Spin Chains as Data, Entanglement and Logic Buses

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Talk Outline

- The motivation for buses of the spin chain form in QC architectures

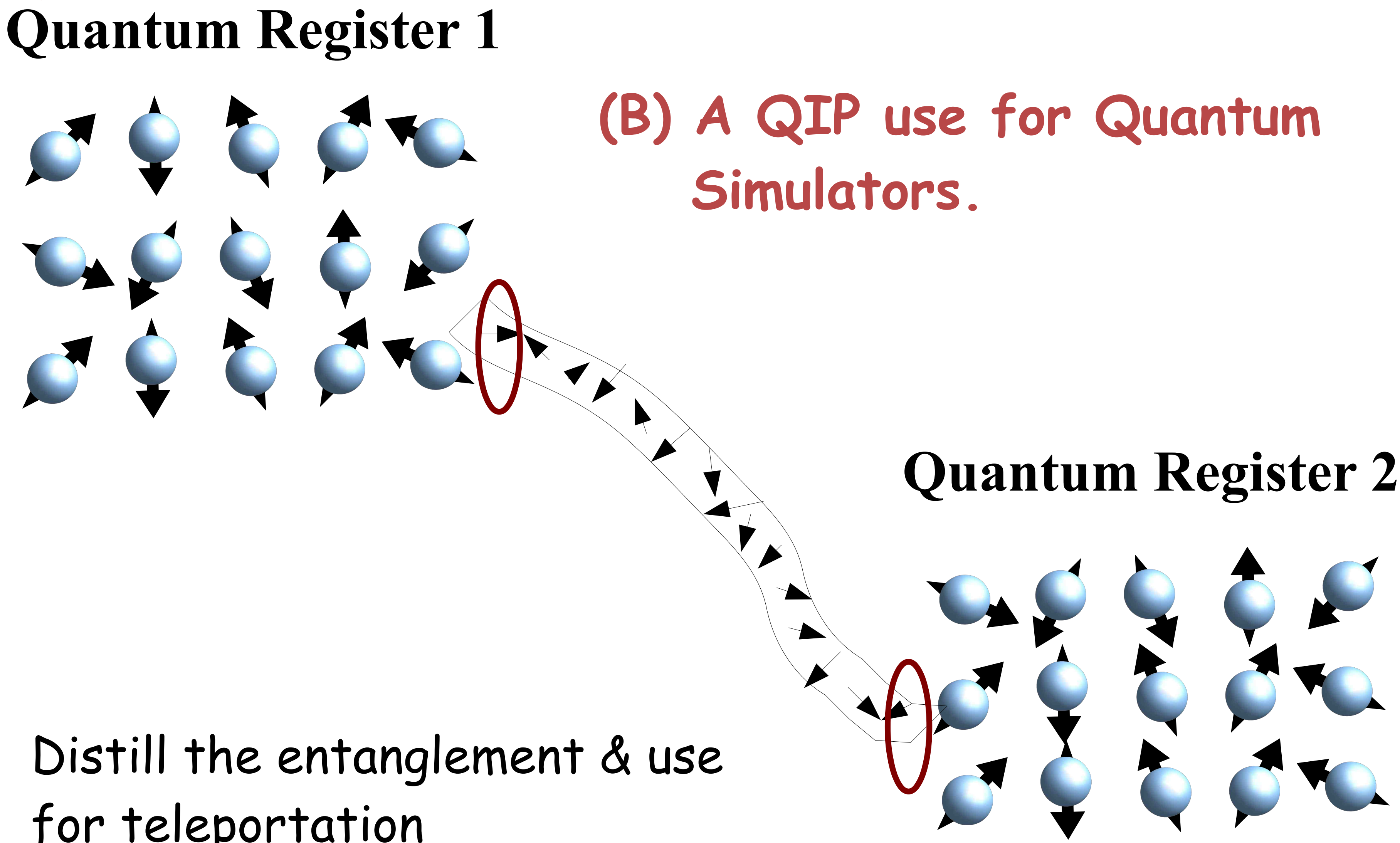
- Brief Review: Spin chains as quantum channels concluding with an optimal minimally engineered scheme.

- Long distance entanglement from "Quenches" followed by unitary dynamics of a spin chain.

- Logic gates mediated by spin chains

- Spin molecules for reversible logic gates

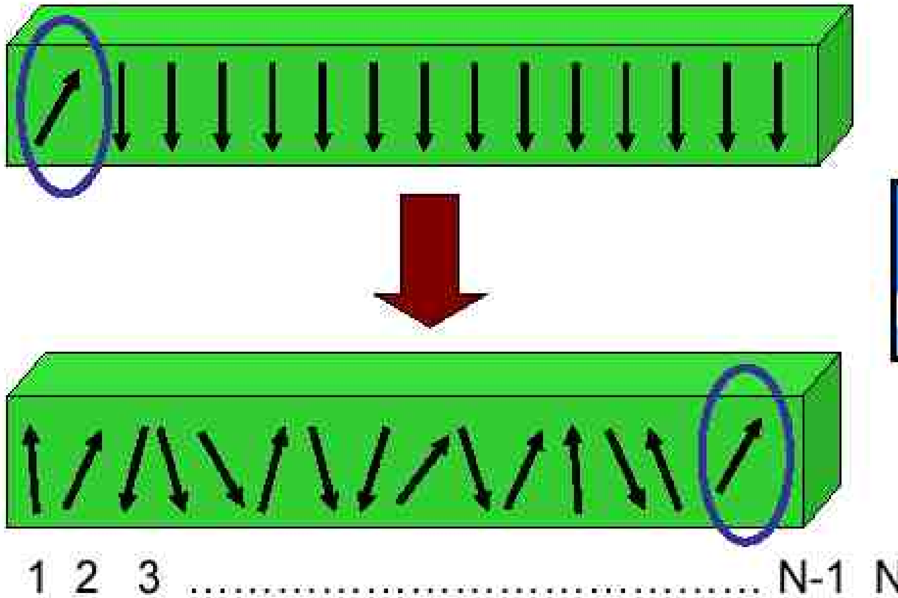
(A) To connect without interfacing different classes of physical systems (e.g. ions and photons) or without physical movement



(B) A QIP use for Quantum Simulators.

Distill the entanglement & use for teleportation

When one wants to use a ferromagnetic chain



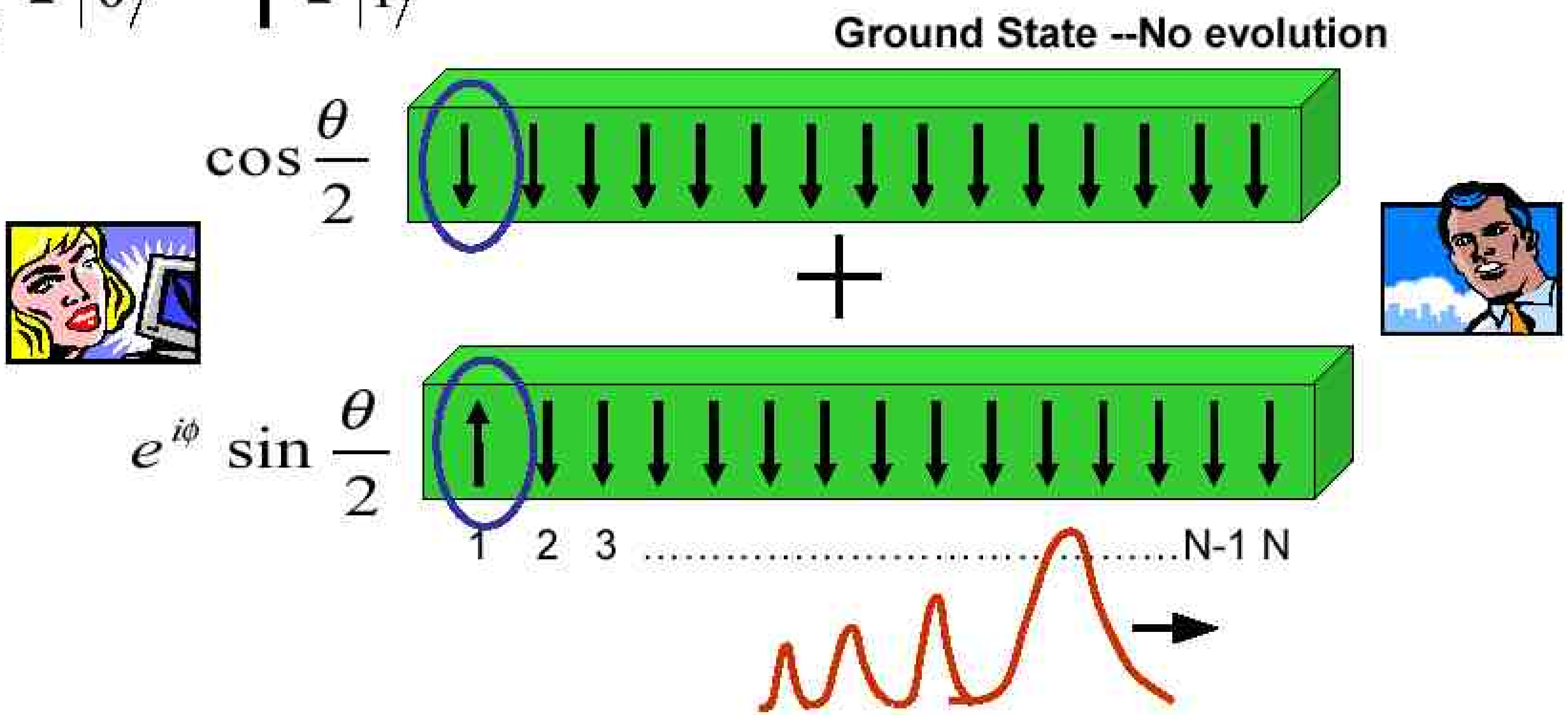
Isotropic Heisenberg

$$H = -\frac{J}{2}\sum_{j=1}^{N-1} \sigma_j \cdot \sigma_{j+1}$$

S. Bose, Phys. Rev. Lett. 91, 207901 (2003).

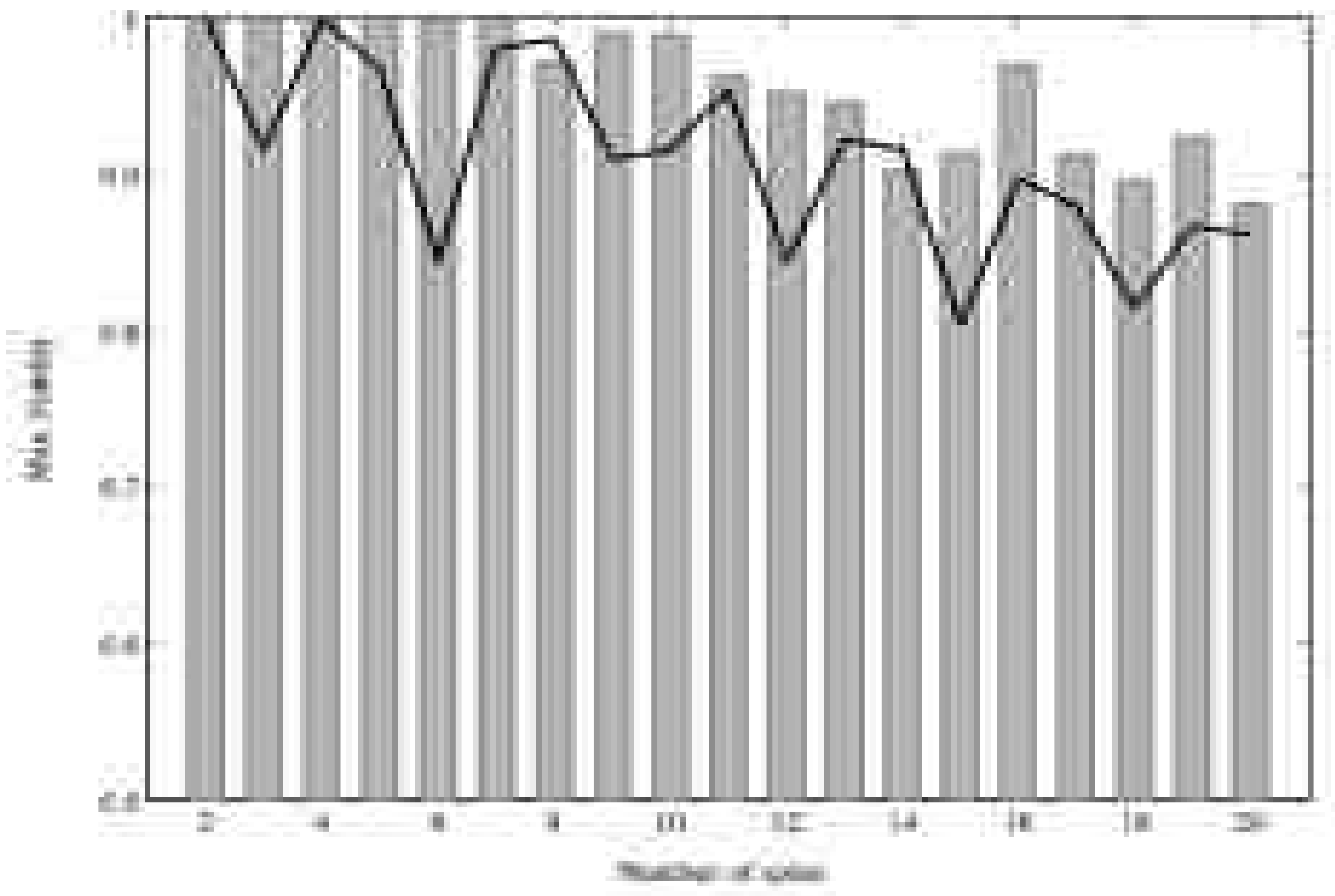
Alice's input state: $|\phi\rangle_1 = \cos\frac{\theta}{2}|0\rangle_1 + e^{i\phi}\sin\frac{\theta}{2}|1\rangle_1$

$\downarrow \equiv |0\rangle \qquad \uparrow \equiv |1\rangle$



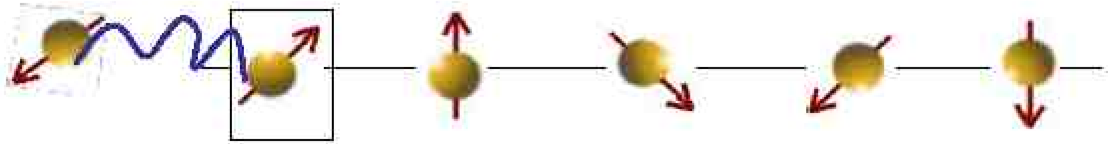
Amplitude Damping Channel

N=4, 8 have very good transfer fidelities (99%) in isotropic Ferromagnets; Better than 2/3 for N=80 and time upto t=4000/J



Yung, Leung, Bose, QIC (2003), for XY model

Entanglement transfer through arbitrarily long chains:



At $t \approx O(\frac{N}{J}), \quad E_C \approx \frac{1}{N^{1/3}}$

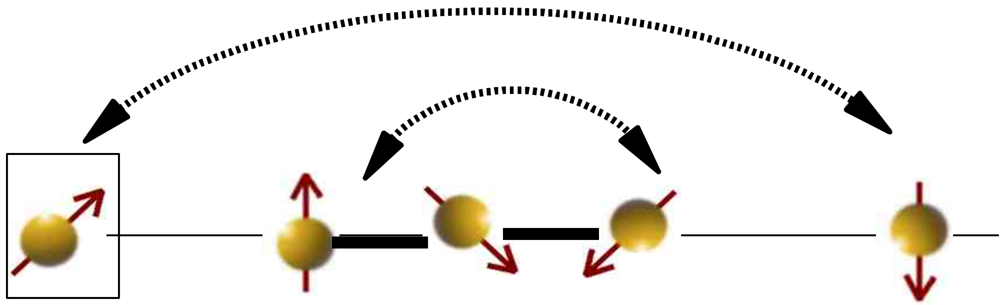
This proves that a spin chain of any length can behave as a quantum channel

Impracticality: *Several transmissions followed by entanglement distillation needed even for transmitting one ebit of entanglement !!!*

S.Bose, PRL 91, 207901 (2003).

This art can be perfected; Several ways of doing so .....

Engineering

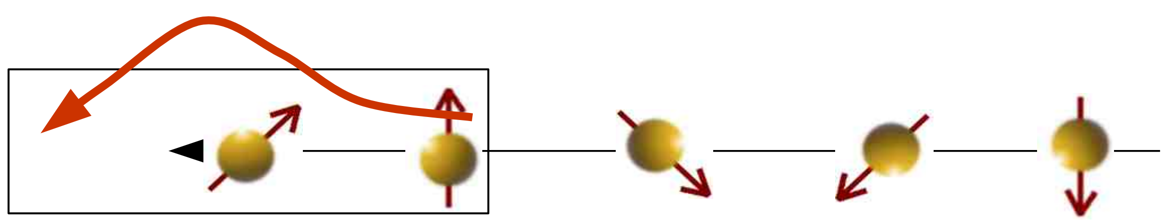


Christandl, Datta, Ekert, Landahl, PRL 92, 187902 (2004)

A thermal/mixed state in the middle can be eliminated by measurements and operations on the end spins only:

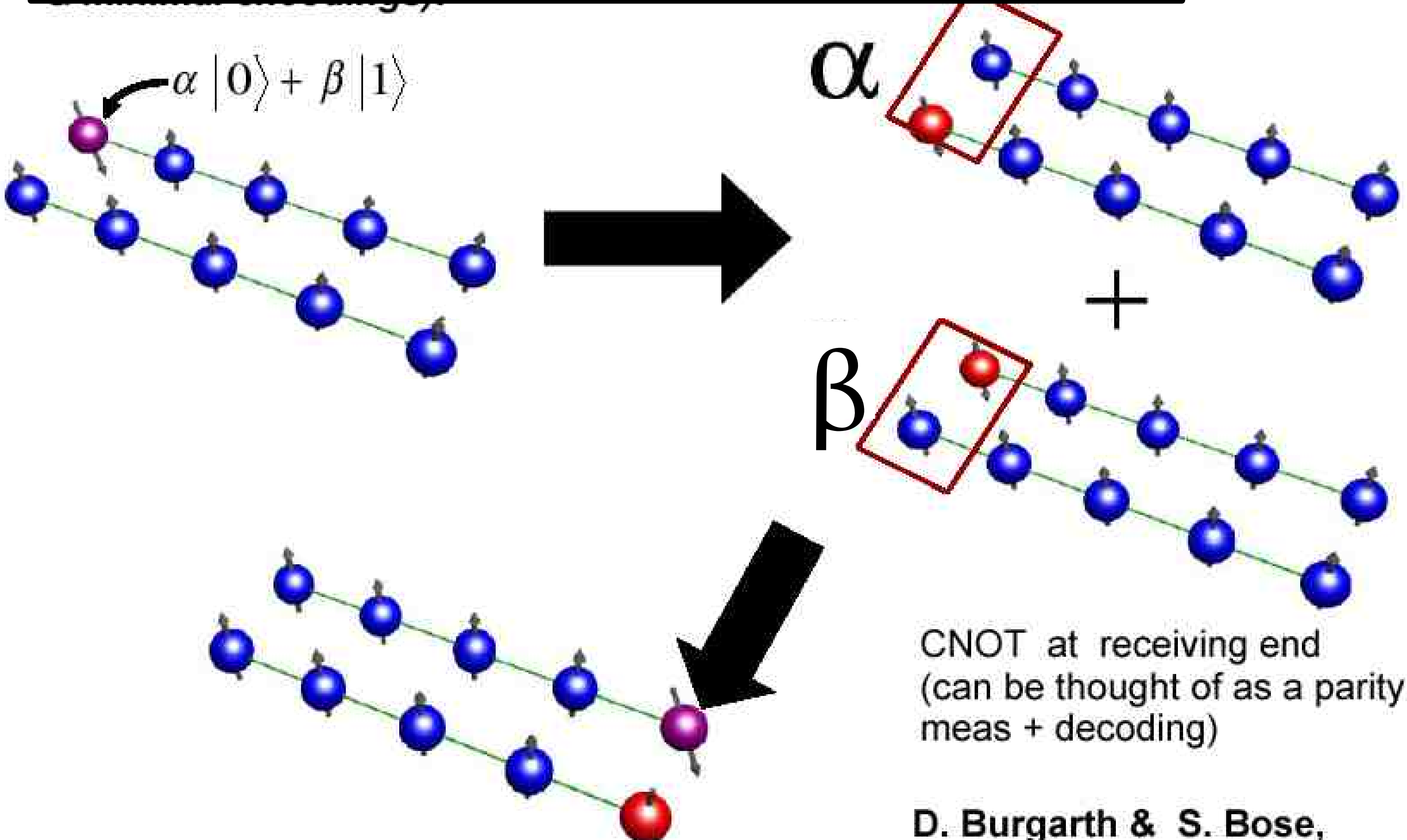
C. Di Franco, M. Paternostro, M. S. Kim, Phys. Rev. Lett. 101, 230502 (2008).

Encoding



Osborne & Linden, PRA 69, 052315 (2004); Haselgrove, PRA 72, 062326 (2005).

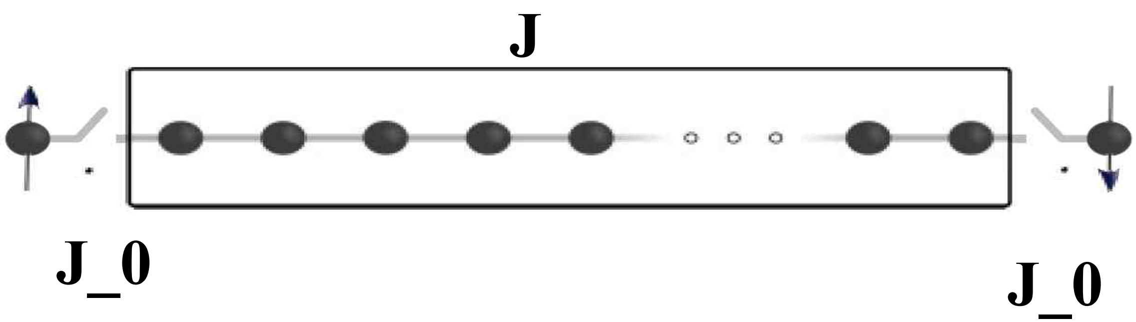
Dual Rail Encoding



D. Burgarth & S. Bose, Phys. Rev. A 71, 052315 (2005)

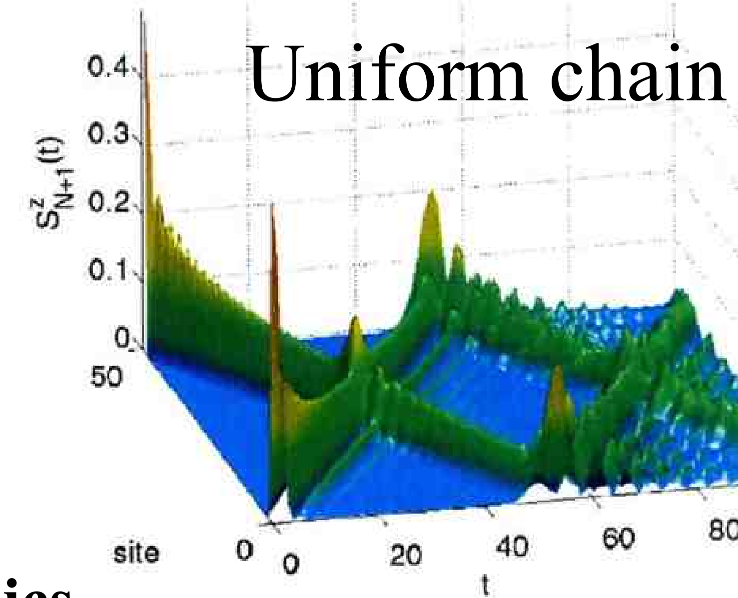
This is a scheme which should not depend on the range of interactions

A minimal (but optimal) coupling at the ends mimicks a fully engineered chain!



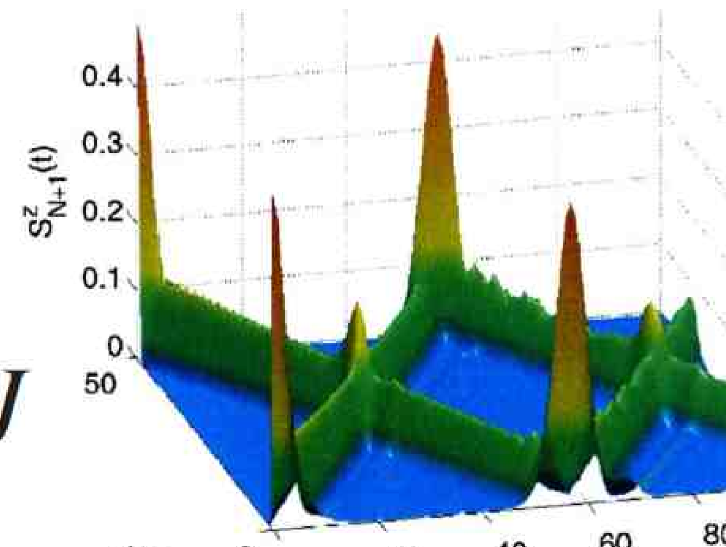
Mirror inversion of end spins on dynamics

Uniform chain



Banchi, Apollaro, Cuccoli, Vaia, Verucchi, PRA 2010.

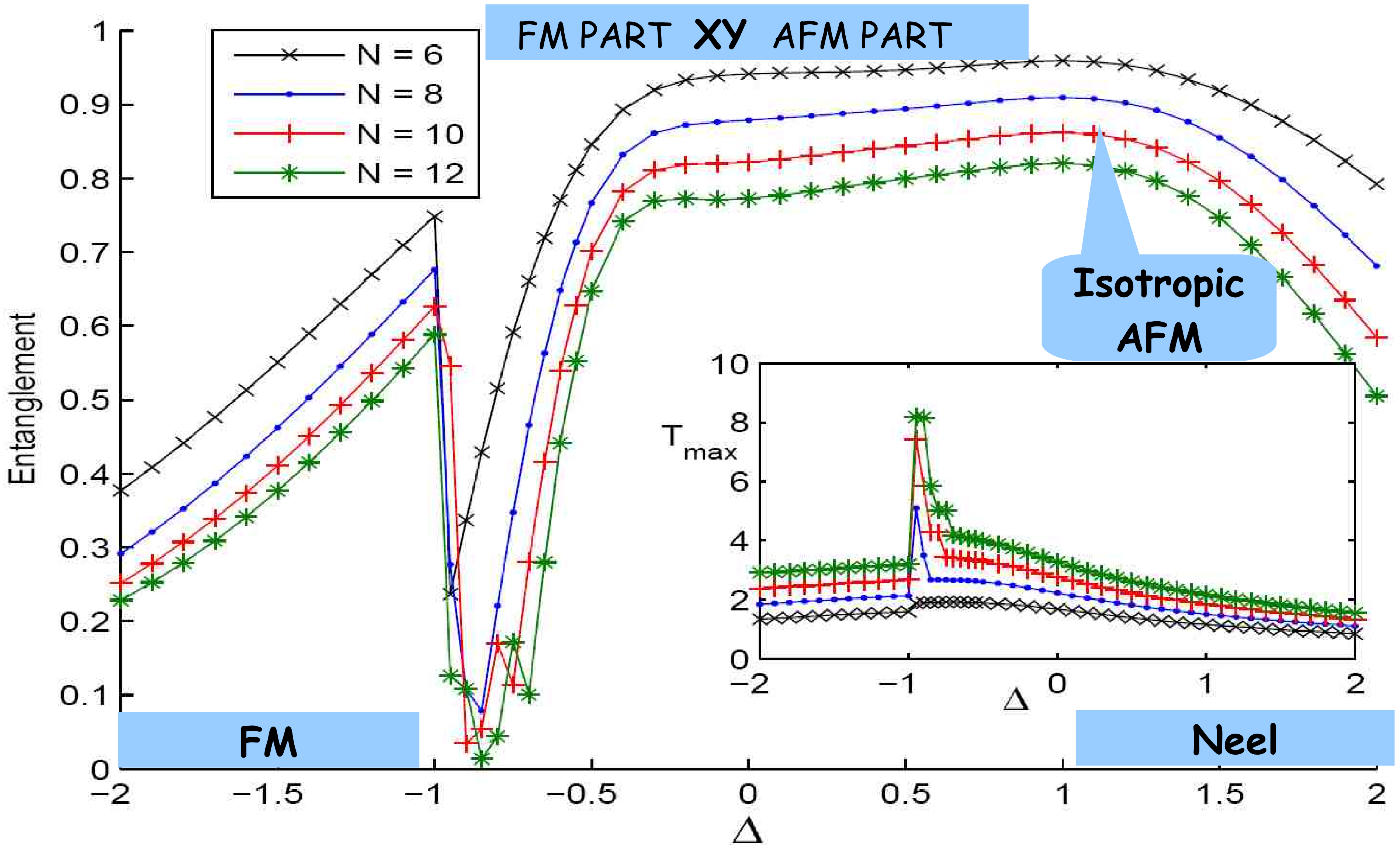
Optimized J_0



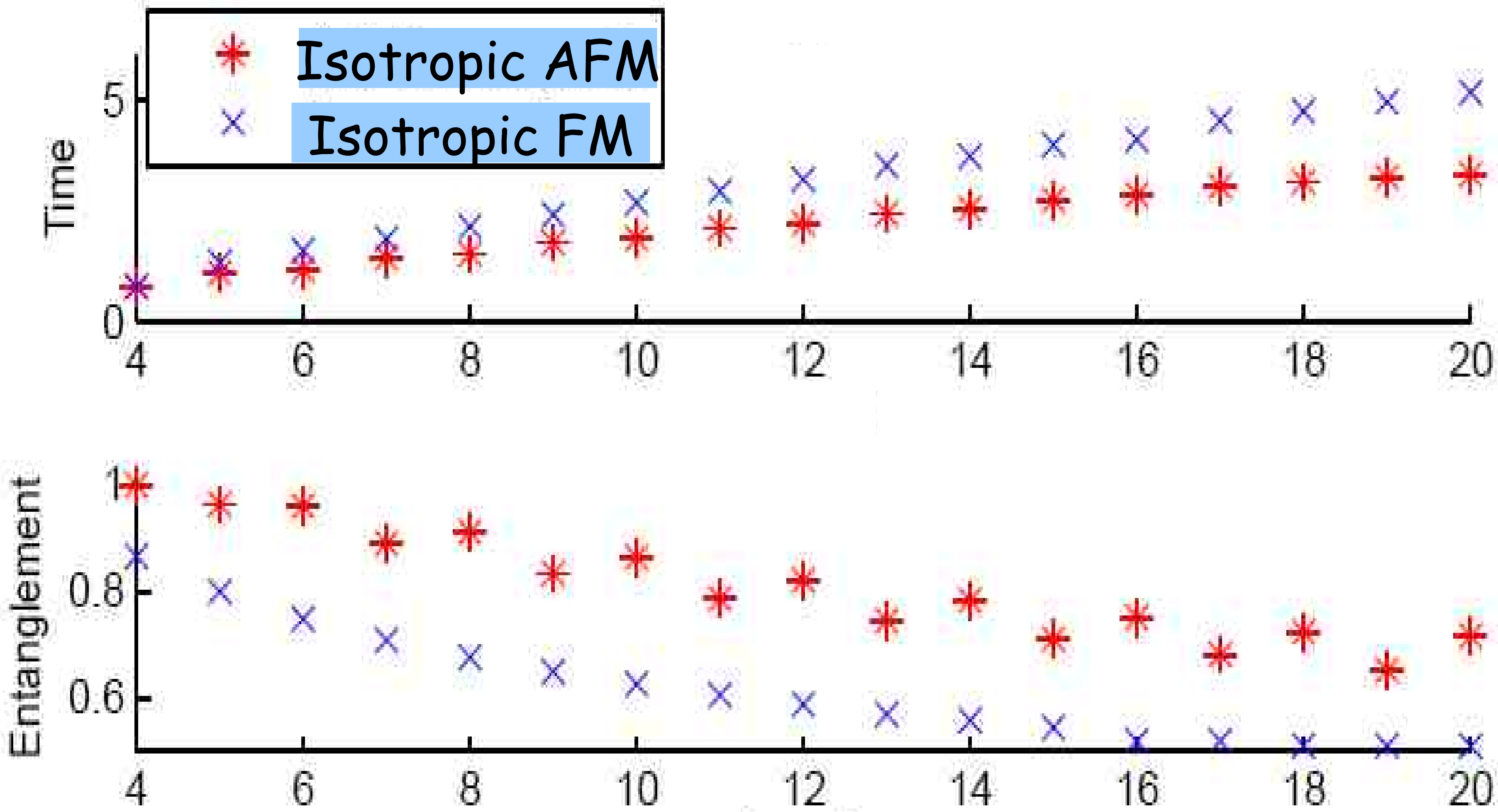
$$J_0^{\mathrm{opt}} \simeq 1.05 J N^{-1/6}$$

$$t^* \simeq (0.25N + 0.52N^{1/3})/J$$

How good a quantum channel is a spin $\frac{1}{2}$ chain in a more general phase?



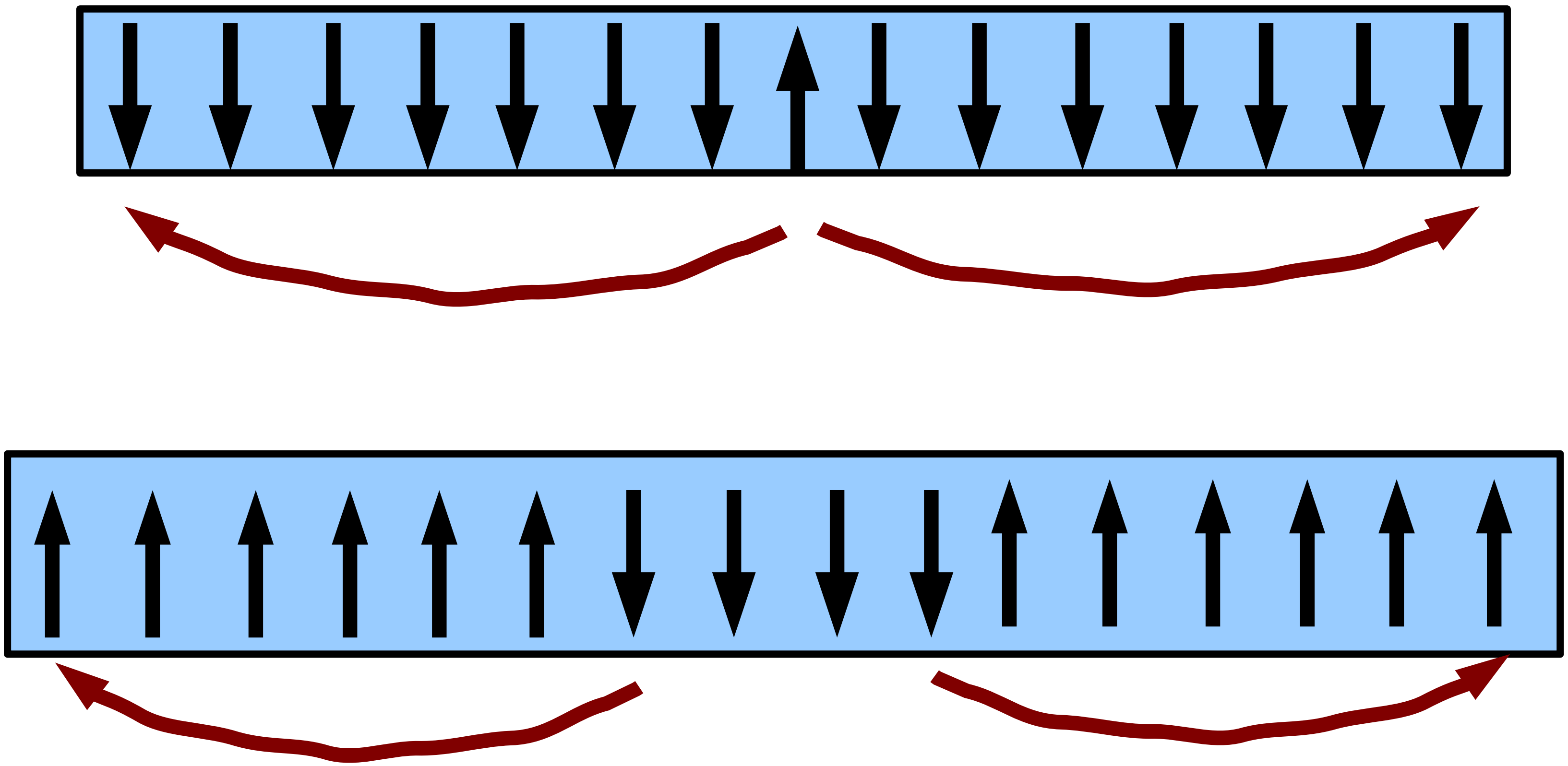
$$H_{ch} = J \sum_{i=1}^{N_{ch}-1} \{\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z\}$$



A. Bayat & S. Bose, PRA (2010)

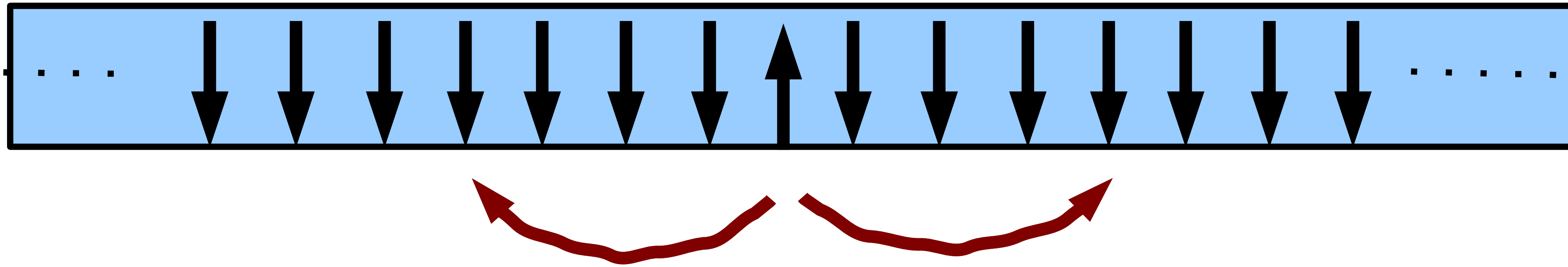
Generating entanglement between the end spins of a chain should be as good a resource

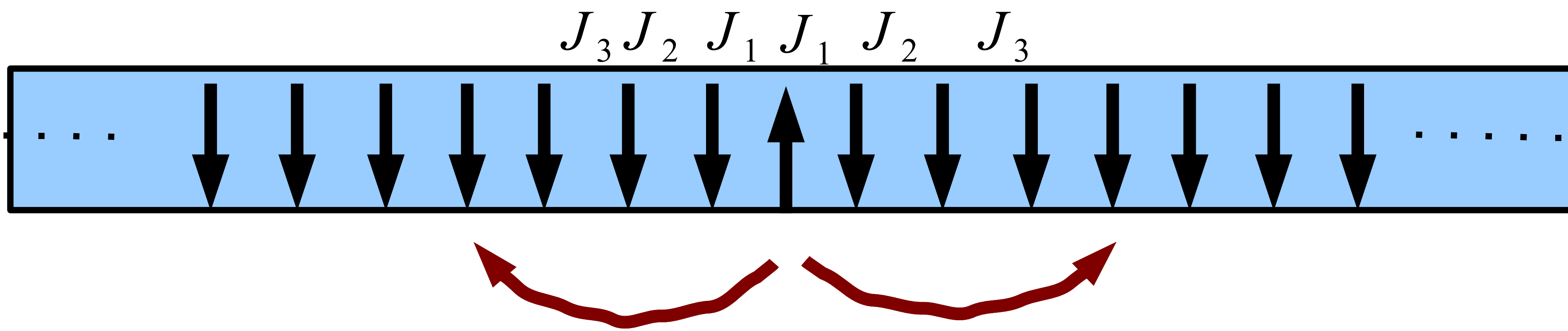
Entanglement resource can also be created by the spin chain dynamics as long as the chain is initialized in a non-eigenstate



Related to the **communication problem**: poor scaling with distance due to dispersion

In general E ~ 14.5/ N^(2/3)



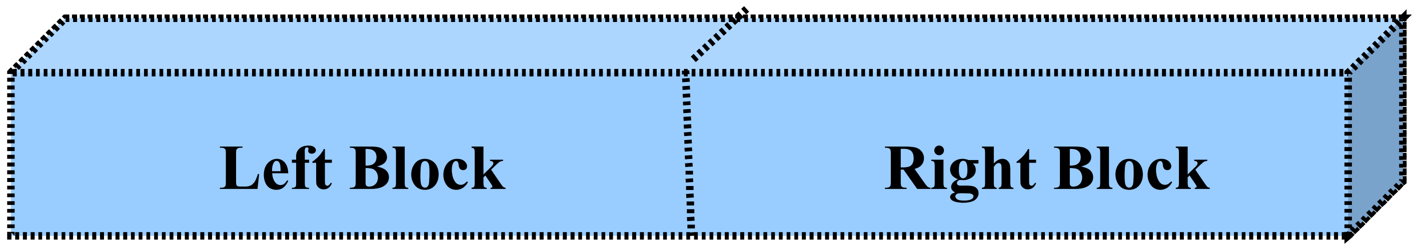


Perfect with full engineering: Yung & Bose, PRA (2005)

Quench is a change of the Hamiltonian. Can that be used to generate substantial entanglement between the ends of a system?

What had already been done some time ago?

Block entanglement due to quench induced dynamics:



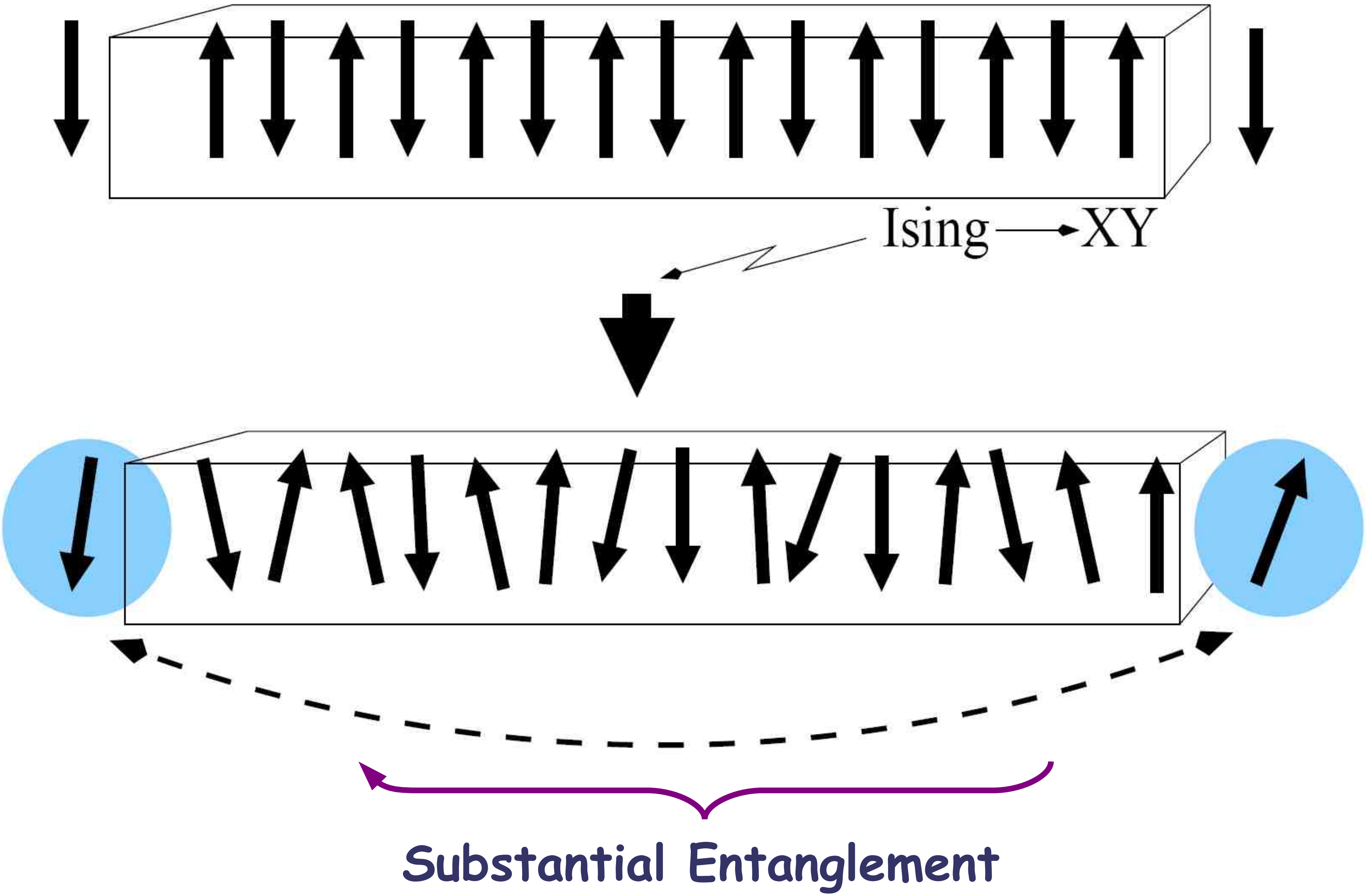
P. Calabrese and J. Cardy, J. Stat. Mech. P04010 (2005).

G. De Chiara, S. Montangero, P. Calabrese, and R. Fazio, J. Stat. Mech. P03001 (2006).

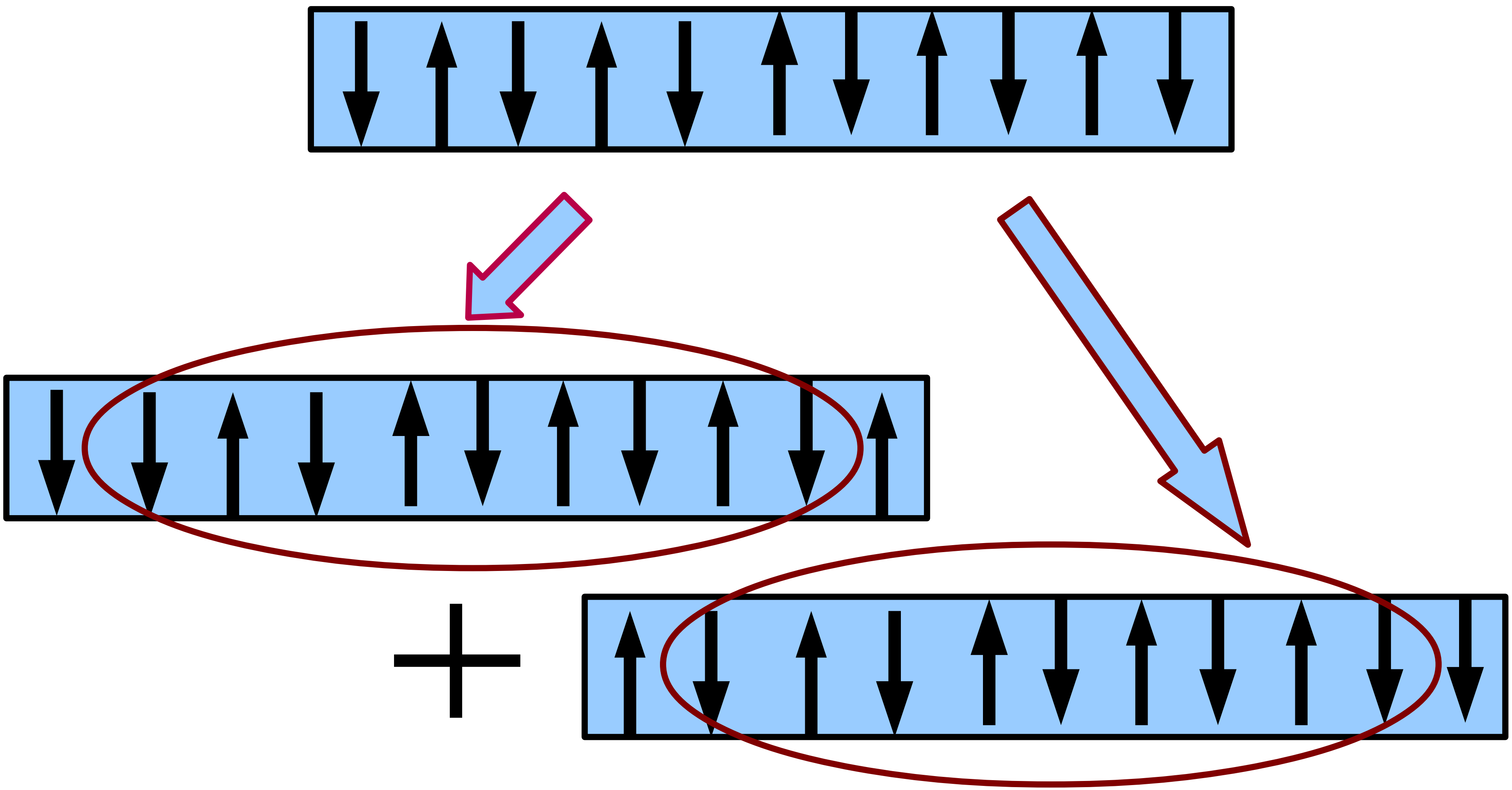
Does not guarantee that individual spins at the ends will be significantly entangled.

Quantum Communication Resource from Quench

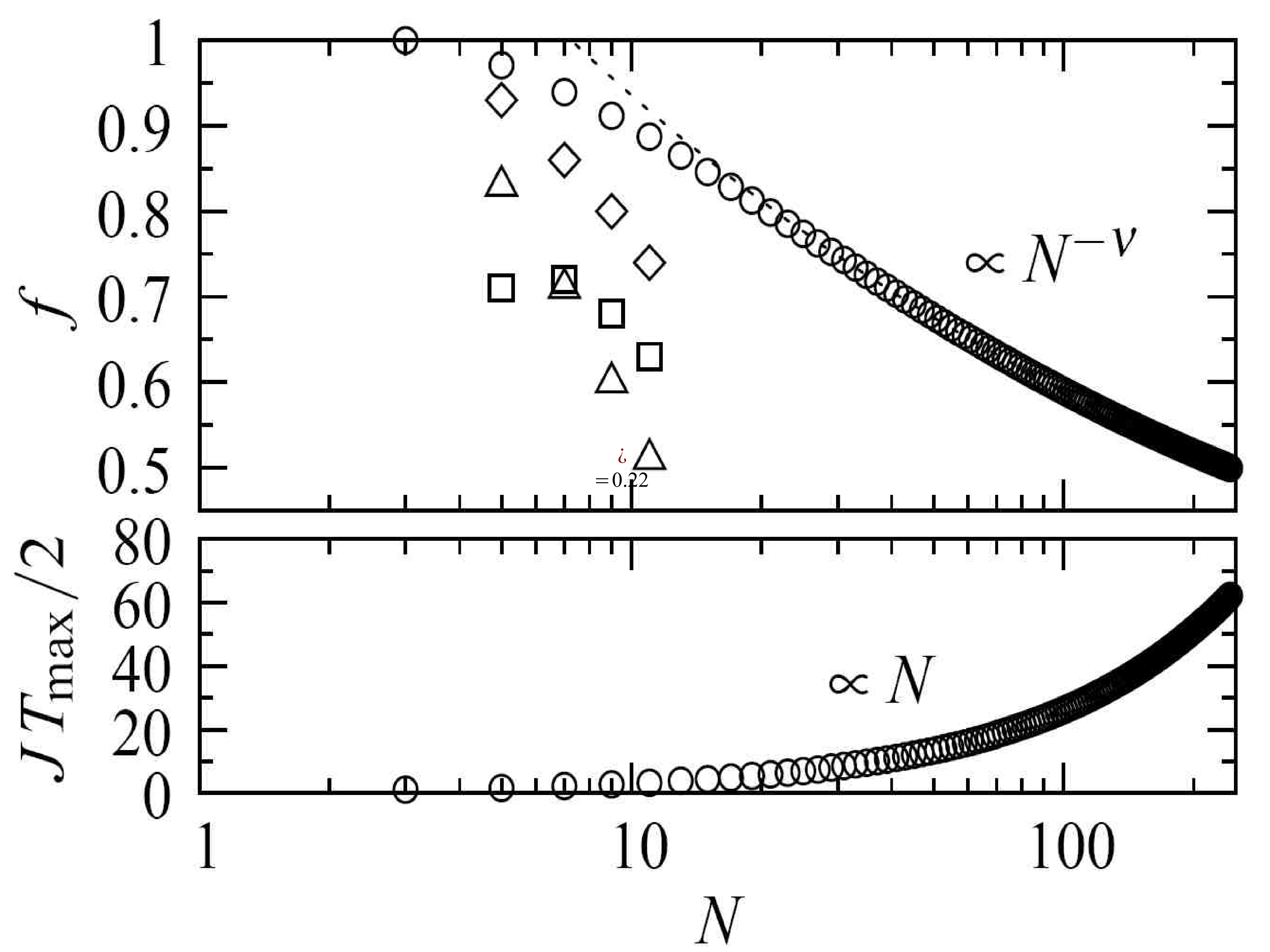
Hannu Wichterich & Sougato Bose, PRA (2009)



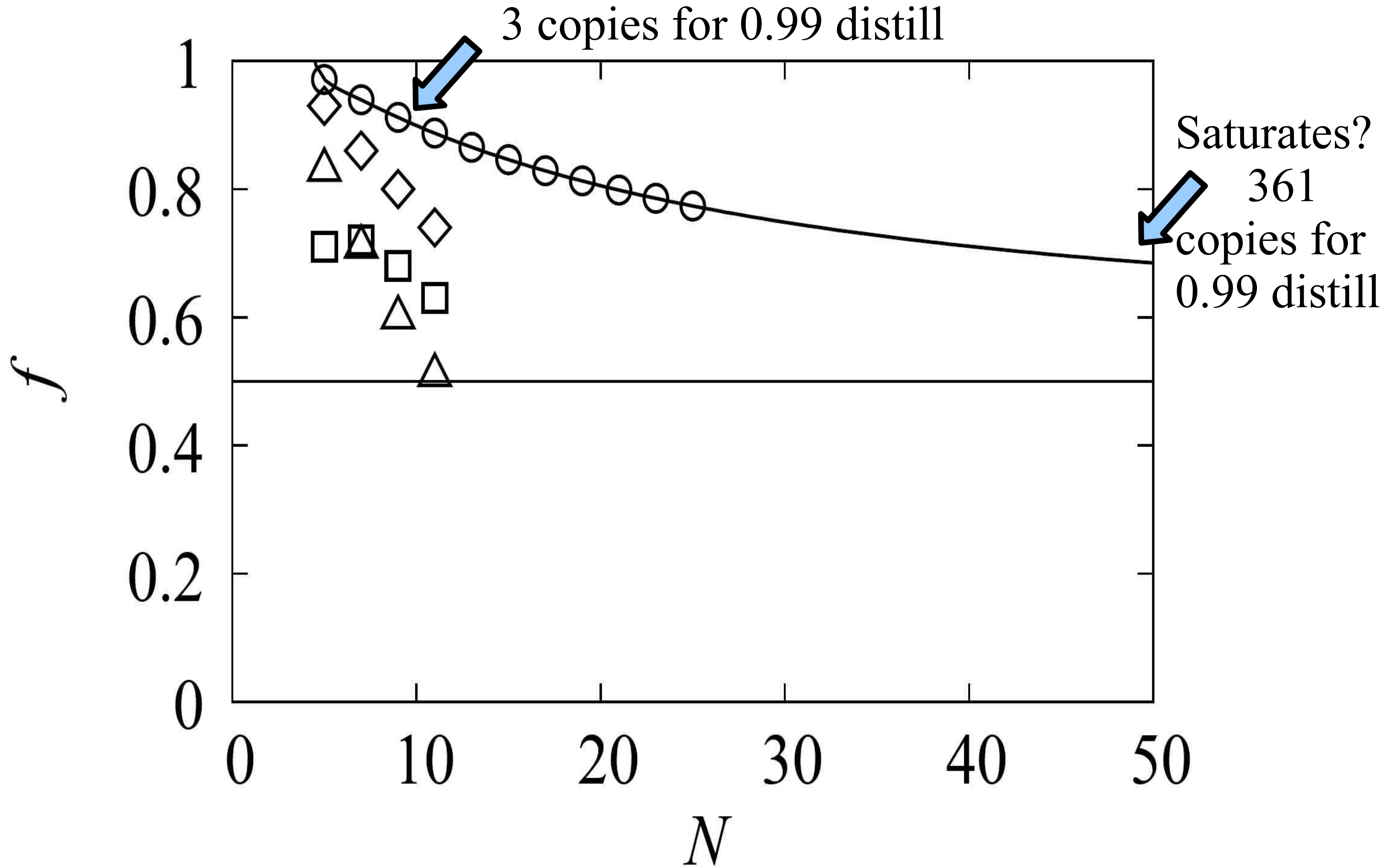
To develop entanglement between ends, complementary blocks need to be in the same state:



Nearest neighbour exchanges only in the above configuration



1
0.9
0.8
0.7
0.6
0.5
f
∝ N^{−ν}
¿
= 0.22
80
60
40
20
0
JT_max/2
∝ N
1
10
100
N


3 copies for 0.99 distill
Saturates?
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0.99 distill
1
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0.4
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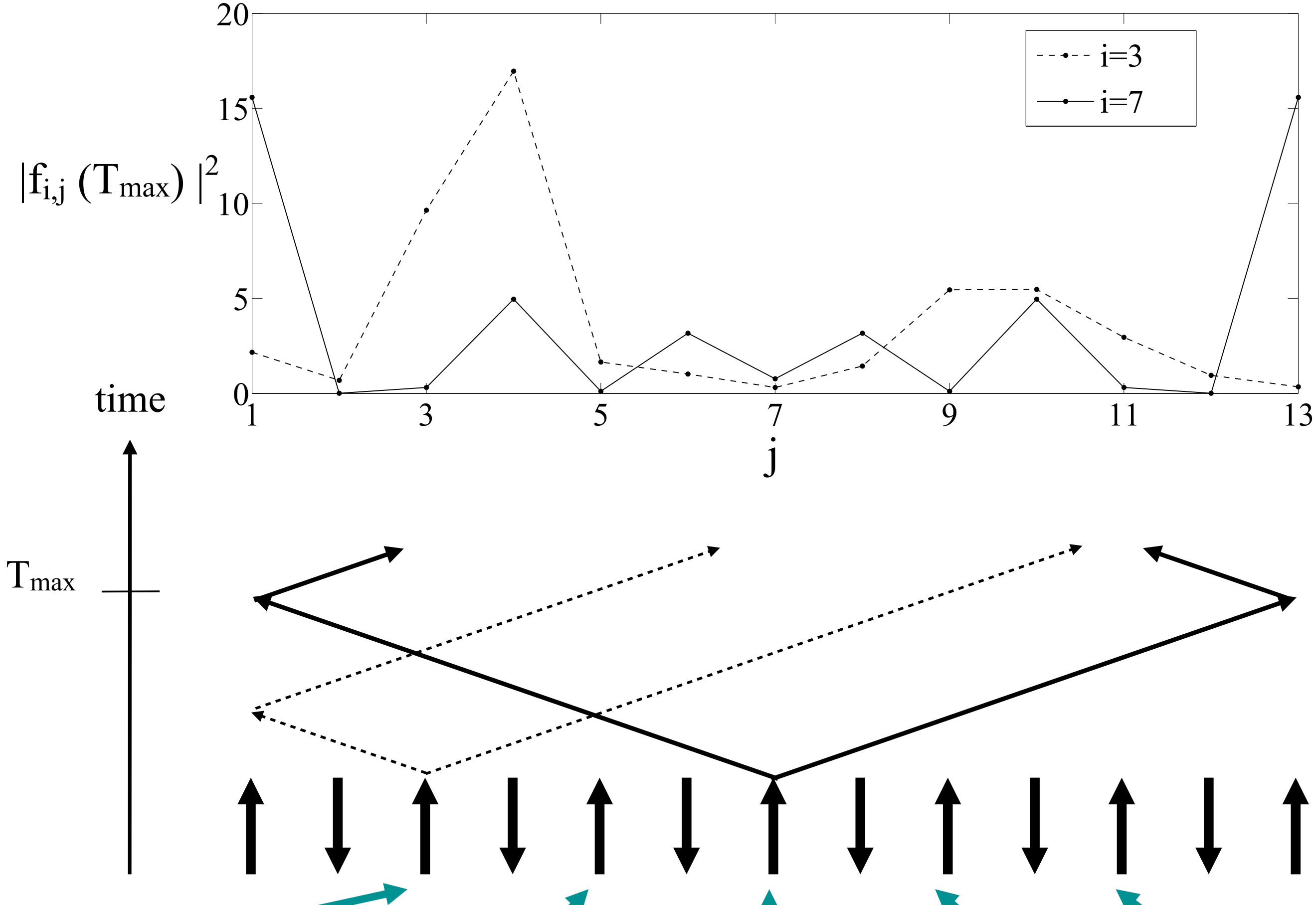
10
20
30
40
50
N

Explanation: A number of sources cooperatively give off diagonal terms of the same sign. Best at ~ T/2 as the sources can make as balanced a contribution to the two ends as they can possibly make.



The fermions do not see each other except for statistics --- whose effects are also cancelled here by the special initial state

$$i(\alpha|10\rangle_{1N}+\beta|01\rangle_{1N})$$

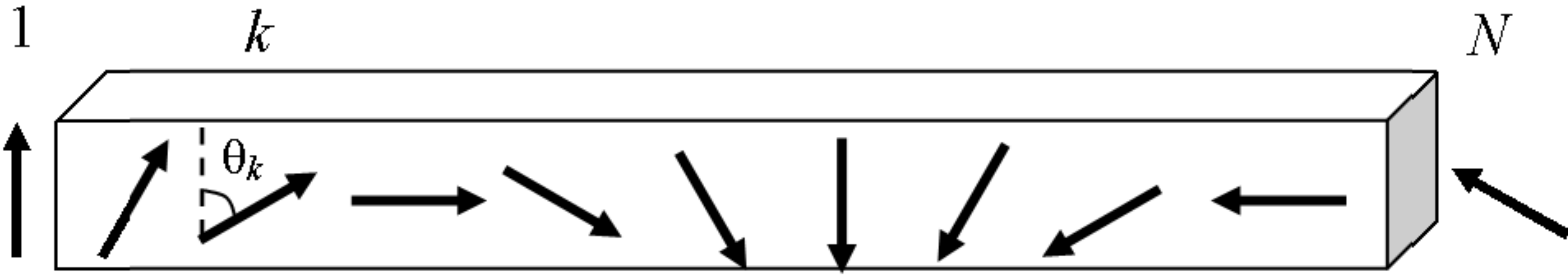
$$i(\alpha'|10\rangle_{1N}+\beta'|01\rangle_{1N})$$

$$i(|10\rangle_{1N}+|01\rangle_{1N})$$

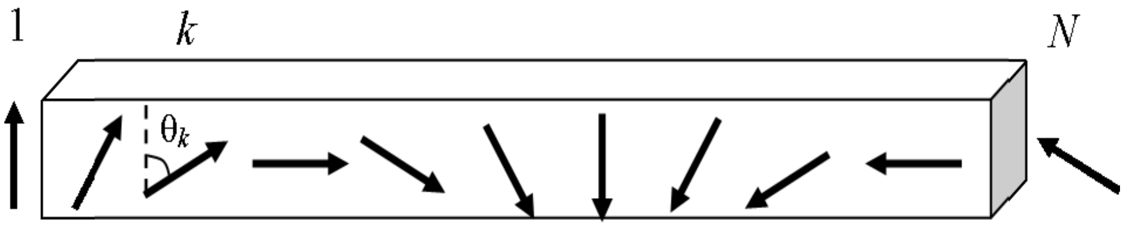
$$i(\beta'|10\rangle_{1N}+\alpha'|01\rangle_{1N})$$

$$i(\beta|10\rangle_{1N}+\alpha|01\rangle_{1N})$$

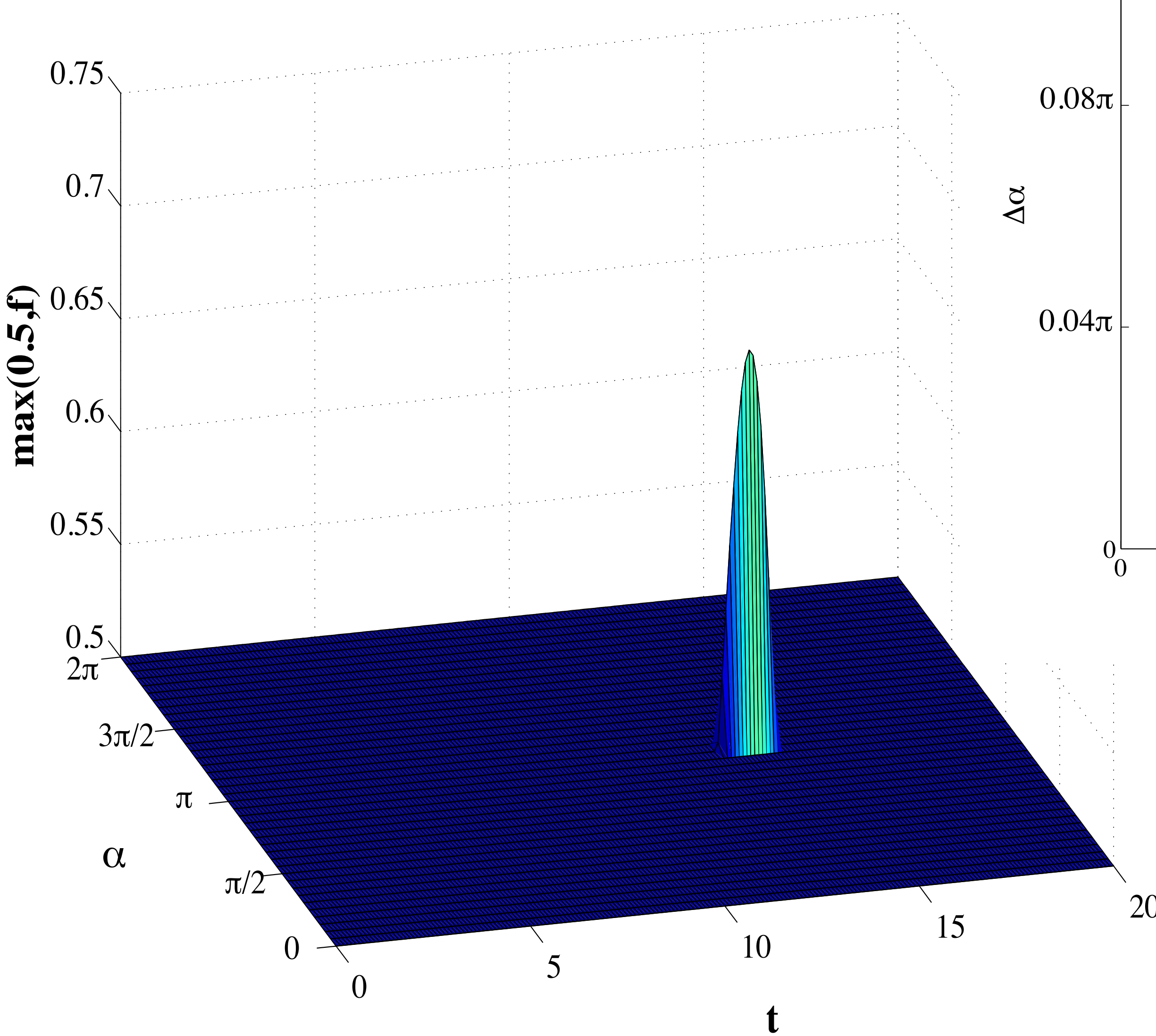
Which other product initial states work (as just an academic question)?

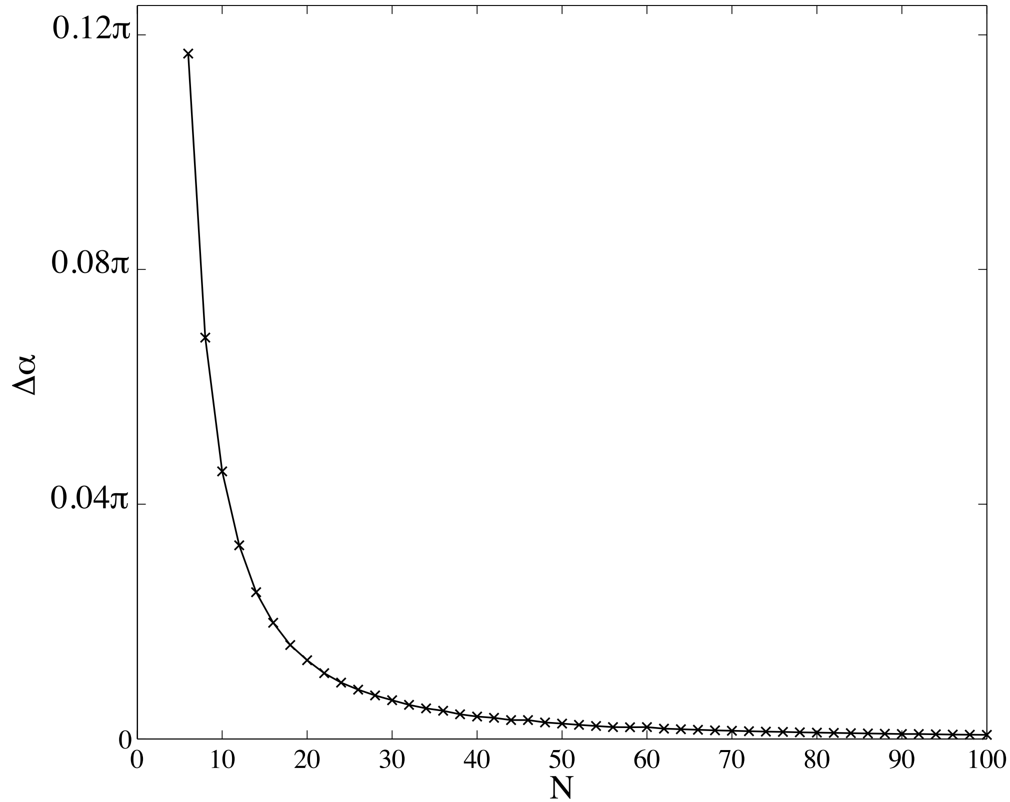


B. Alkurtass, H. Wichterich and S. Bose, PRA (*in submission*)



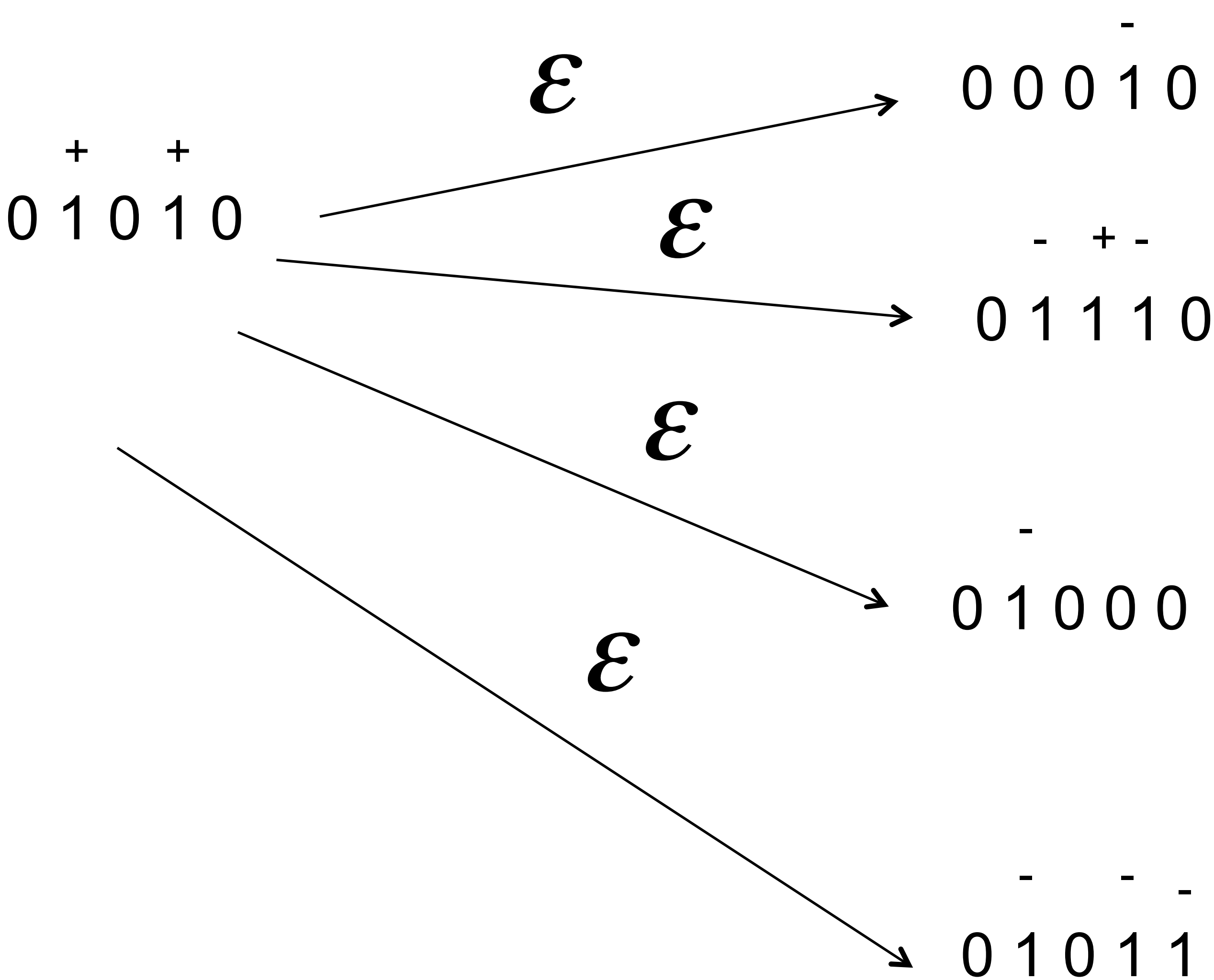
Literally only for Neel Order!!





B. Alkurtass, H. Wichterich and S. Bose, PRA (in submission)

Why so thin a spread around the Neel state?



When $N\varepsilon \approx 1$ entanglement should vanish

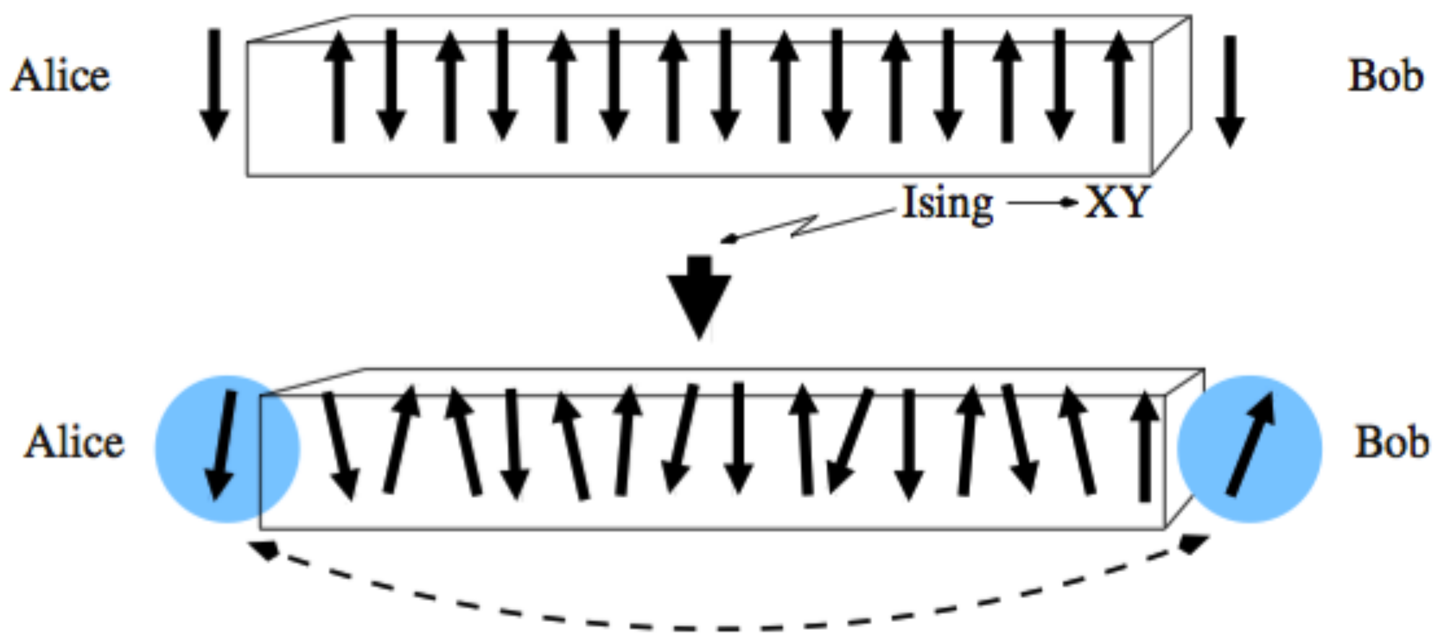
Decrease with distance is a problem. Can we optimize further?

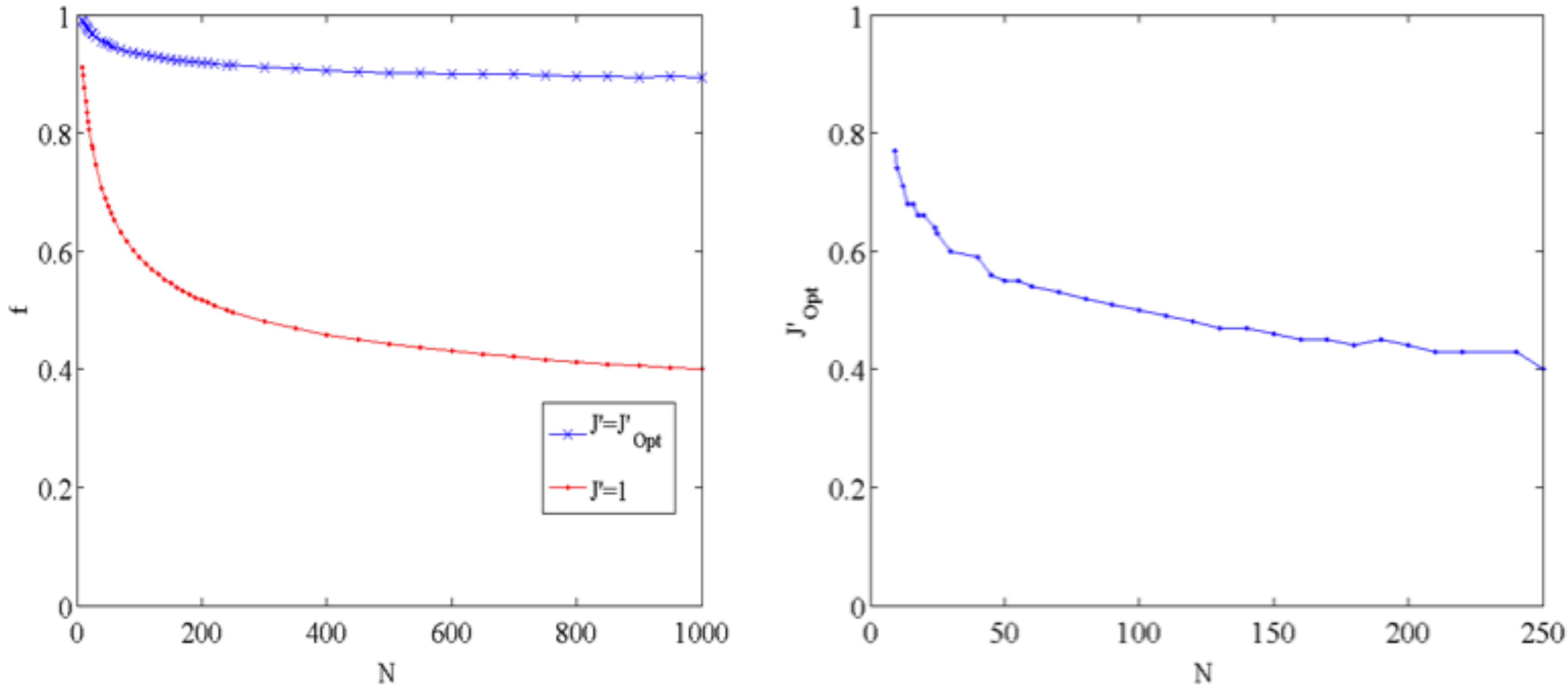
$$H_0 \xrightarrow{t=0} H_1$$

$$H_0 = \sum_{i=1}^{N-1} \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

Optimized Weaker Coupling

$$H_1 = \frac{J}{2} \sum_{i=2}^{N-2} \left(\hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y \right) + \frac{J'}{2} \sum_{i=1,N-1} \left(\hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y \right)$$





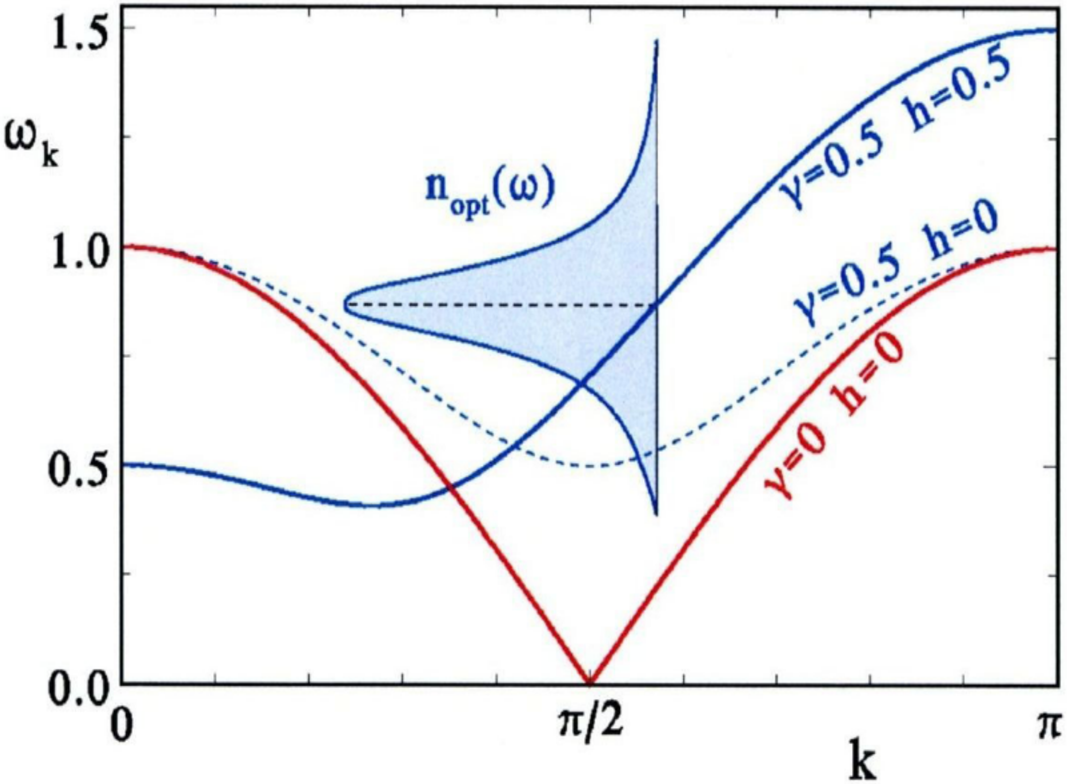
Distance independent entanglement !

Entanglement between spins 1,N versus chain length

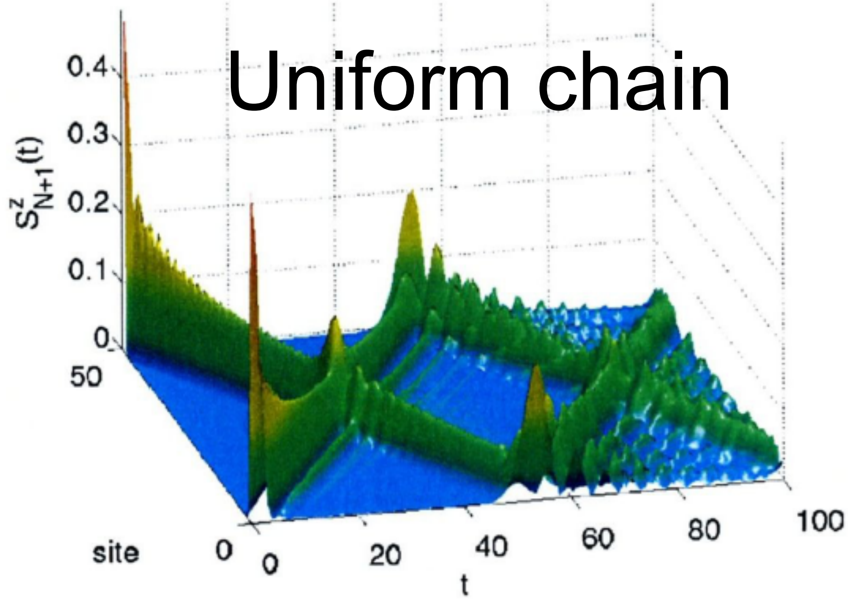
Optimal J'

$$J'_{Opt} \sim N^{-1/6}$$

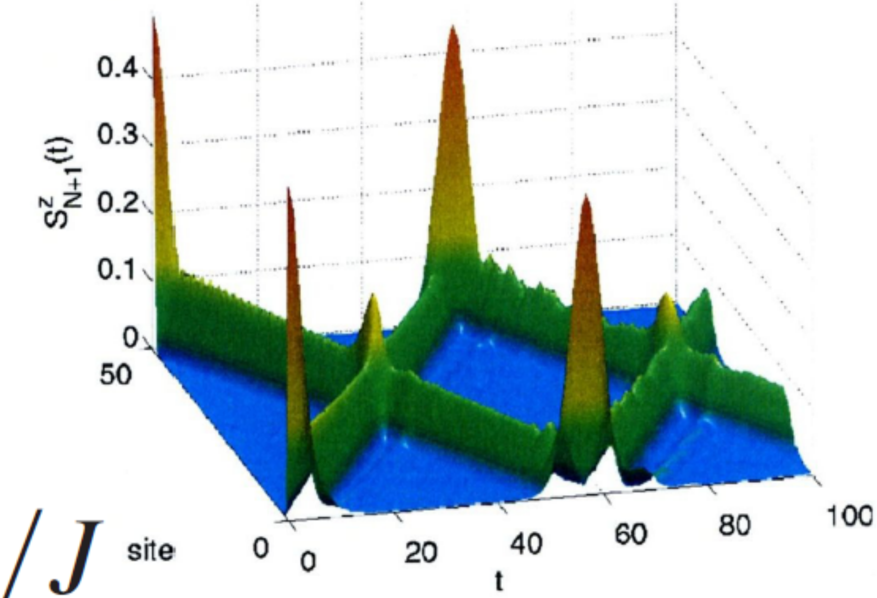
Alkurtass, Bose (in preparation)



Uniform chain



Optimized J_0



Banchi, Apollaro, Cuccoli, Vaia, Verucchi, PRA 2010.

$$J_0^{\mathrm{opt}} \simeq 1.05 J N^{-1/6}$$

$$t^* \simeq (0.25N + 0.52N^{1/3})/J$$

$$\left\langle c_1^+(t)c_N(t)\right\rangle = \sum_{m=odd} f_{1m}^*(t) f_{mN}(t)$$

$$= \sum_{m=odd} \left\langle 1\right| e^{iHt} \left|m\right\rangle\left\langle m\right| e^{-iHt} \left|N\right\rangle$$

1010

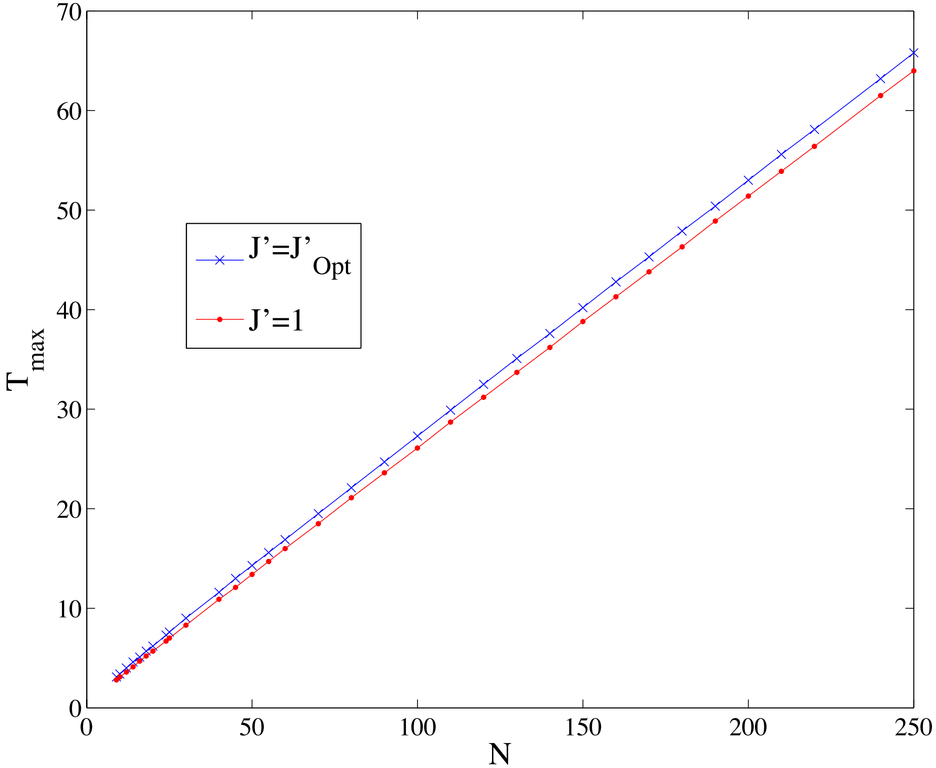
$$= \sum_{m=odd} \left\langle 1\right| e^{-iHt} \left|m\right\rangle\left\langle m\right| e^{-iHt} \left|N\right\rangle$$

$$= \frac{1}{2}\sum_{m=all} \left\langle 1\right| e^{-iHt} \left|m\right\rangle\left\langle m\right| e^{-iHt} \left|N\right\rangle$$

Half the time required for state transfer!

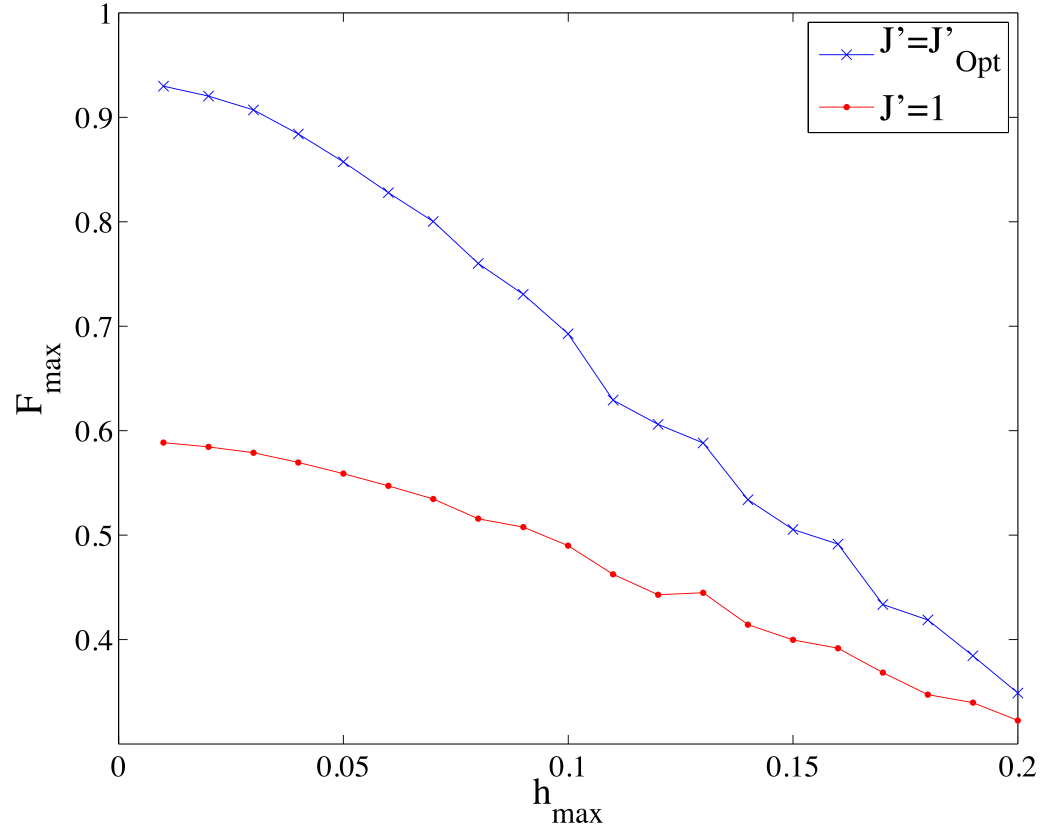
$$= \frac{1}{2} f_{1N}(2t)$$

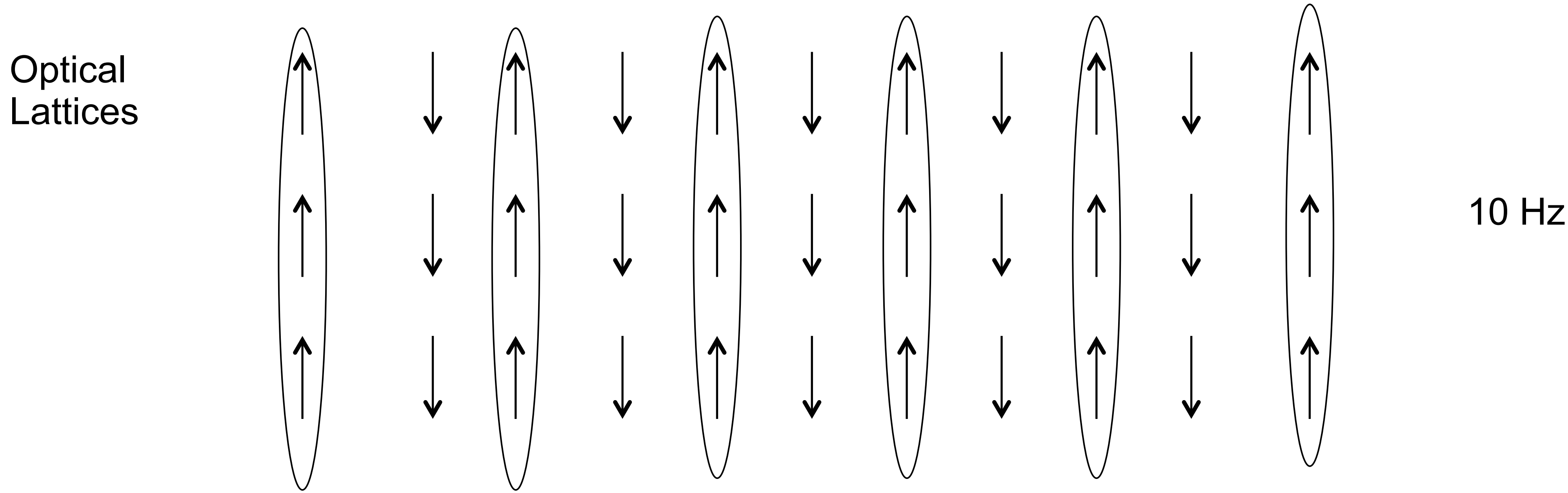
Alkurtass & Bose, *In preparation*.

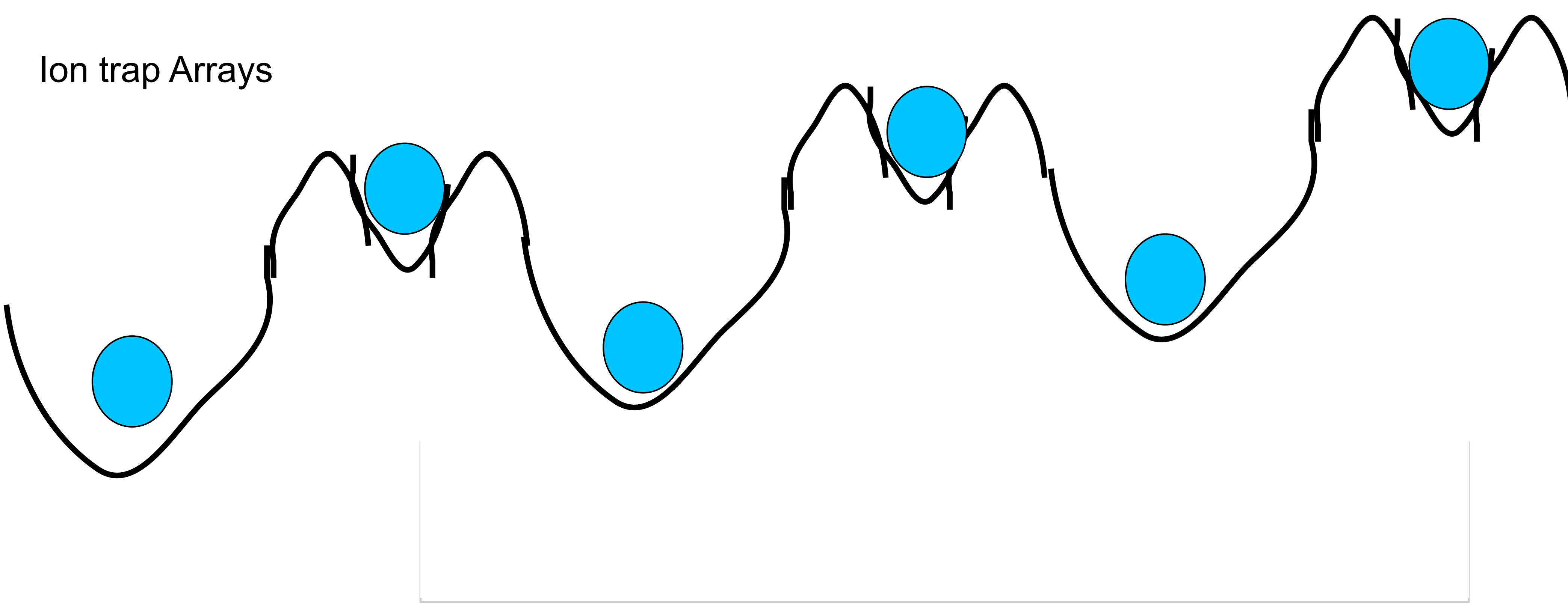


Time

Dephasing

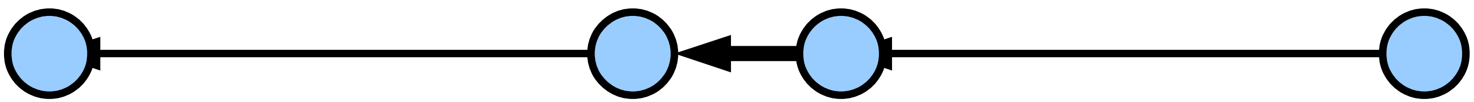



Optical
Lattices
10 Hz


Ion trap Arrays

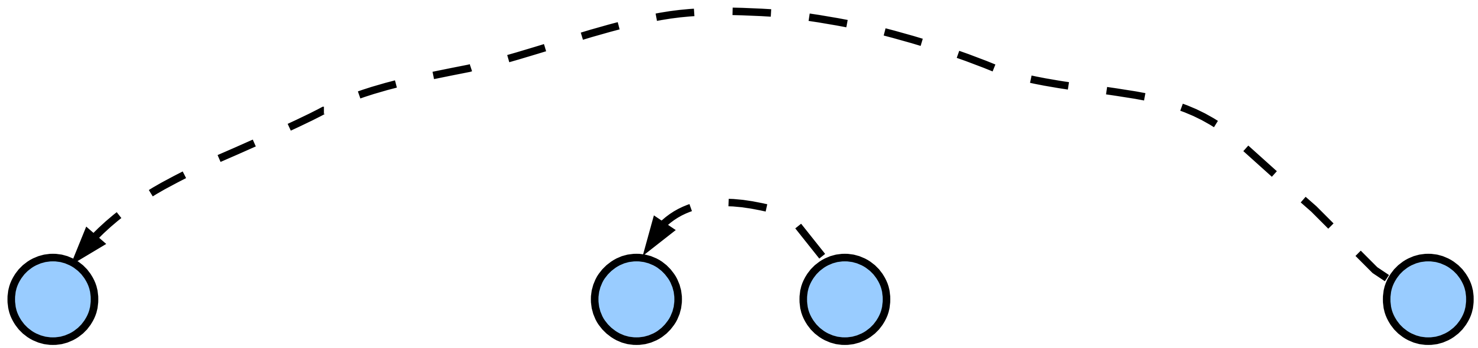
$$|gs\rangle = |\psi^-\rangle \otimes |\psi^-\rangle$$

$$|gs\rangle = |\psi^-\rangle \otimes |\psi^-\rangle$$



$$J_m = J_1' + J_2'$$

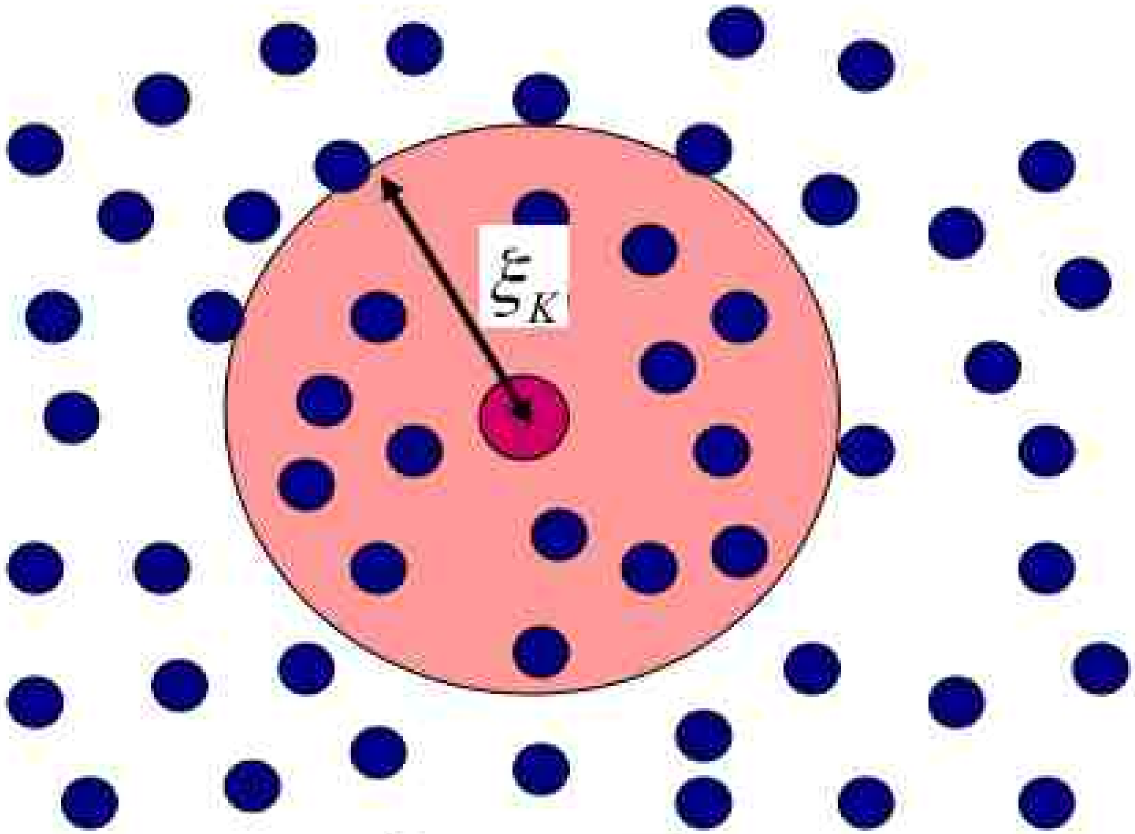
$$|gs\rangle = |\psi^{-}\rangle \otimes |\psi^{-}\rangle$$



$$|\psi(t)\rangle = e^{-iE_{S_1}t}|S_1\rangle\langle S_1|gs\rangle + e^{-iE_{S_2}t}|S_2\rangle\langle S_2|gs\rangle$$

$$\begin{aligned}|\psi(t)\rangle &= \frac{-i\sin(2J_m t)}{2}(|0011\rangle + |1100\rangle) - \frac{\cos(2J_m t)}{2} \\ &\times (|1001\rangle + |0110\rangle) + \frac{e^{i2J_m t}}{2}(|0101\rangle + |1010\rangle)\end{aligned}$$

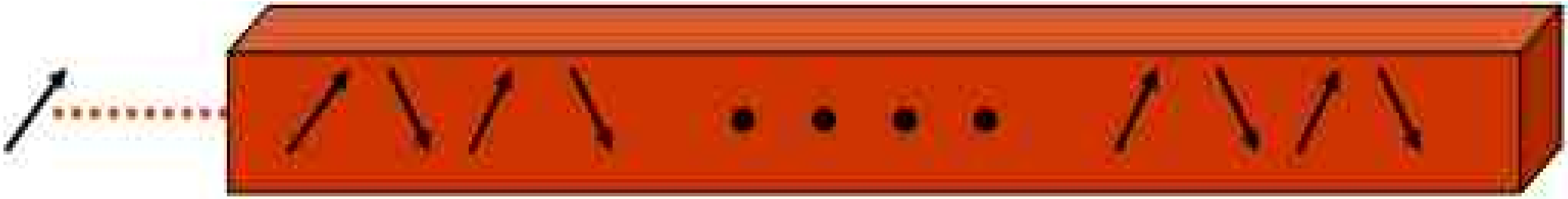
Kondo Physics:



$$\xi_K \Leftrightarrow T_K$$

Despite the gapless nature of the Kondo system, we have a length scale in the model

Kondo Spin Chains:



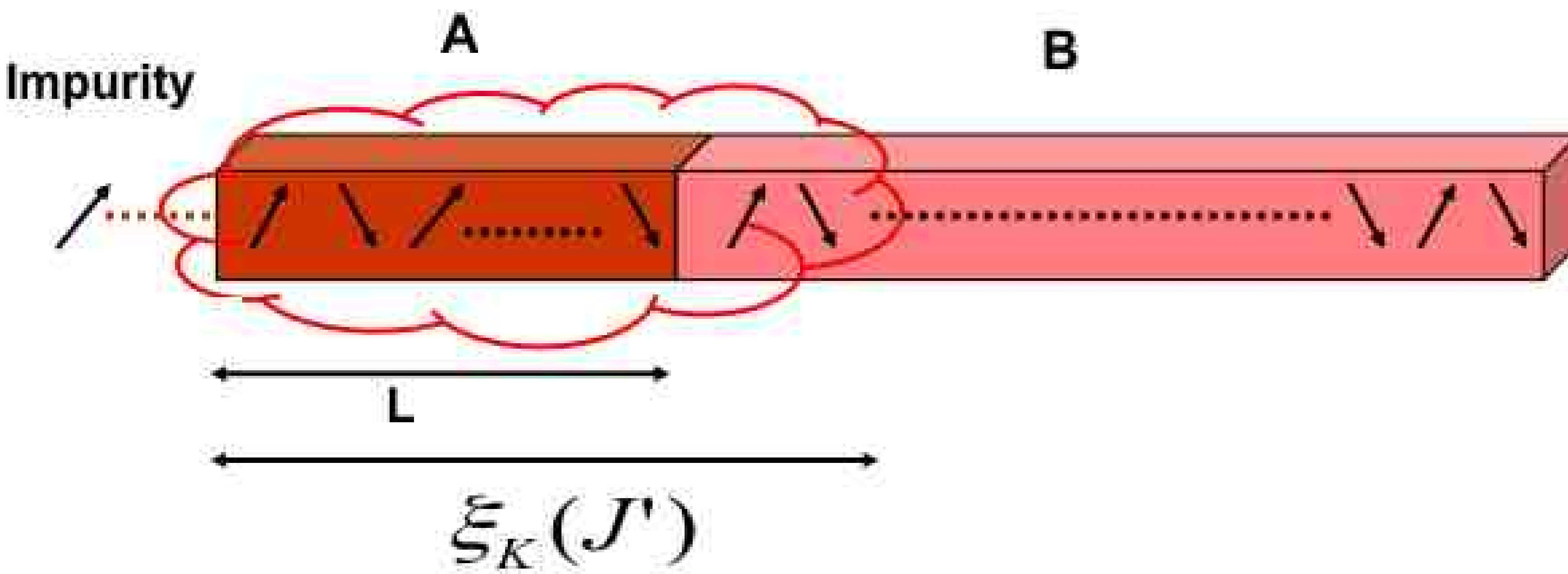
$$H = J'(J_1\sigma_1.\sigma_2 + J_2\sigma_1.\sigma_3) + \sum_{i=2} J_1\sigma_i.\sigma_{i+1} + J_2\sigma_i.\sigma_{i+2}$$

$$\frac{J_2}{J_1} < J_2^c = 0.2412: \quad \text{Kondo (gapless)}$$

$$\frac{J_2}{J_1} > J_2^c: \quad \text{Dimer (gapfull)}$$

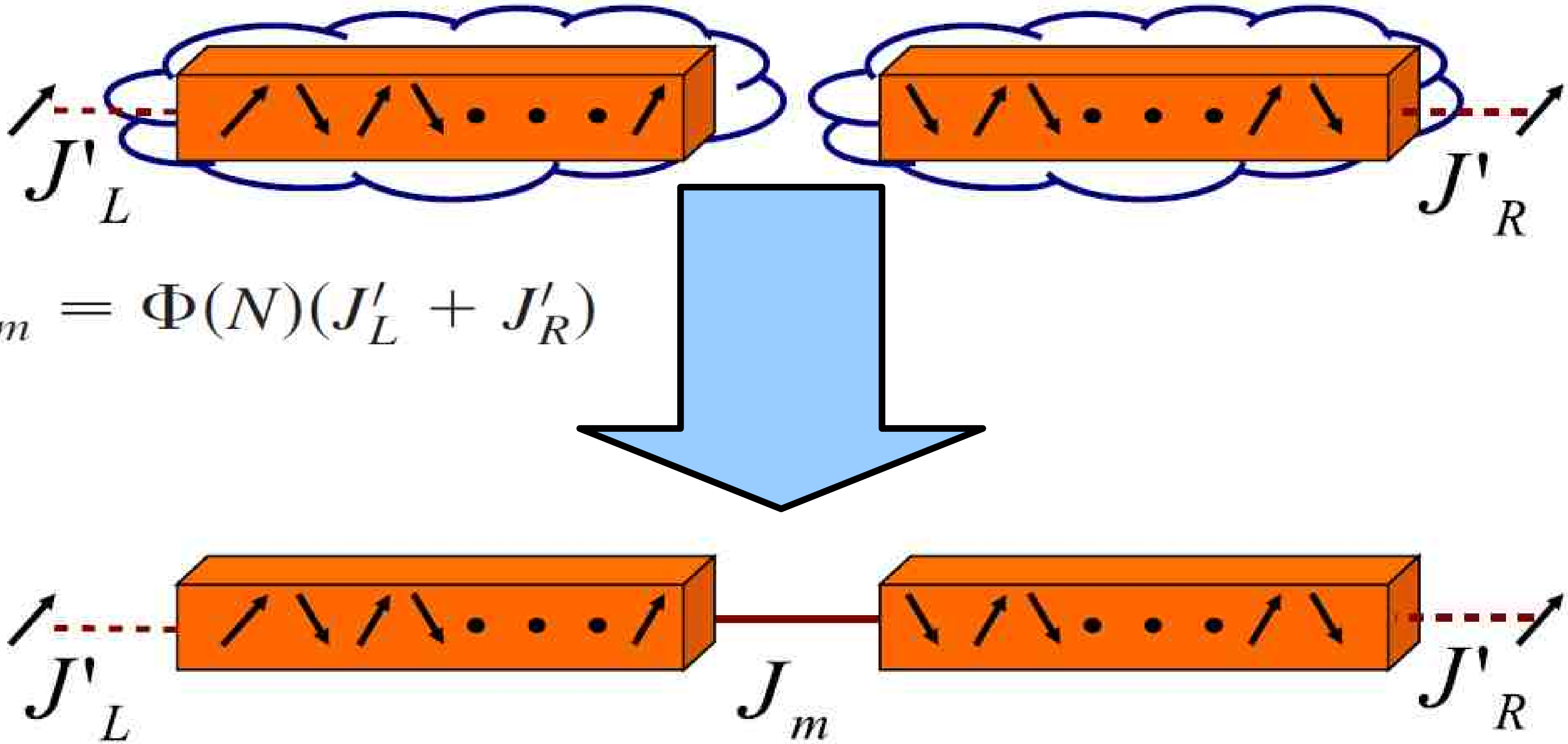
E. S. Sorensen *et al.*, J. Stat. Mech., P08003 (2007)

Entanglement Structure in the Kondo Model:



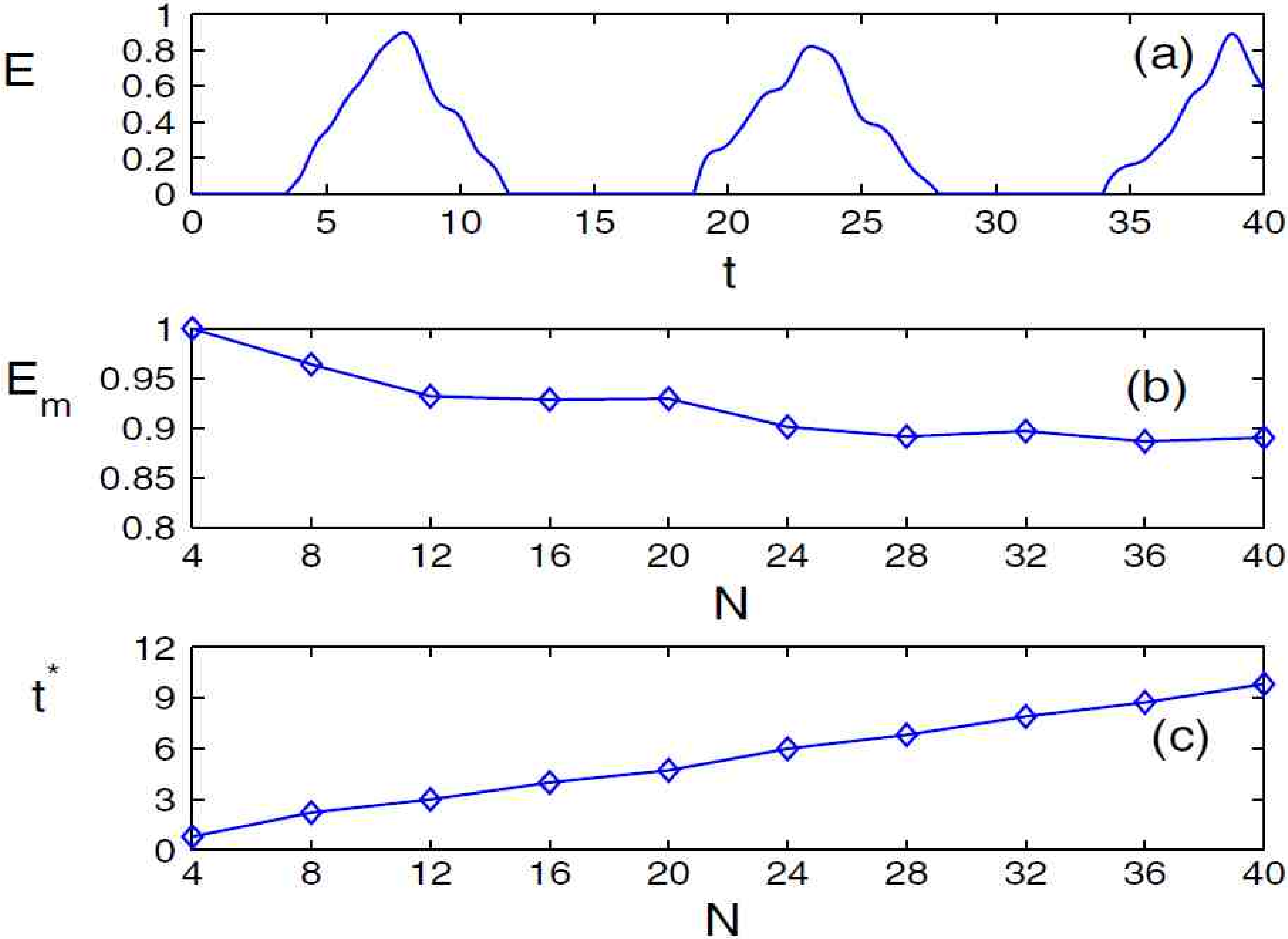
$$\begin{cases} L < \xi_K : \quad E_{SA} < 1 \Rightarrow E_{SB} > 0 \\ L = \xi_K : \quad E_{SA} = 1 \Rightarrow E_{SB} = 0 \\ L > \xi_K : \quad E_{SA} = 1 \Rightarrow E_{SB} = 0 \end{cases}$$

Literally generalizing the 4 spin case!

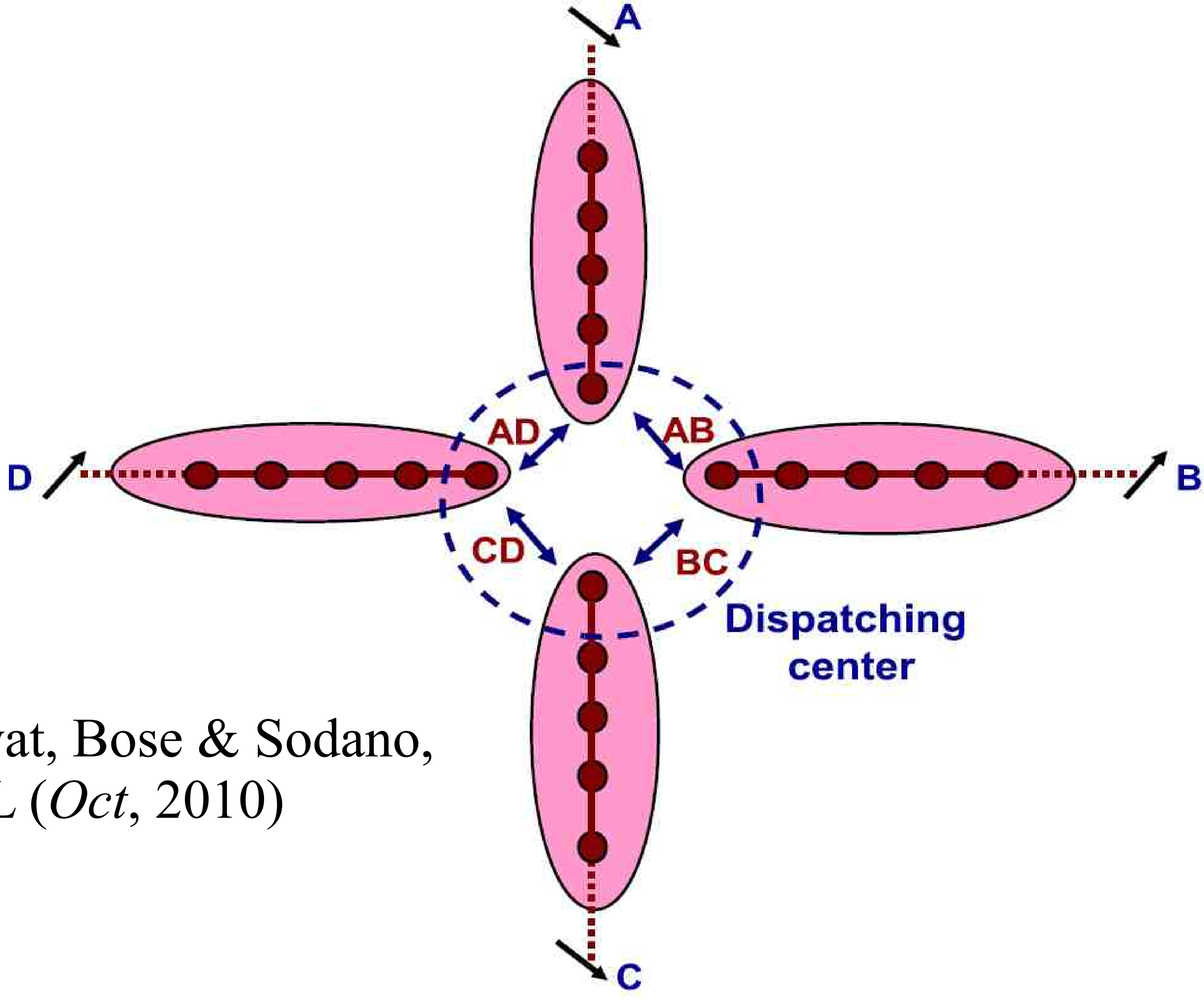


$$J_m = \Phi(N)(J'_L + J'_R)$$

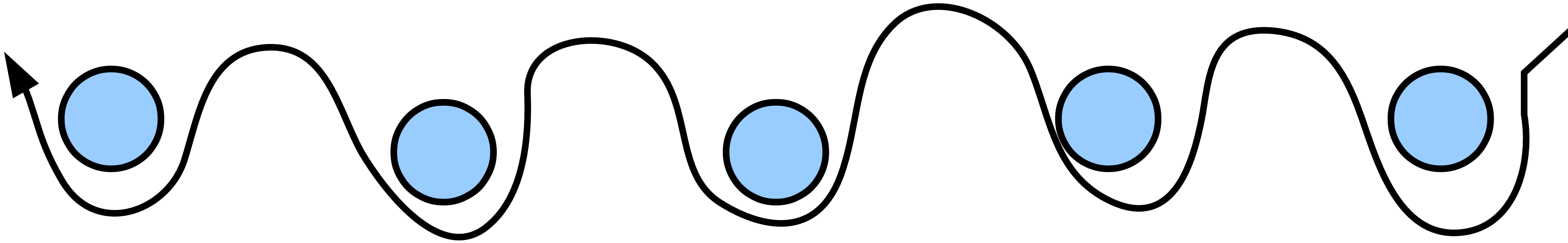
Bayat, Bose & Sodano, PRL (*Oct*, 2010)



E
(a)
t
E
m
(b)
N
t
*
(c)
N



Bayat, Bose & Sodano,
PRL (*Oct*, 2010)



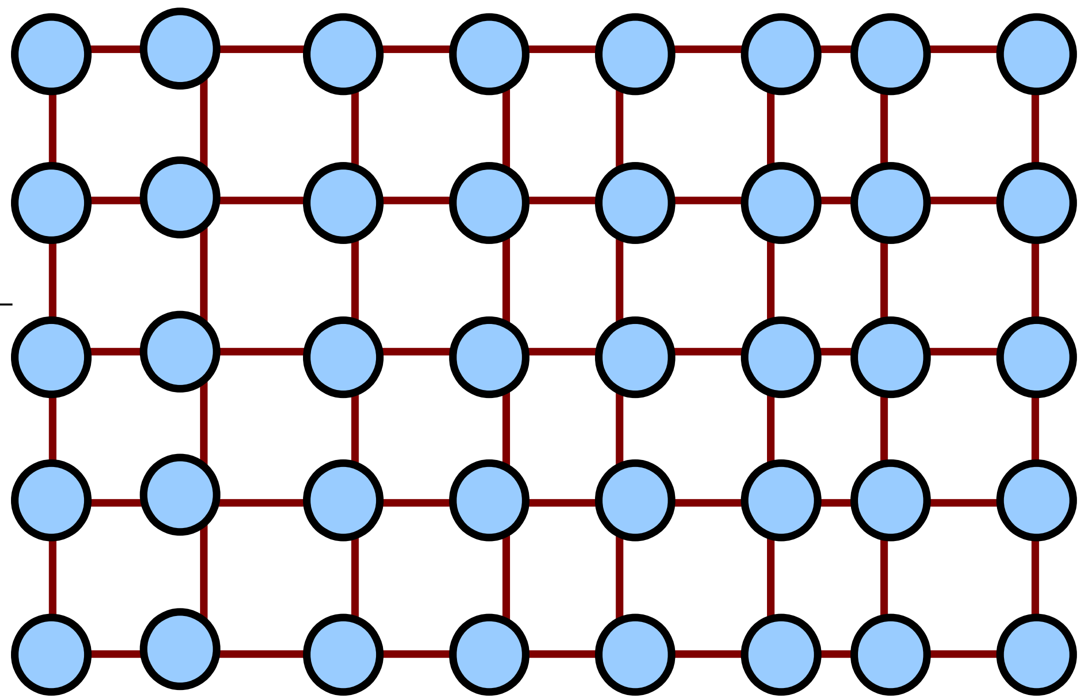
Hubbard models, Strongly correlated spin systems etc.

Cold atoms are ideal for
non-dissipative non-equilibrium
Dynamics (closed system *dynamics* of a quantum
Many-body system). I will discuss two **Quantum Information Applications of their dynamics**

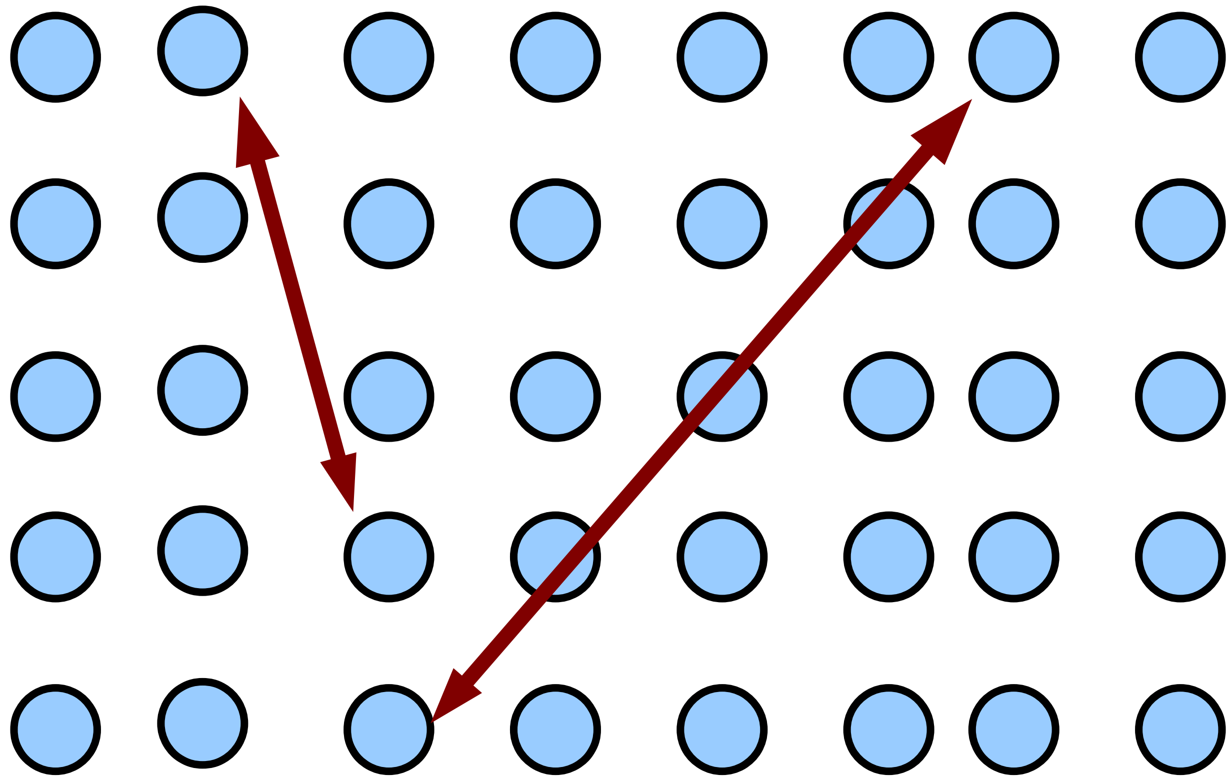
Cold atoms have already been proposed as a system for quantum computaion

But one obstacle: Nearest neighbour interactions & the lack of specificity of interactions

Still a certain form of quantum computation is known to be possible (measurement based quantum computing) – but this requires many measurements



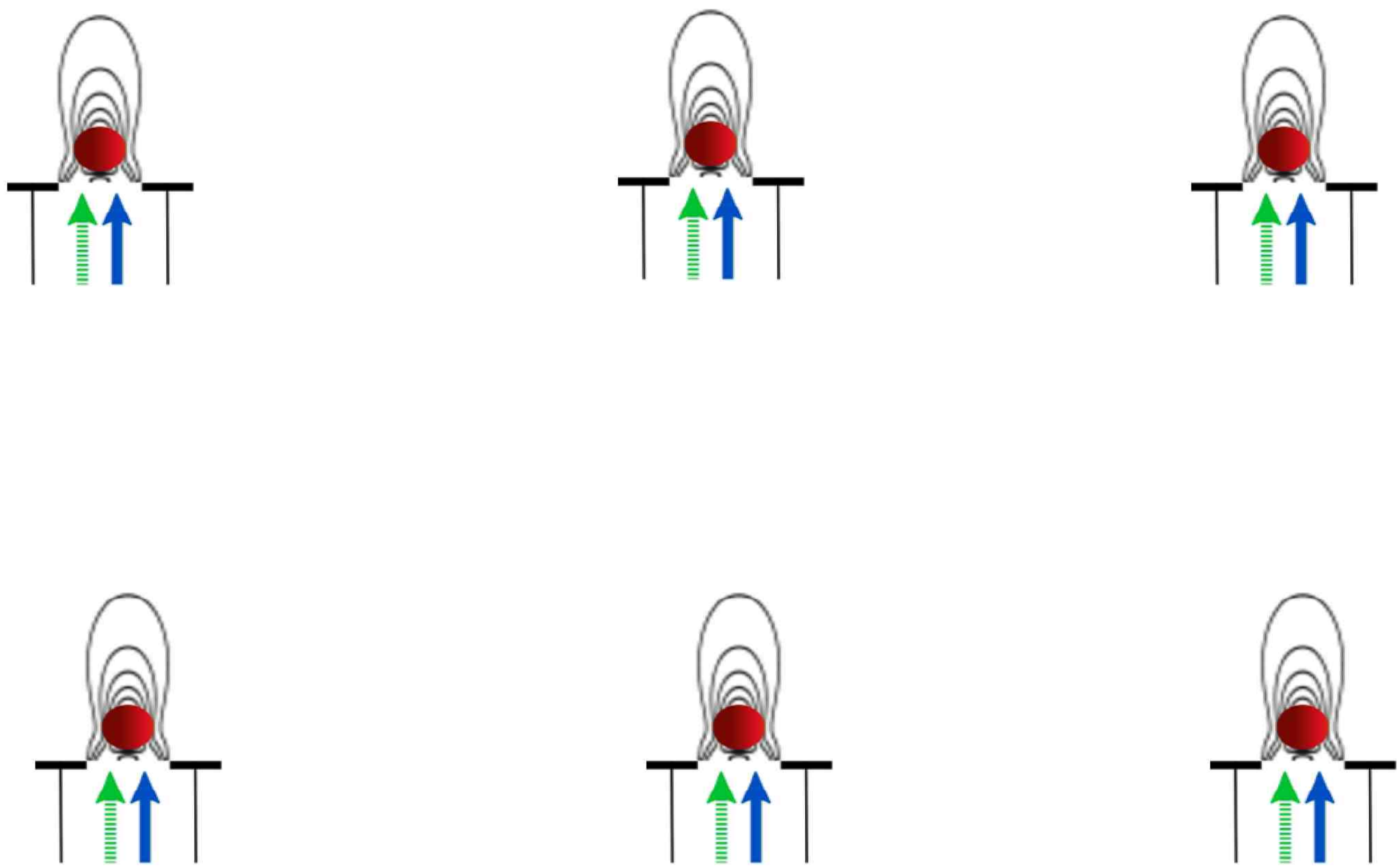
For example you cannot do:



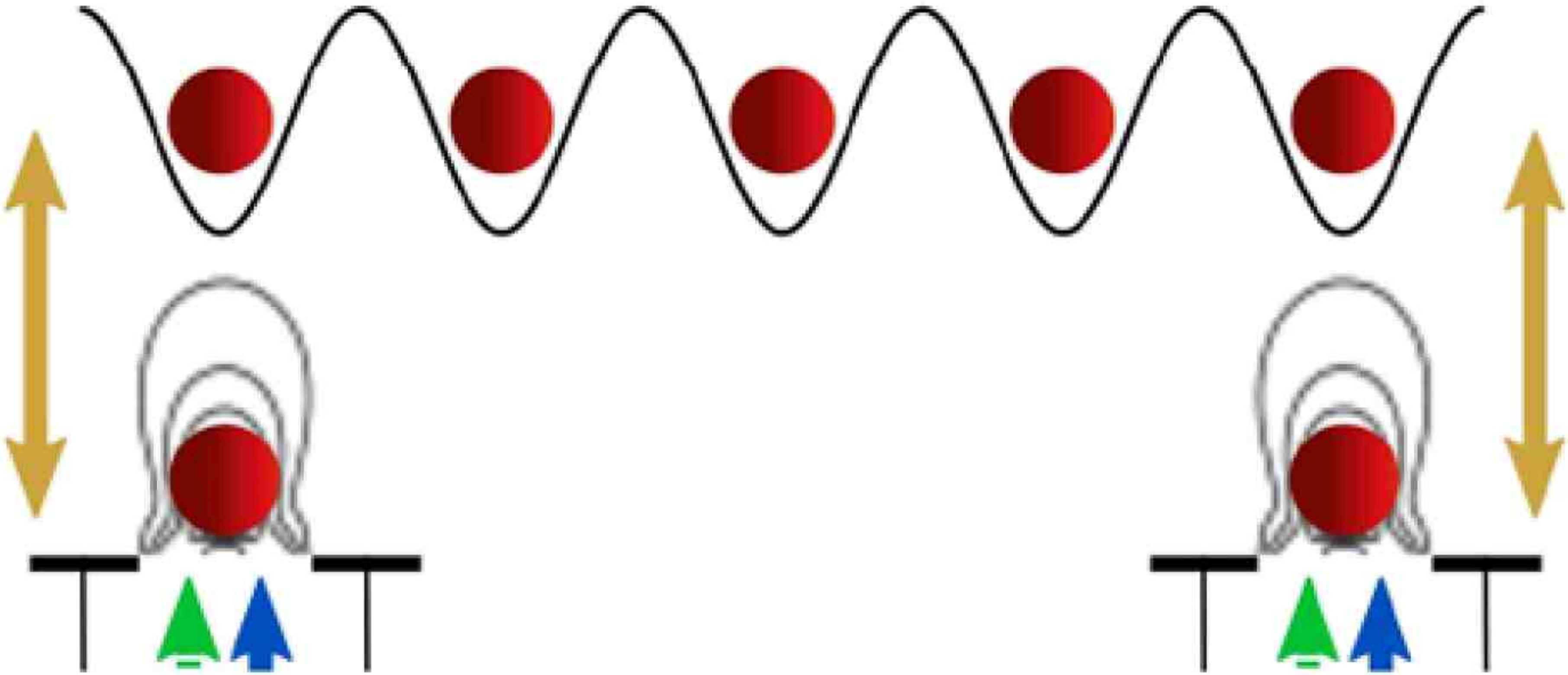
General problem for any implementation, where the specificity of gates between arbitrary pairs of qubits are absent *without* physical movement.

What kind of gates are required? **Gates that can**

So we propose to take them all apart from each other – so that *normally no one interacts with no one now!*



So what is the idea?



Banchi, Bayat, Verrucchi, Bose, Phys. Rev. Lett. 106, 140501 (2011)

Three ingredients needed from recent history:

Ingredient 1:

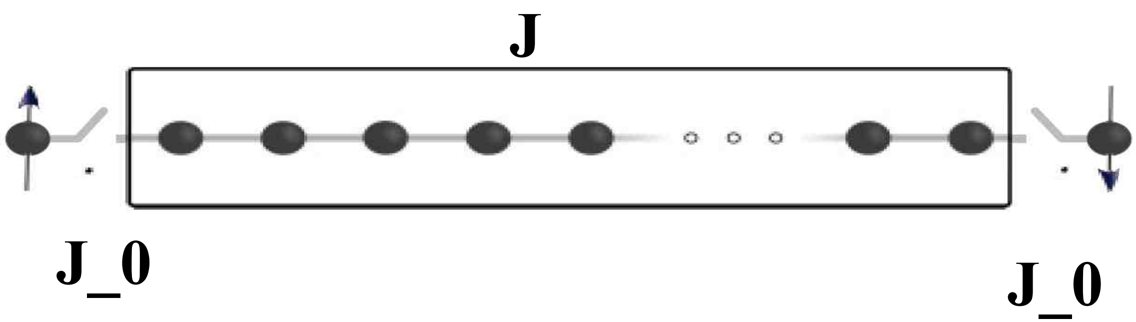
Spin Chains can be realized with cold atoms in optical lattices: (Duan-Demler-Lukin 2003, Garcia-Ripoll, Cirac)

Atomic internal levels: say, two hyperfine grund levels |g_1> and |g_2> stand for the spin-1/2 levels |up> and |down> (qubit levels |0> and |1>).

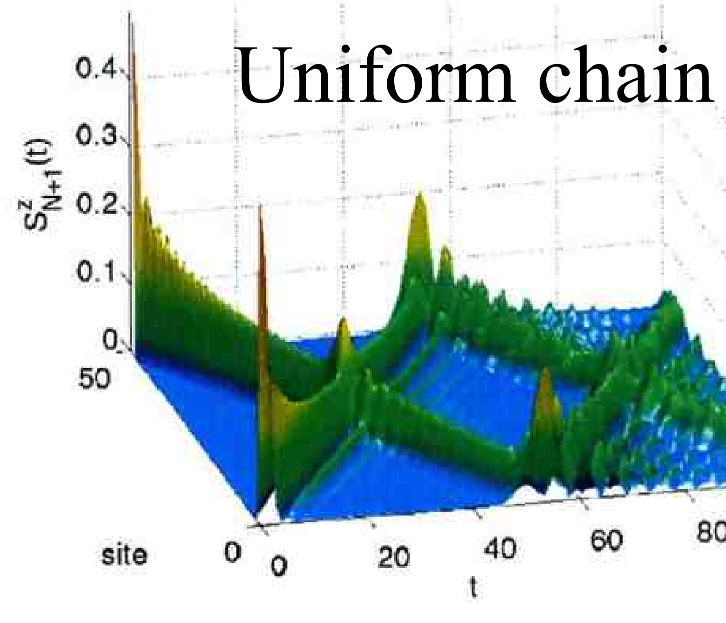
Realizable spin models:

$$H_M = J \sum_{n=1}^{N-1} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \lambda \sigma_n^z \sigma_{n+1}^z)$$

Ingredient 2:



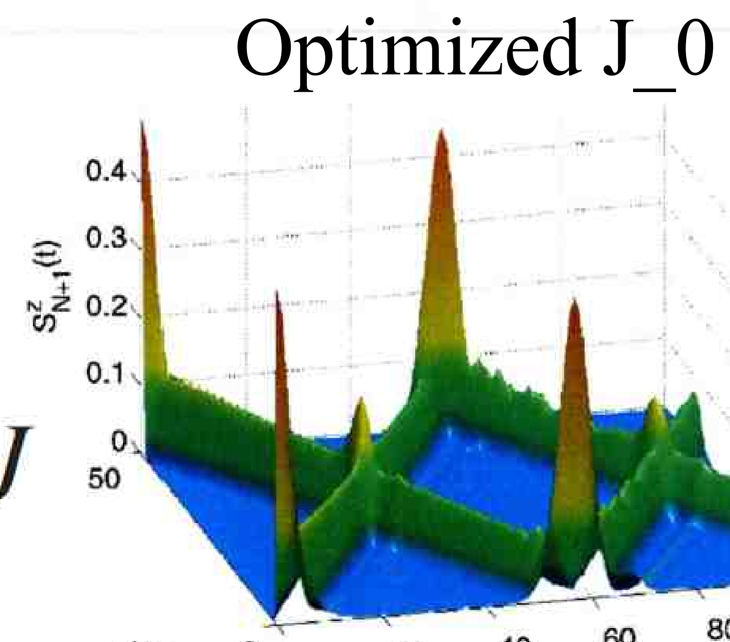
Mirror inversion on dynamics



Banchi, Apollaro, Cuccoli, Vaia, Verucchi, PRA 2010.

$$J_0^{\mathrm{opt}} \simeq 1.05 J N^{-1/6}$$

$$t^* \simeq (0.25N + 0.52N^{1/3})/J$$



Ingredient 3: Quantum Gates from Fermion Exchange

Yung & Bose, PRA 2005, Clark, Moura-Alves, Jaksch, NJP 2005
Yung, Benjamin, Bose, PRL 2006, Yao, Lukin et al, PRL 2011

Suppose there is a mechanism of swap

Indeed, for the mirror inverting XY quantum chains

$$|000000\rangle$$

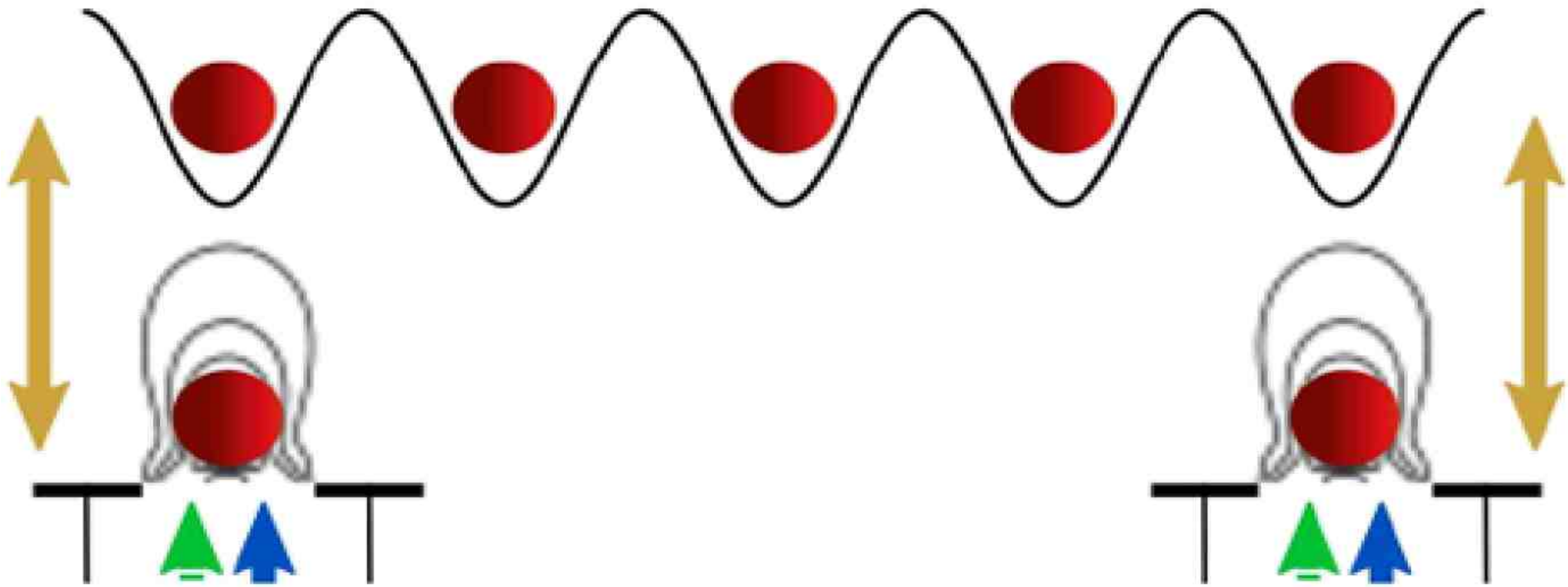
$$|100000\rangle \equiv c_1^\dagger |000000\rangle$$

$$|000001\rangle \equiv c_6^\dagger |000000\rangle$$

$$|100001\rangle = c_1^\dagger c_6^\dagger |000000\rangle$$

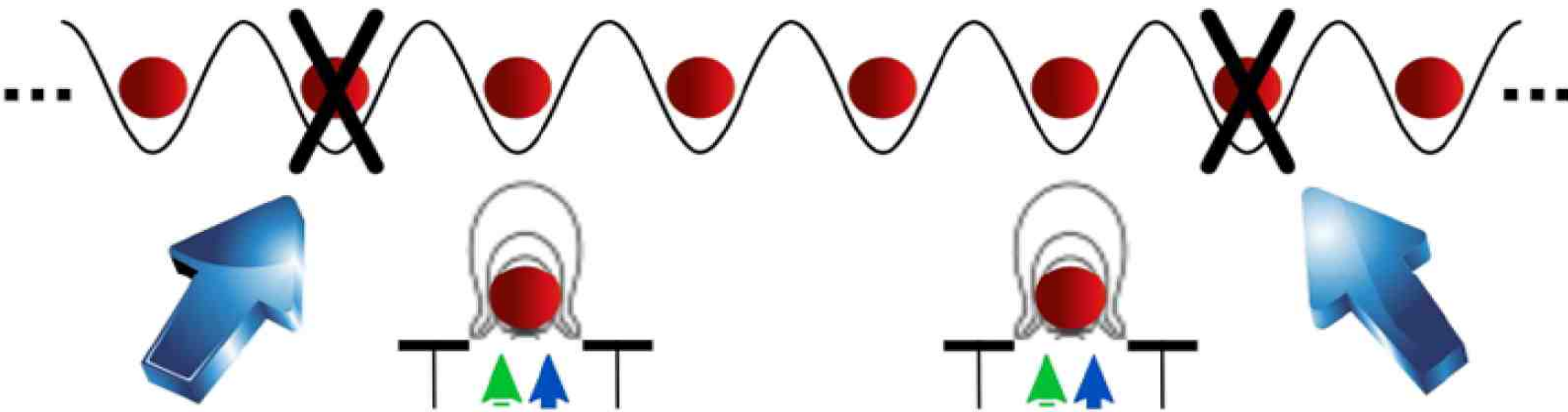
Q. Gate on mirror inversion (Controlled phase gate)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



$$H_M = J \sum_{n=1}^{N-1} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \lambda \sigma_n^z \sigma_{n+1}^z)$$

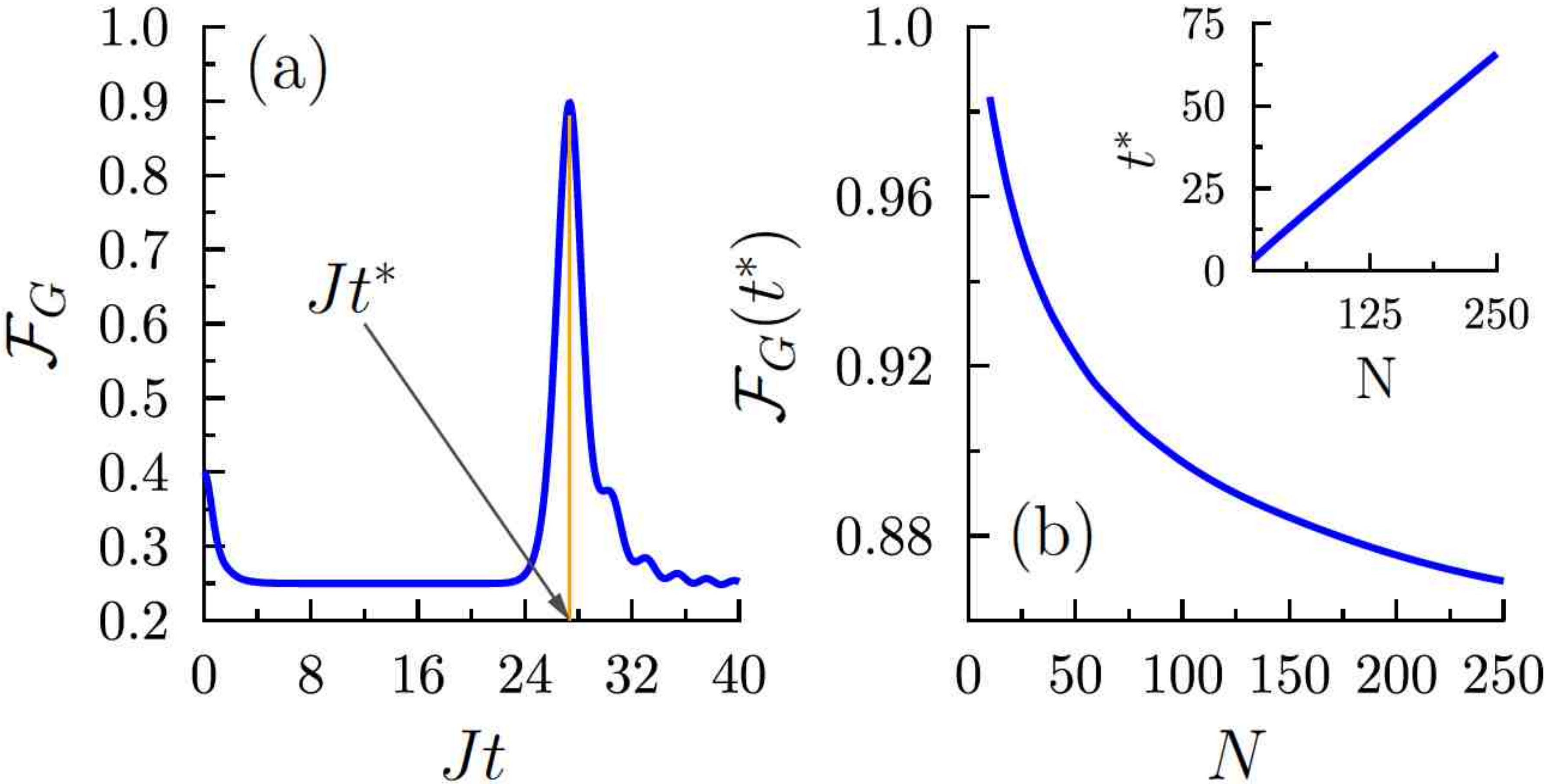
$$H_I = J_0 \sum_{n=0,N} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \lambda \sigma_n^z \sigma_{n+1}^z)$$



In practice --- as we want to interface ANY pair of trapped atoms

Banchi, Bayat, Verrucchi, Bose, Phys. Rev. Lett. **106**, 140501 (2011)

$$J_0^{\rm opt} \simeq 1.05 J N^{-1/6}\, , \; t^* \simeq (0.25N + 0.52 N^{1/3})/J$$

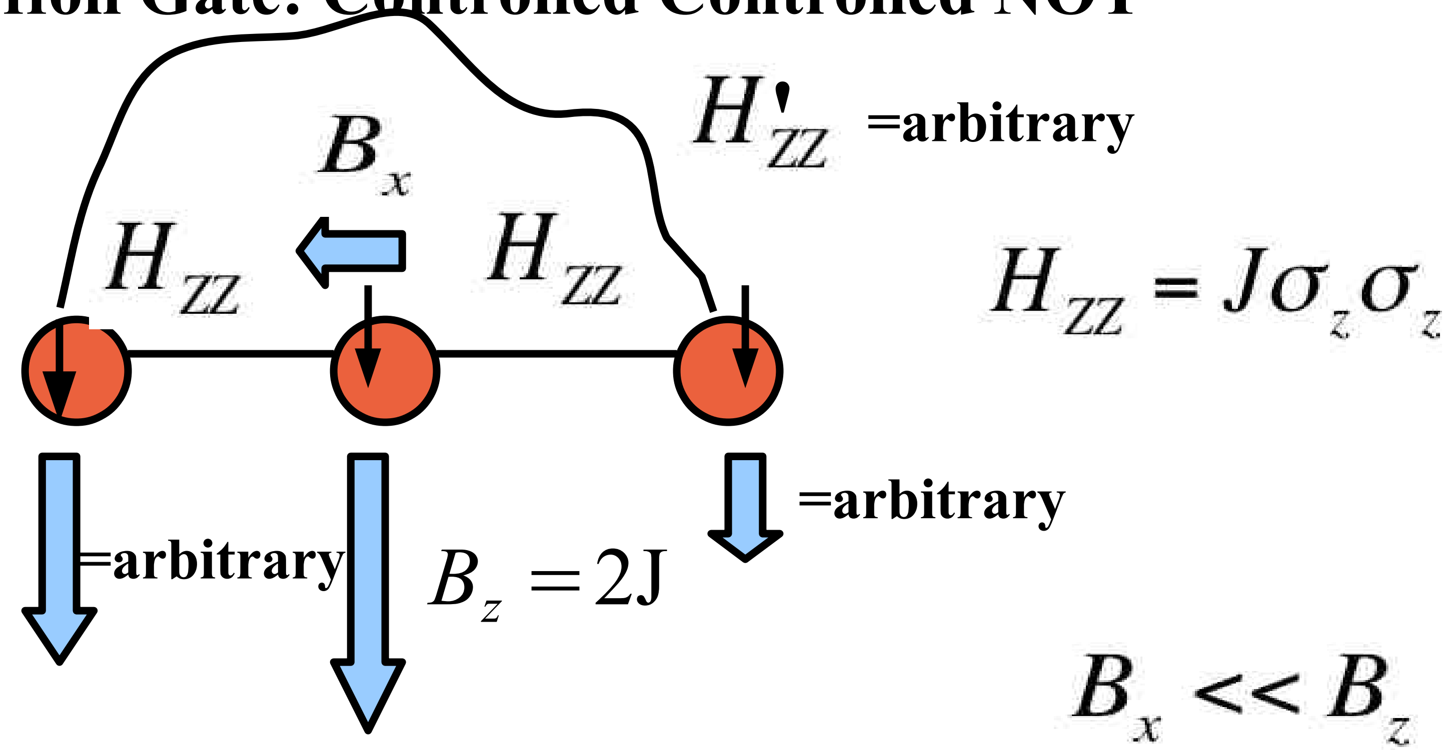


$$\mathcal{F}_G(t) = \int d\psi \, \langle\psi|G^\dagger \mathcal{E}_t[|\psi\rangle\langle\psi|]G|\psi\rangle$$

Agenda: Why not do classical computation by quantum non-adiabatic dymanics? (Reversibility, Speed, Minimal Dissipation, Miniaturization)

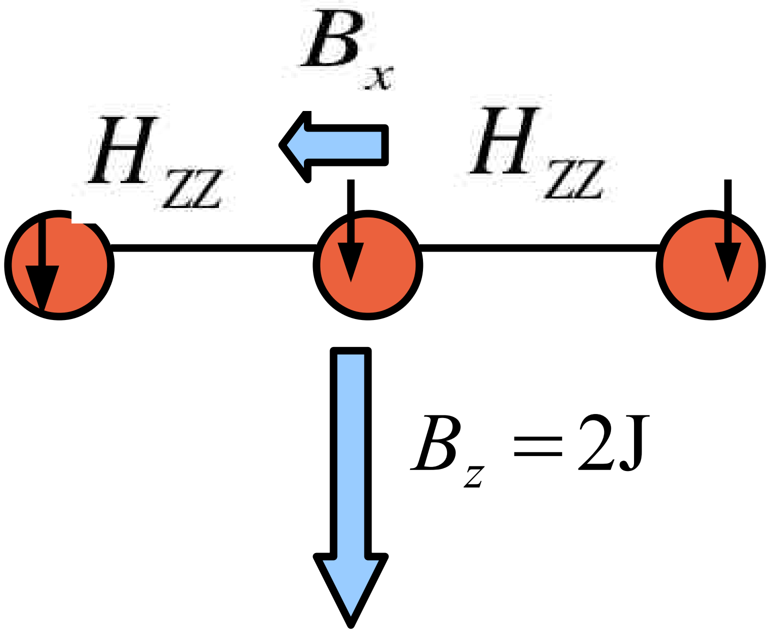
Spin Molecuels for 3-bit Gates for Reversible Computation

Toffoli Gate: Controlled Controlled NOT



B. Antonio, G. W. Morley, J. Randall, W. Hensinger, S. Bose, *in preparation*

Toffoli Gate: Controlled Controlled NOT (I have removed all the unnecessary terms):



$$H_{ZZ} = J\sigma_z\sigma_z$$

$$B_x << B_z$$

$$\nu = 2\pi\, 100\, KHz$$

Separation = 14 μm

Magnetic field gradient =100 T/m, we have $J = 2\pi\, 1.6\, KHz$

Effective magetic fields on spin 2 will be generated by a microwave field.

Future:

1. Spin networks (perpetually coupled spins placed in fixed locations) for computation. – Is it possible? Perhaps couplings and positions inspired by real systems.

2. Is it practically useful in QIP in certain implementations? (compare to shuttling, to photon-atom interfaces)

Collaborators in work reported today:
Bayat, Banchi, Verrucchi, Sodano, Wichterich, Alkurtass, Yung.

Support: EPSRC (previously), Royal Soc. (previously), ERC.

REVIEW REFERENCES:

1. S. Bose, Contemp. Phys. **48**, 13 (2007).
2. S. Bose, A. Bayat, P. Sodano, L. Banchi, P. Verrucchi, Springer Lecture Notes 2013 (published, but not on arXiv).