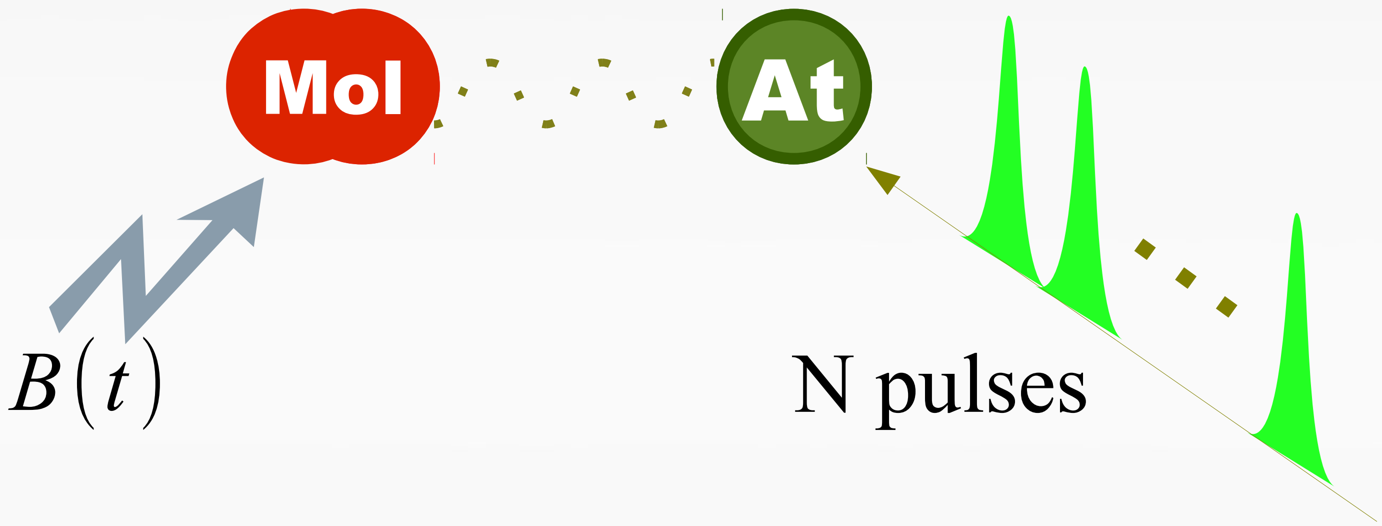
$$H_{eff} \sim \sum_{i,j} \sigma_i^z J_{ij} \sigma_j^z$$

Quantum simulation



A. Friedenauer et al, Nature Physics 4, 757 - 761 (2008)
R. Islam et al, Nature Comm. 2, 377 (2011); ibid, Science 340 (2013)
J. W. Briton et al, Nature 484, 489 (2012)

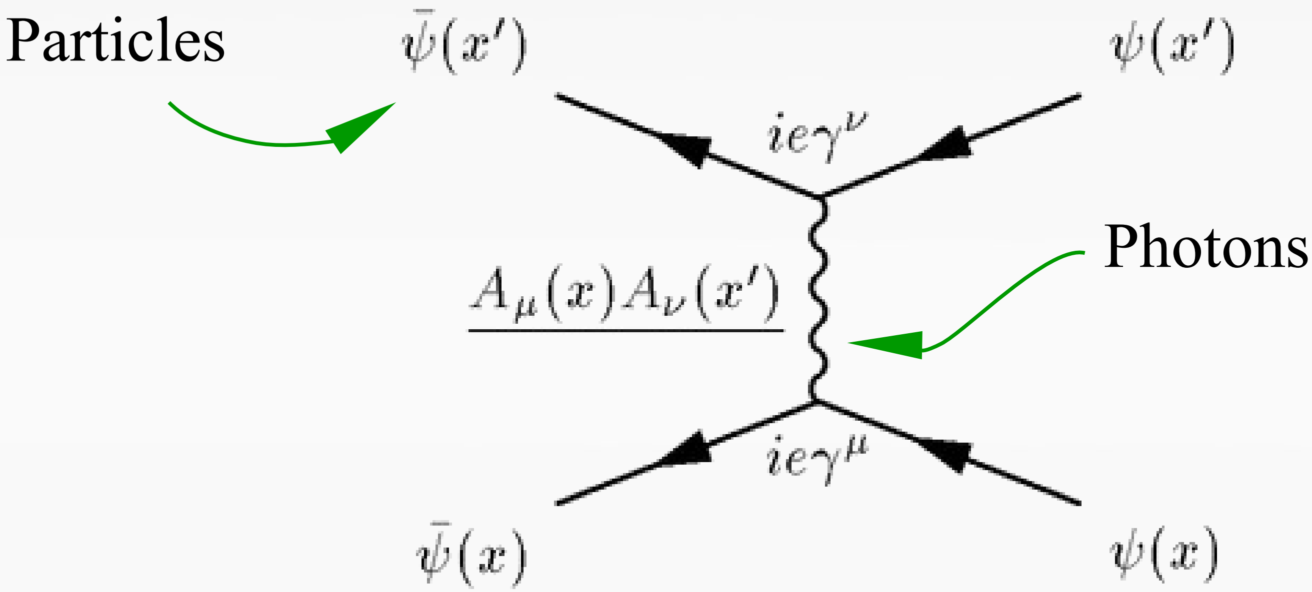
Signal intensification



Different ions may experience different forces

$$\varphi = \mathrm{Im}\int_0^T \int_0^t G(|t-t'|) f_1(t) f_2(t')\,\mathrm{d}t\mathrm{d}t'$$
$$\propto B \times N$$

Boson-mediated interactions



A very general framework: c-QED, ions, photonics,...

When the dynamics of the bosons is as fast as the particle-boson interaction, we cannot eliminate them perturbatively.

Proof sketch

Many intermediate results:

- Proof of a LR for the harmonic oscillator model with and without long-range interactions
  - Tighter bounds than existing literature
- "Interaction-like" picture removes the influence of the oscillators in the ion's local dynamic.
- Integrate out the bosonic d.o.f. with the propagator.
- Sum up the resulting recurrence only for the spins.
- Feed back to the oscillators for further bounds.

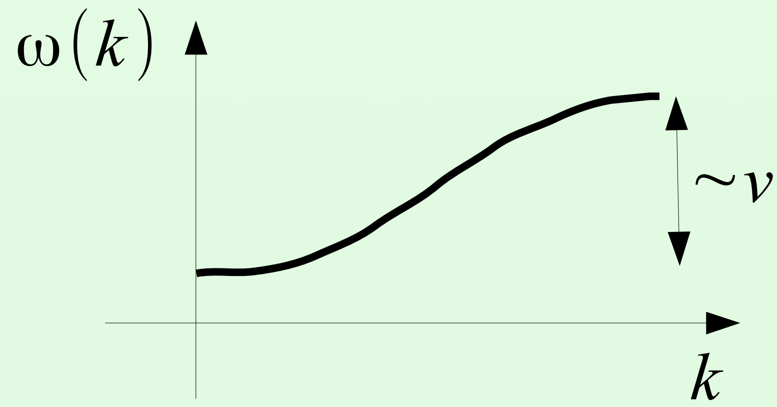
Two theory results

#1 An improved L-R bound for the ion phonons

$$\|[R_i(t), R_j(0)]\| \leq \frac{\exp(v\,t)}{d_{i,j}^{\eta}}$$

Long range interaction

$$H_{\text{int}} \sim \sum_{ij} \frac{1}{|x_i - x_j|^3}$$



Two theory results

#2 A resulting L-R bound for the interacting ions / spins

$$\left\|[\sigma_i(t),\sigma_j(0)]\right\| \le \frac{\exp(vt)}{d_{i,j}^{\eta}}\left(e^{g^2\frac{t}{v}}-1\right)$$

Bosonic LR repeated

$$\left\|[R_i(t),R_j(0)]\right\| \le \frac{\exp(vt)}{d_{i,j}^{\eta}}$$

Speed of information exchange

coupling g vs *speed* v

Bound saturation

#2 A resulting L-R bound for the interacting ions / spins

$$\left\|\left[\sigma_i(t), \sigma_j(0)\right]\right\| \leq \frac{\exp(vt)}{d_{i,j}^{\eta}}\left(e^{g^2 \frac{t}{\gamma}} - 1\right)$$

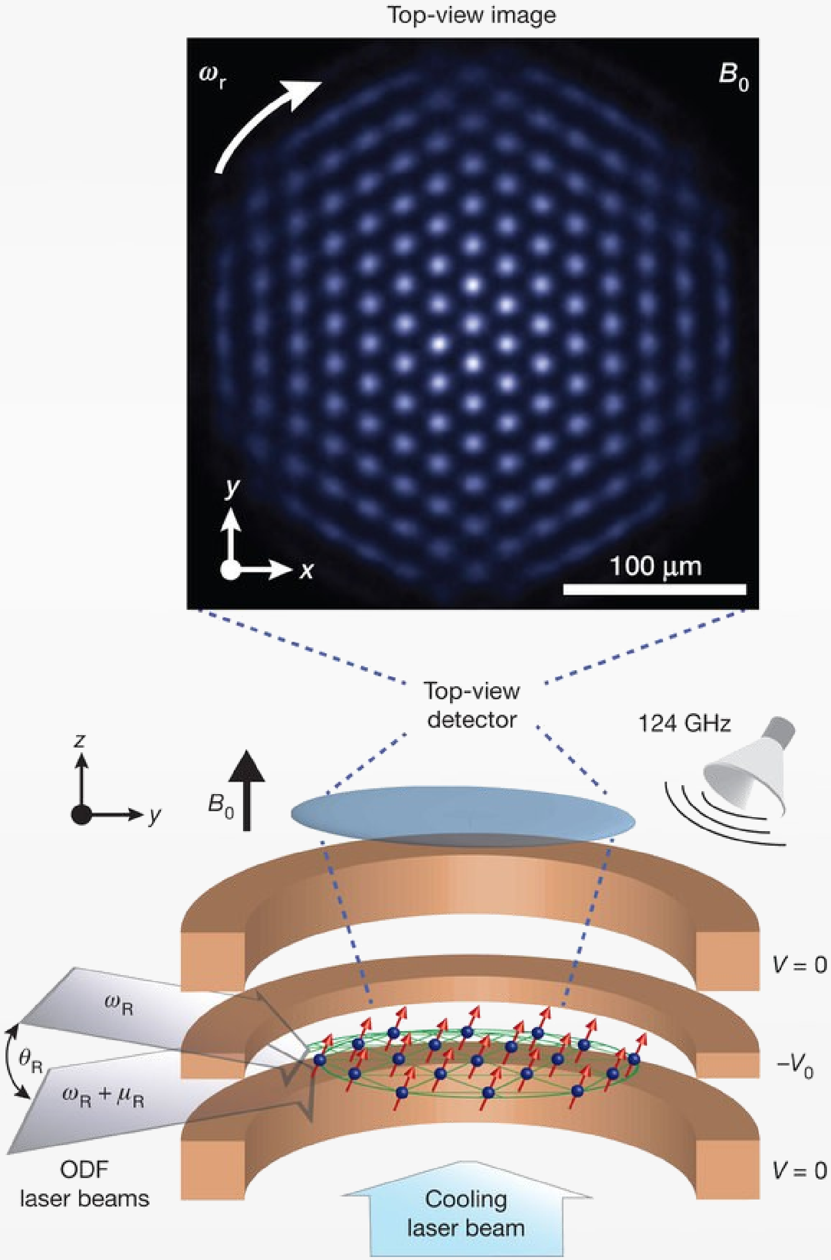
The bound is saturated in the

impulsive regime

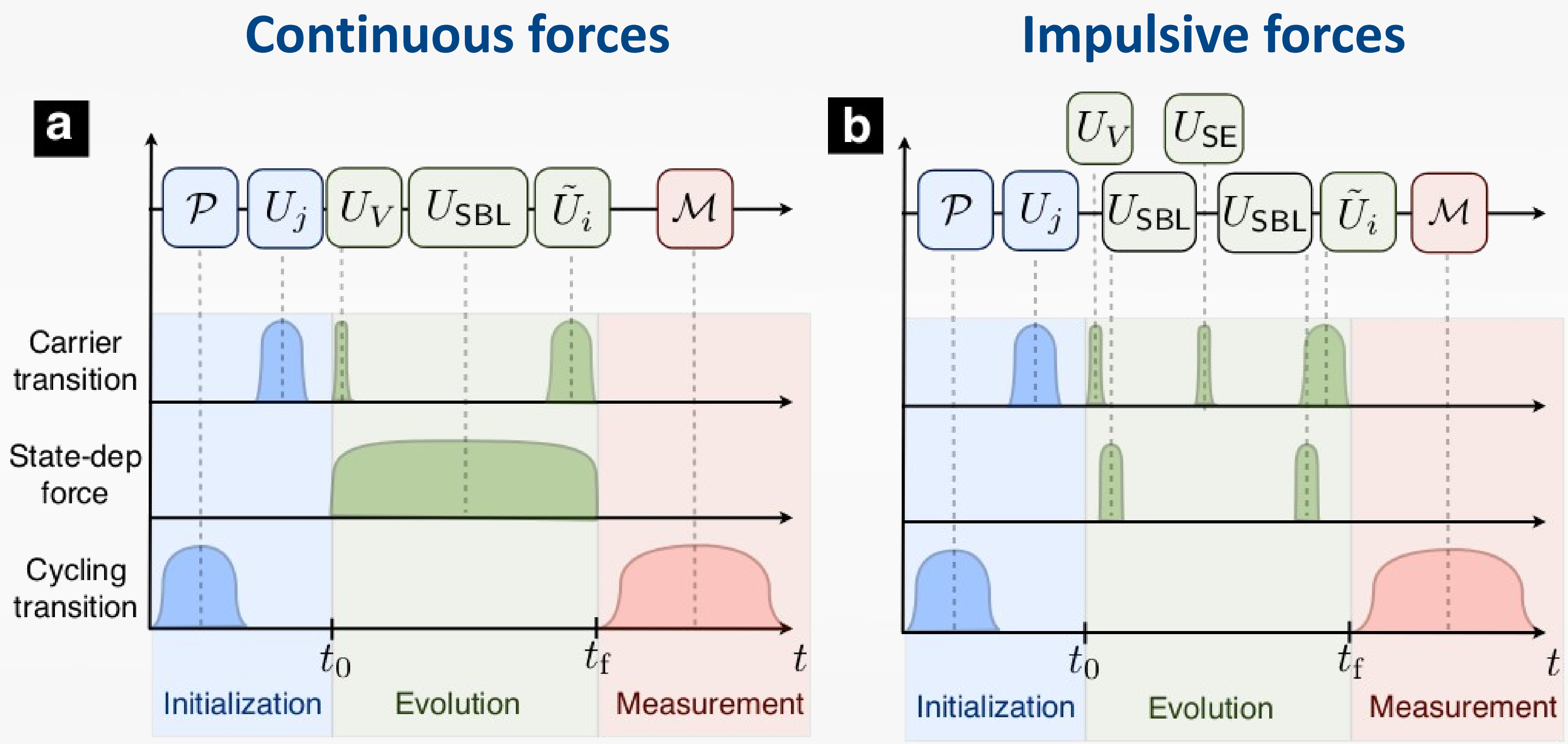
(When the forces act instantaneously)

Trapped ions

*J. Britton et al, Nature **484**, 489–492 (2012)*

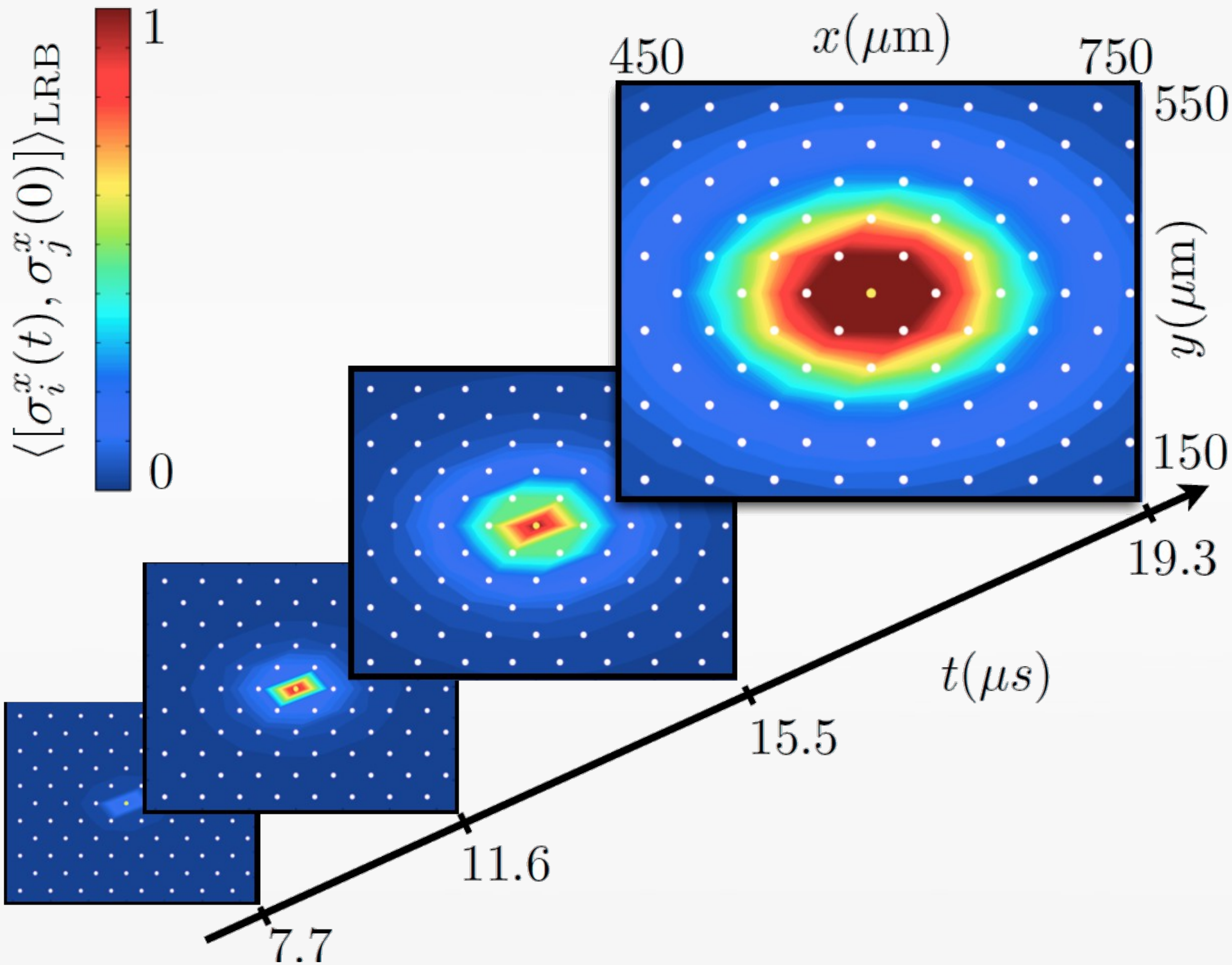


Measurement protocol



$$\langle B(t)\rangle \rightarrow \langle B(t)\rangle + \frac{\varepsilon}{\hbar}\langle [B(t), A(0)]\rangle + O(\varepsilon^2)$$

Correlation spread



Generalized LR bounds

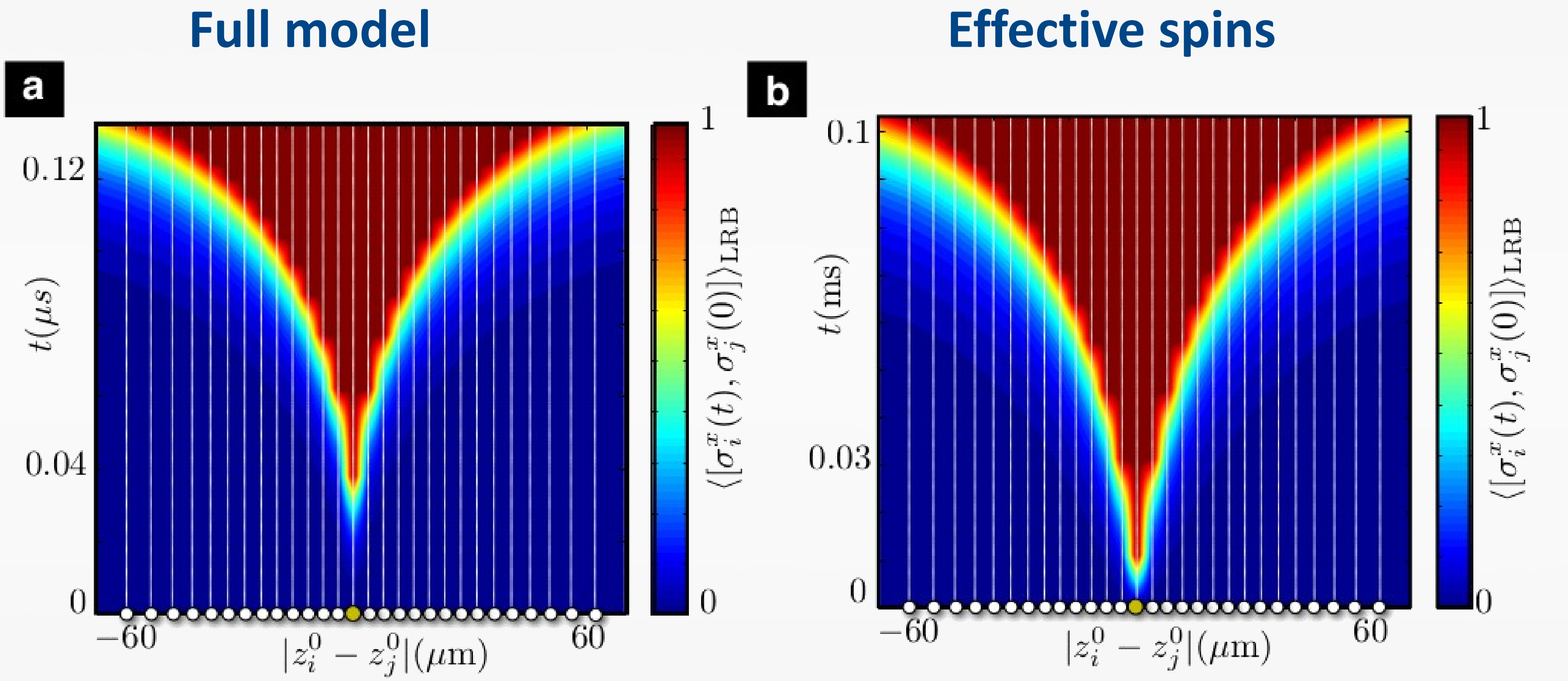
$$\left\|\left[A_x(t), B_y(0)\right]\right\| \leq c \exp\left(v t - d_{x,y}\right)$$

Outside an effective light-cone with speed v correlations are exponentially attenuated.

$$\left\|\left[A_x(t), B_y(0)\right]\right\| \leq c \exp\left(v t\right) \frac{1}{d_{x,y}^{\eta}}$$

A milder version needed for long range interactions.

Correlation spread

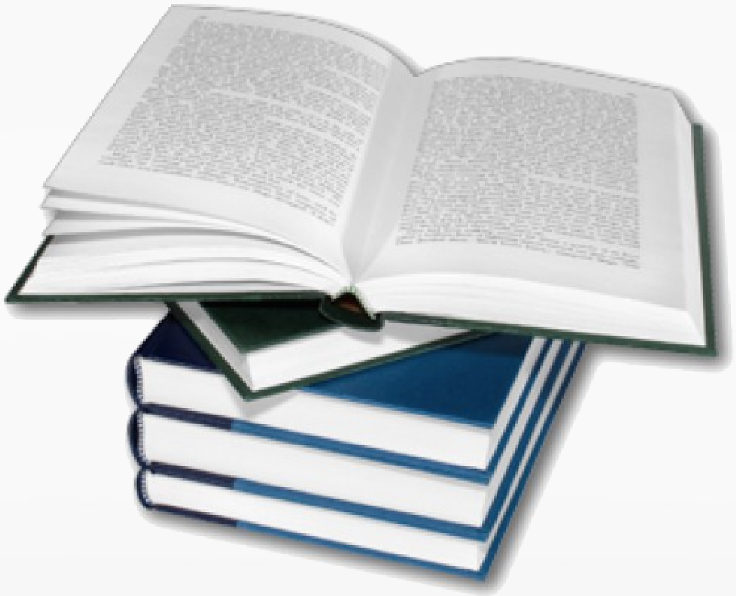


Approximate light cones because of long range interaction.

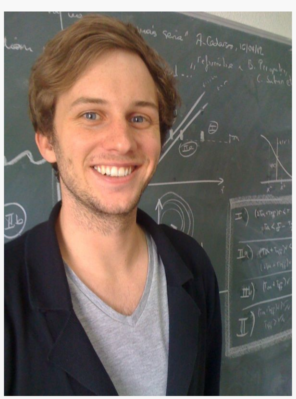
The full model is faster, because it involves the bosons in a non-perturbative fashion

Summary

- Spin-boson model of interest beyond quantum dissipation.
- New mathematical tools for analyzing correlation spread in these systems.
- Experimental possibility of measuring the propagation of correlations.



Acknowledgements



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