Non-equilibrium quantum simulations wih ion traps:
Quantum Heat Transport

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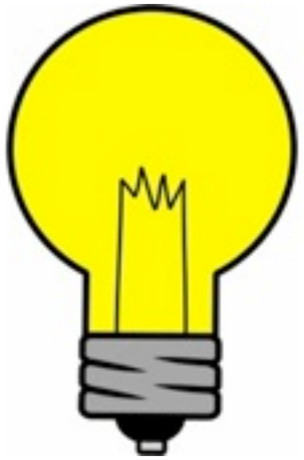
Motivation



Miniaturization of electronic devices

- ✱ quantum information processing
- ✱ electronic transport at the **nanoscale**
  - **(semi) classical transport**
  - **quantum transport**

(semi) Classical Transport

✓ mimic the behavior of electronic components at reduced sizes

↪ cost ↓↓ , efficiency ↑↑ , dissipation ↓↓ .

Quantum Transport

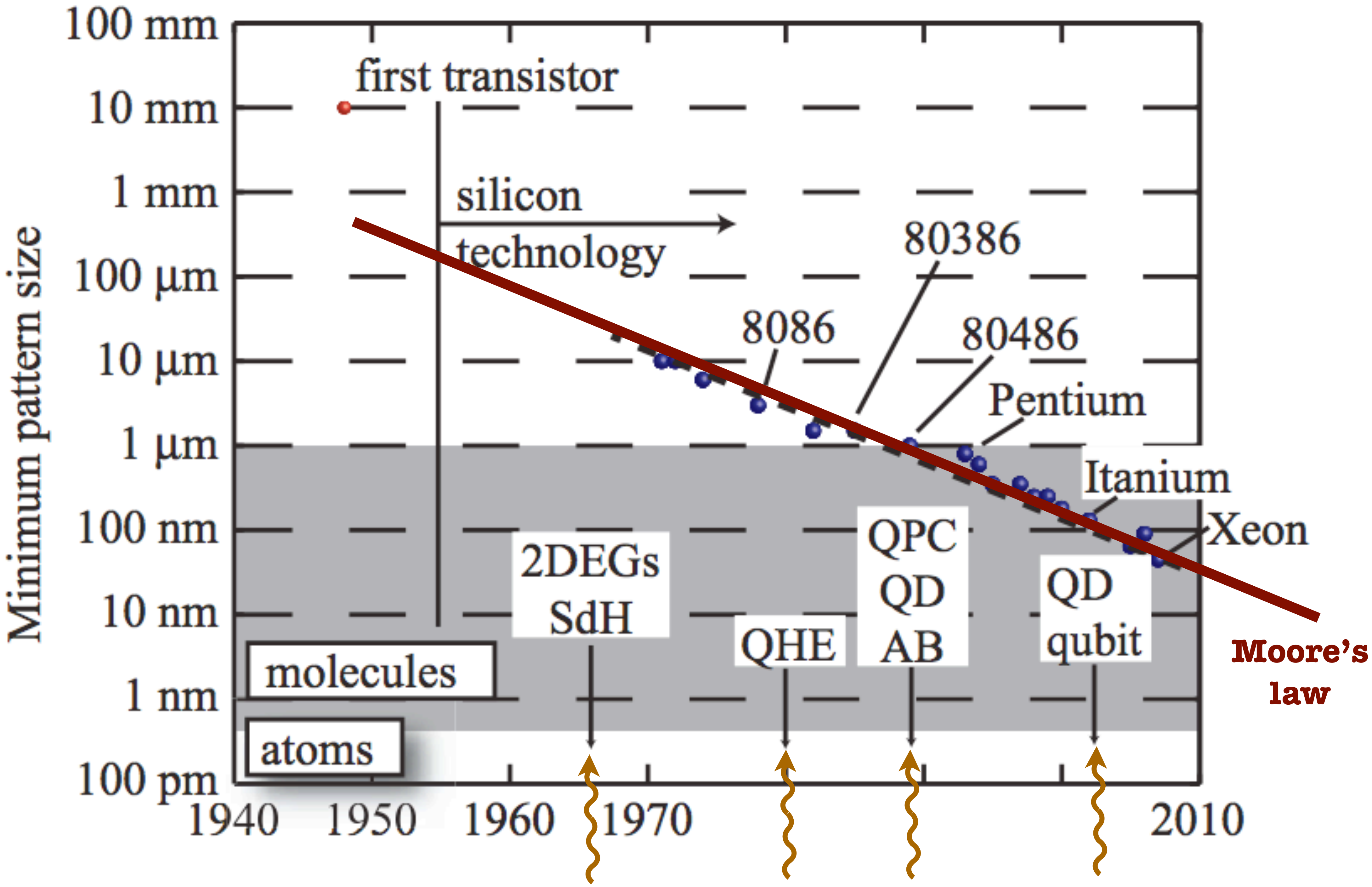
✓ exploit quantum features of the transport

↪ novel functionalities with technological applications

↪ discover fundamental phenomena (new physics)

Transport that preserves coherence (interference effects, quantized conductance)

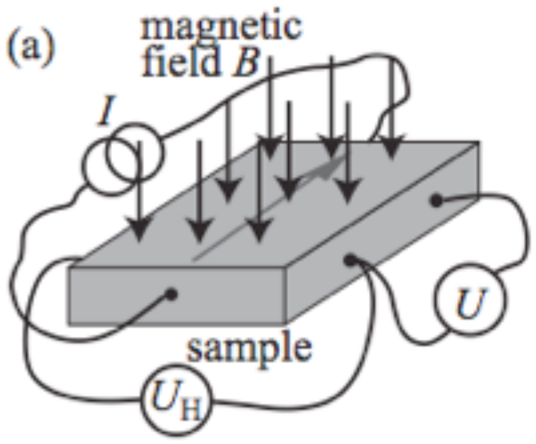
Miniaturization of computer processor's chips over time



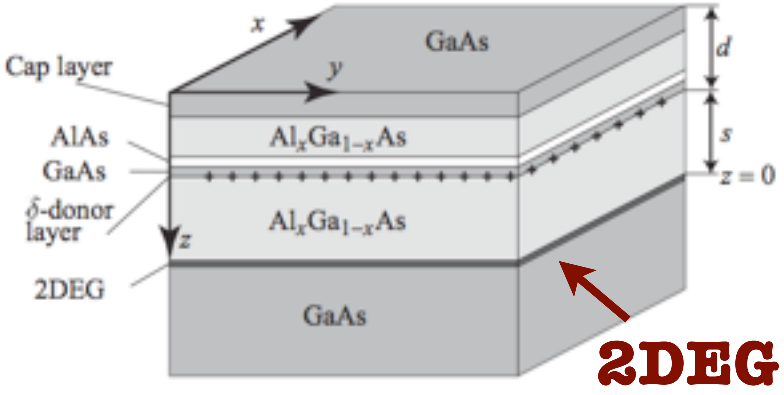
Milestones in the field of quantum transport

T. Ihn, *Semiconductor nanostructures* (Oxford, 2010).

e.g. **Quantum Hall Effect**: transport in the **2DEG** subjected to strong magnetic fields (**QHE**)

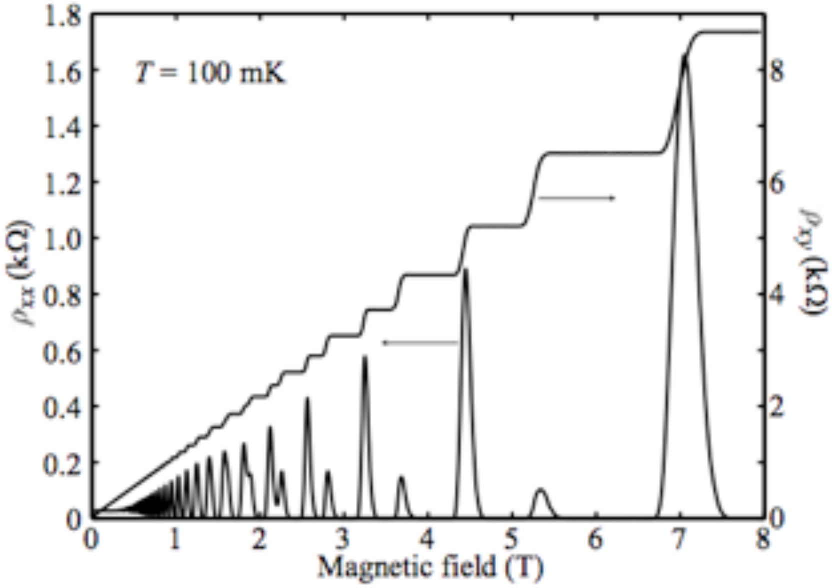


transport toolbox



2DEG

Semiconductor heterostructures



Quantum Hall plateaux

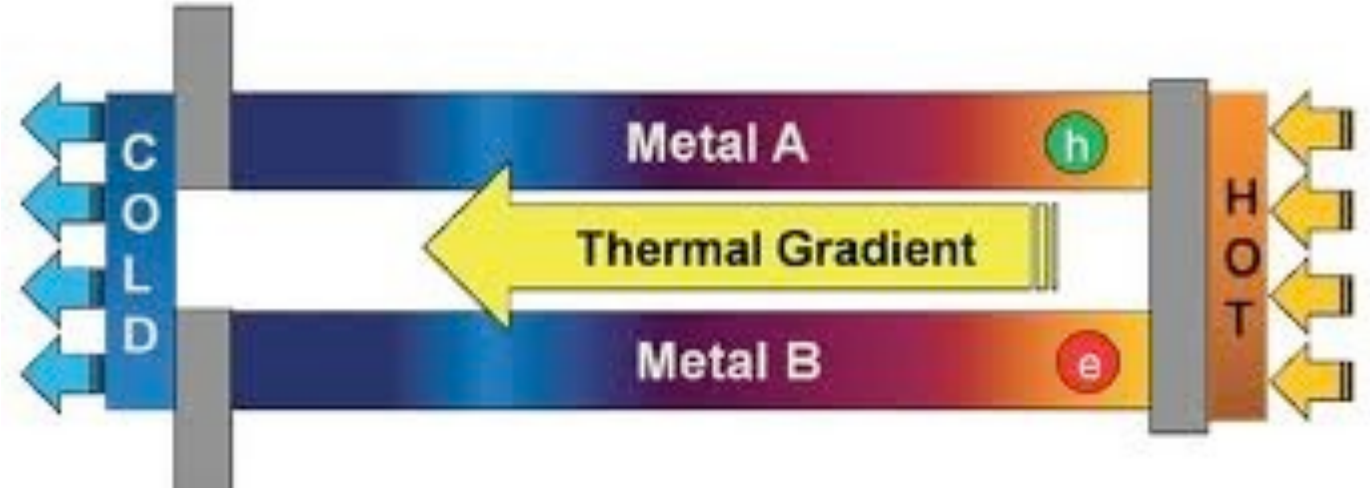
- **nanoscale:** thickness of the electron gas 3-4nm
- **quantum features:** Quantization of the electric conductivity

$$\sigma_{xy} = \frac{e^2}{h} n \qquad n \in \mathbb{Z}$$

Transport coherence preserved in the sample's edges
(scattering by non-magnetic impurities forbidden)

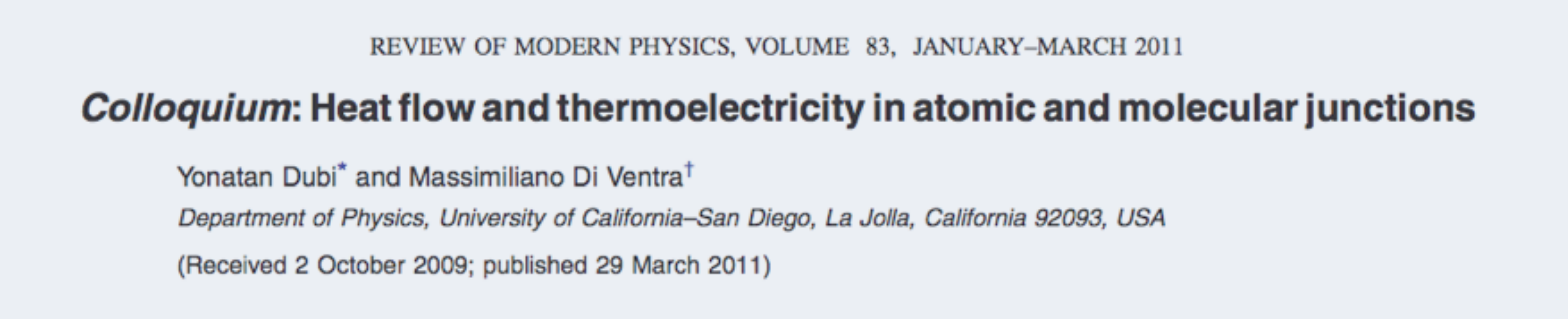
- **novel functionality:** robust quantization of conductance serves as a standard of resistance
- **fundamental phenomena:** The **QHE** defies the standard picture of Landau symmetry-breaking (it is an instance of topological order)

What about heat transport?



At the nanoscale, heat transport should also be dominated by quantum effects with peculiarities caused by the different quantum statistic of carriers (**phonons**)

However ...



REVIEW OF MODERN PHYSICS, VOLUME 83, JANUARY–MARCH 2011

Colloquium: Heat flow and thermoelectricity in atomic and molecular junctions

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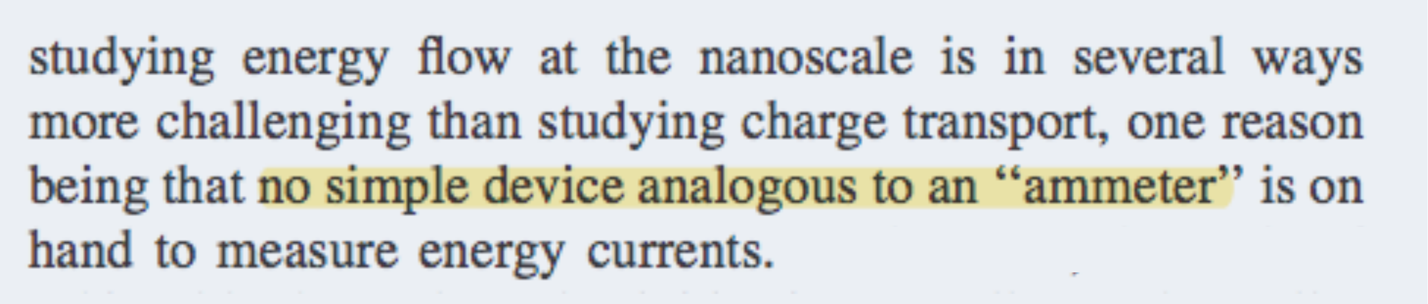
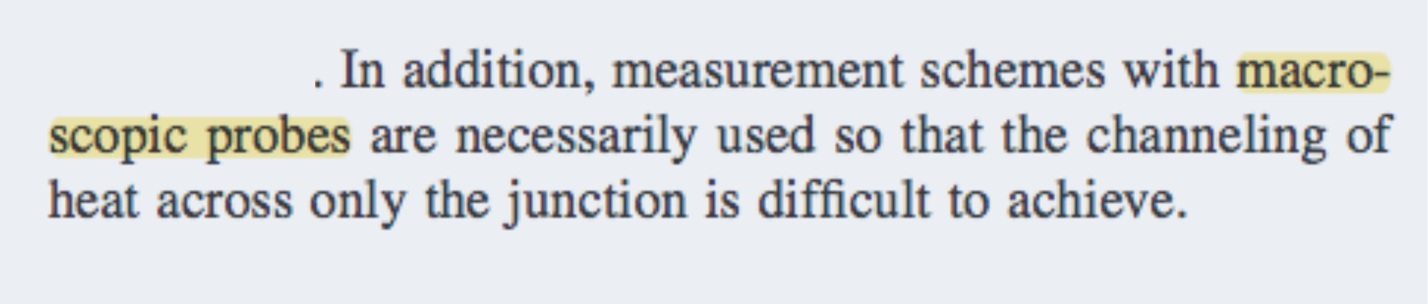


studying energy flow at the nanoscale is in several ways more challenging than studying charge transport, one reason being that no simple device analogous to an "ammeter" is on hand to measure energy currents.

No devices to measure heat currents

One typically measures temperatures & infers conductances (model-dependent)

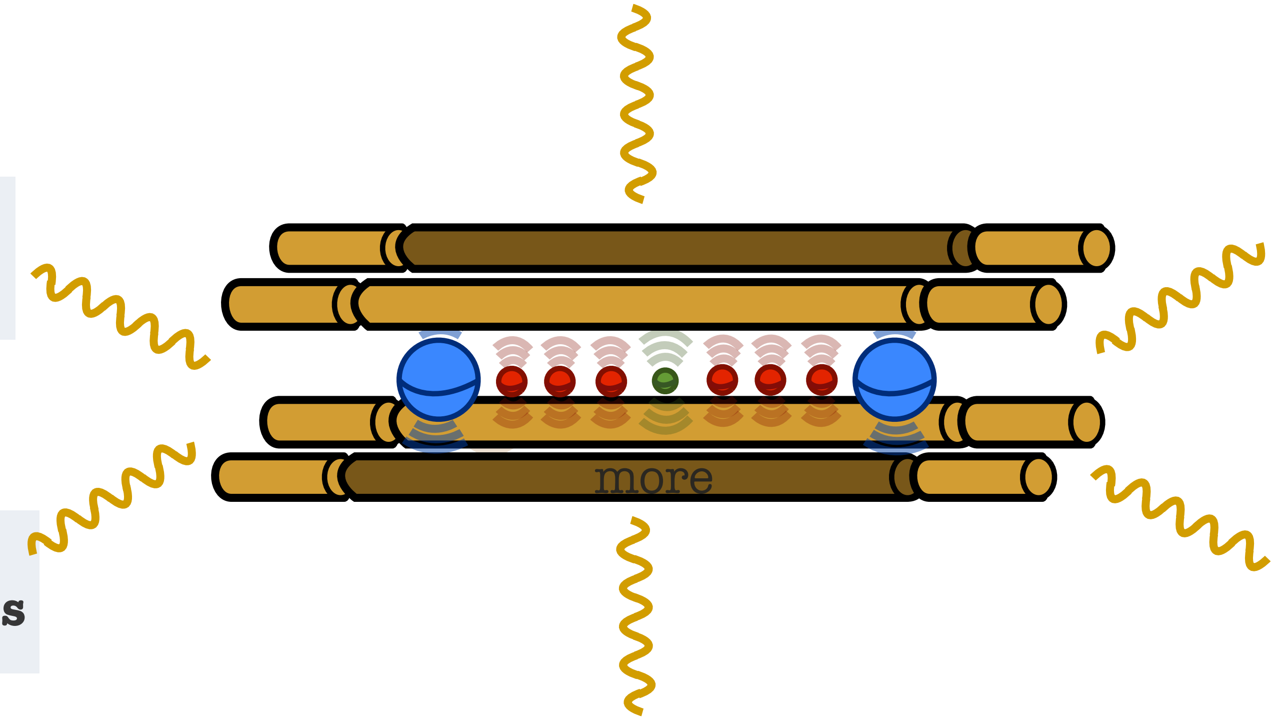


. In addition, measurement schemes with macroscopic probes are necessarily used so that the channeling of heat across only the junction is difficult to achieve.

Temperature probes are invasive

measurements are typically affected by the probe

Our goal is to construct a **transport toolbox** based on a **mixed-species ion crystal**, which allows us to treat heat currents in the same footing as electric ones.



heat reservoirs

single-atom heat switch

temperature & current probes

bosonic quantum wires

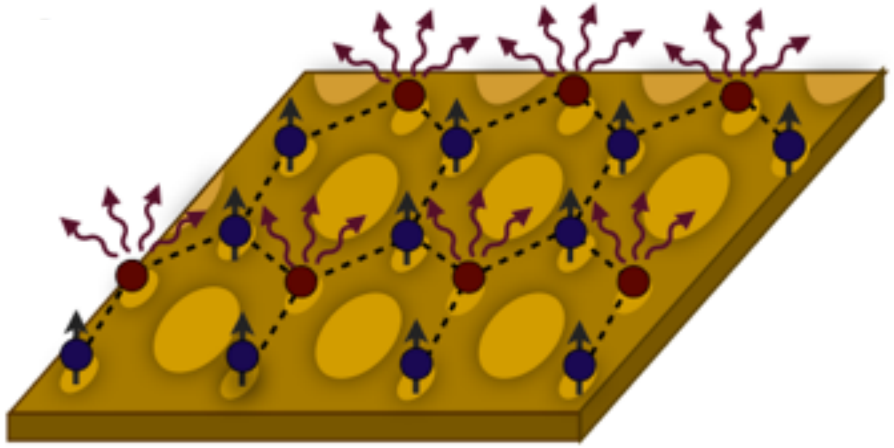
bosonic quantum dots

phonon battery

From a more general perspective, we would like to use trapped ions as a **quantum simulator of non-equilibrium phenomena** (in this talk = non-equilibrium steady-state heat currents)

Propagation of vibrational excitations	T. Pruttivarasin, et al., NJP. **13**, 075012 (2011).
Thermalization of laser-cooled ion chains	G-D Lin, et al, NJP. **13,** 075015 (2011).
Quantum heat engine	O. Abah, et al., PRL **109**, 203006 (2012).

Sympathetic Dissipative Gadgets



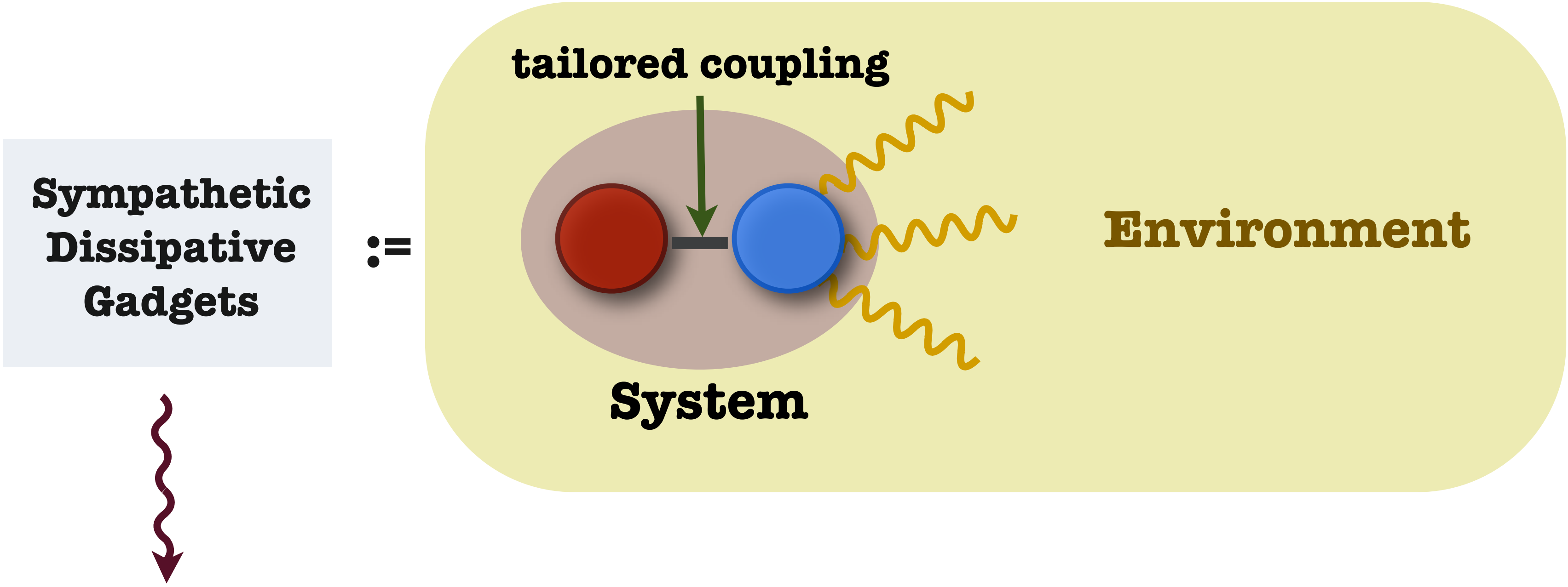
“ Exploit dissipation as a tool for quantum information processing (QIP) “

Well-known examples could be

- **optical pumping** to initialise the **state** used to encode information

We want to harness dissipation not only as a tool to prepare initial states, but also as

- gadget to make new **Quantum simulators** based on **dissipative control**



Sympathetic Dissipative Gadgets :=

tailored coupling

Environment

System

“ Use dissipation of some auxiliary degree of freedom 🔵 to control the irreversible dynamics of the target degree of freedom 🔴 “

A key idea throughout this talk is that, if the **auxiliary degree of freedom decays on a much faster time-scale than any other process in the system**

strong dissipation Γ_0

$$\tilde{\mathscr{L}} = \tilde{\mathscr{L}}_0 + \tilde{\mathscr{L}}_1$$

slower dynamics

- For the timescales of interest, the **auxiliary degree of freedom is effectively projected onto the steady state** of the dissipative process

$$\rho \rightarrow \rho_{\rm aux}^{\rm ss} \otimes \rho_{\rm tar}(t) \qquad t \gg \Gamma_0^{-1} \qquad\qquad \tilde{\mathscr{L}}_0(\rho_{\rm aux}^{\rm ss}) = 0$$

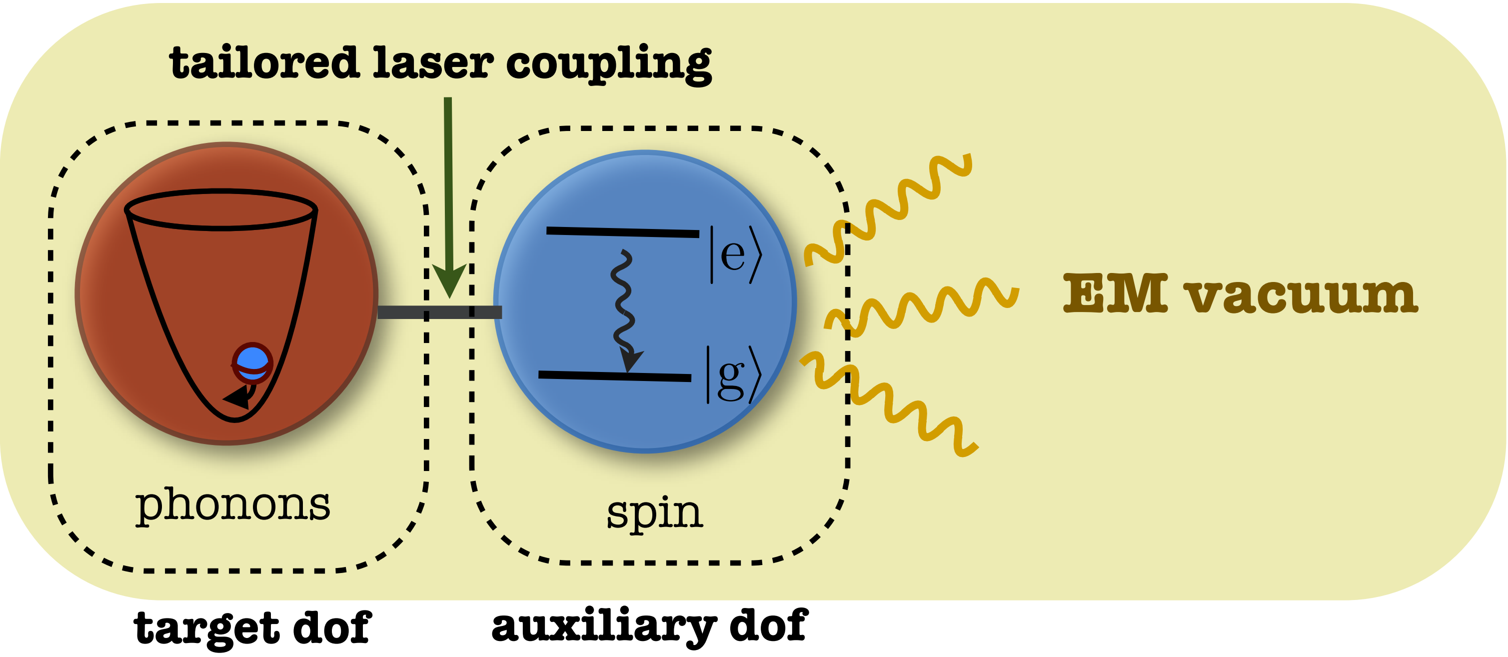
- The **target degree of freedom evolves by virtual processes that take the auxuliary dof away from the above steady state** (formalised in terms of projection-operator techniques)

$$\frac{{\rm d}\rho_{\rm tar}}{{\rm d}t} = \mathscr{L}_{\rm eff}(\rho_{\rm tar})$$

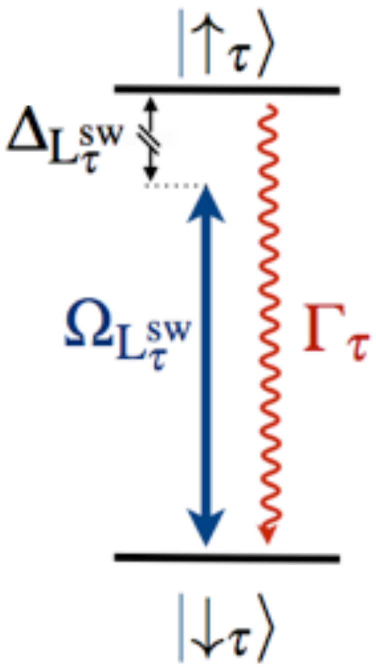
S. Chaturvedi, et al, ZPB **35,** 297 (1979).

Laser cooling can be understood as a sympathetic dissipative gadget

Laser cooling of trapped ions :=



If the **spin** corresponds to a **dipole-allowed transition** (very gast decay), and the **spin-motion coupling** comes from a **standing-wave laser** (ion in the node) tuned to the first sideband:



$$\rho \to \rho_{\rm ss}^{\tau} \otimes \rho_{\rm vib}(t) \qquad \rho_{\rm ss}^{\tau} = |{\downarrow_{\ell_\tau}}\rangle\langle{\downarrow_{\ell_\tau}}|$$

The **phonons** evolve on a slower time-scale by virtual excitation/decay of the spin steady state

laser heating

$$\mathscr{L}_{\rm eff}^{\ell_\tau}(\rho_{\rm vib}) = \Gamma_{\ell_\tau}^{+}(a_{\ell_\tau}^{\dagger}\rho_{\rm vib}a_{\ell_\tau} - a_{\ell_\tau}a_{\ell_\tau}^{\dagger}\rho_{\rm vib}) + \\ + \Gamma_{\ell_\tau}^{-}(a_{\ell_\tau}\rho_{\rm vib}a_{\ell_\tau}^{\dagger} - a_{\ell_\tau}^{\dagger}a_{\ell_\tau}\rho_{\rm vib}) + {\rm H.c.}$$

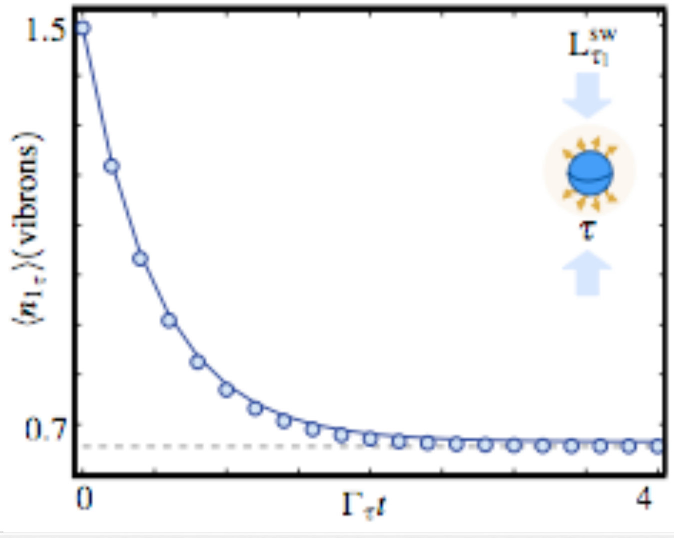
laser cooling

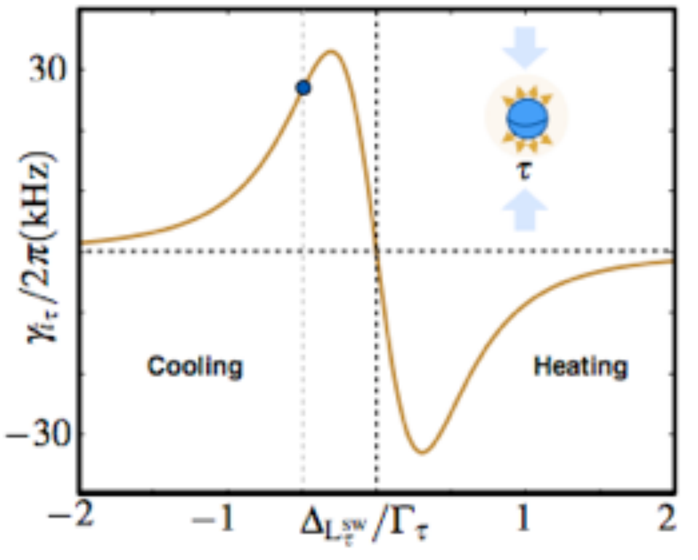
exponential damping controled by laser-coupling power spectrum

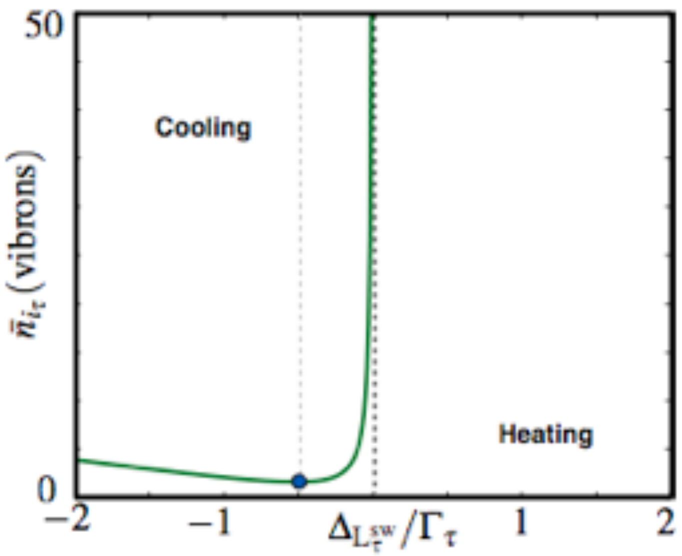
J. I. Cirac, et al., PRA **46,** 2668 (1992).

$$S_{F_{\ell_\tau},F_{\ell_\tau}}(\omega) = \int_0^\infty {\rm d}t \langle \tilde{F}_{\ell_\tau}(t)\tilde{F}_{\ell_\tau}(0)\rangle_{\rm ss} e^{i\omega t}.$$

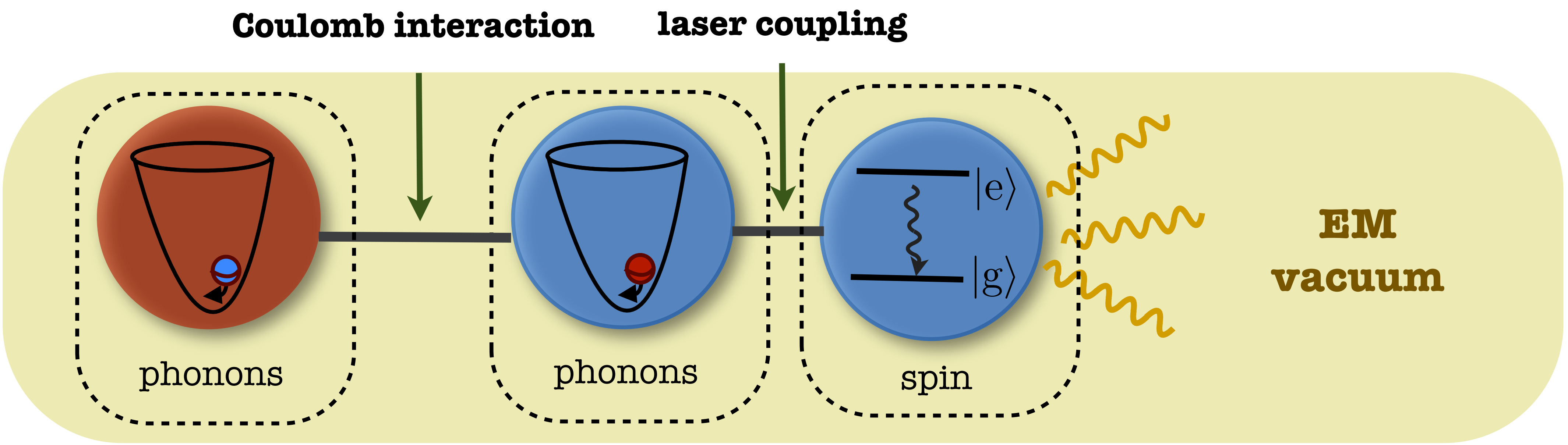
$$\Gamma_{\ell_\tau}^{-} = S_F(\omega_{\ell_\tau}) > S_F(-\omega_{\ell_\tau}) = \Gamma_{\ell_\tau}^{+}$$







For quantum transport, we will be interested in **sympathetic cooling of a mixed-species ion crystal**

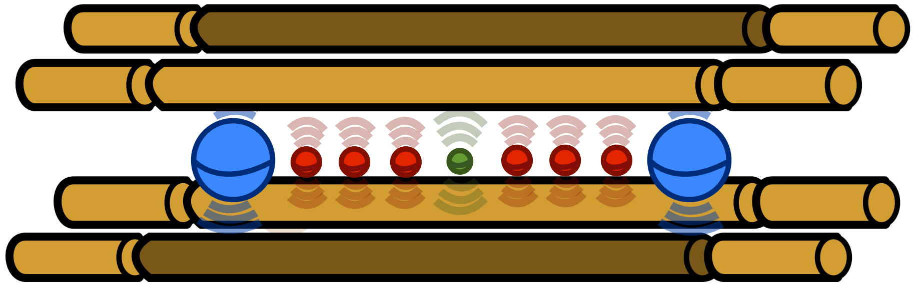


target species
(e.g. $^{25}\mathrm{Mg}^{+}$)

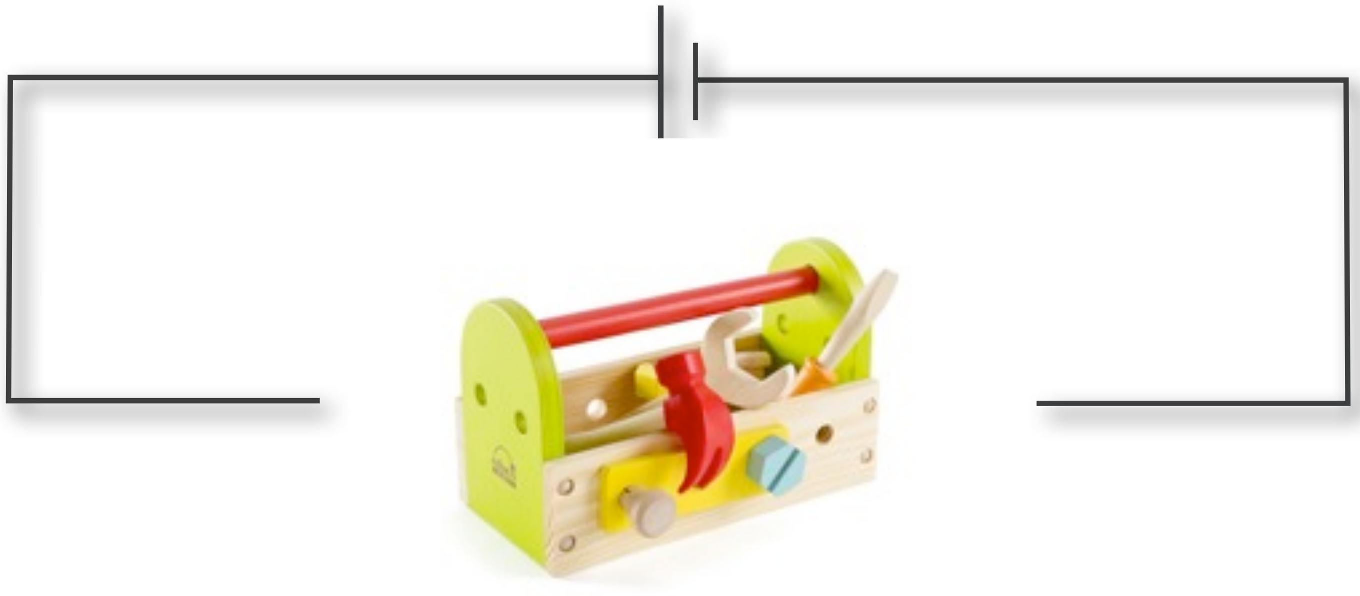
auxiliary laser-colled species
(e.g. $^{24}\mathrm{Mg}^{+}$)

Simulate **heat reservoirs**
phonon batteries

Simulate **quantum wires**
quantum dots
quantum switches
quantum probes



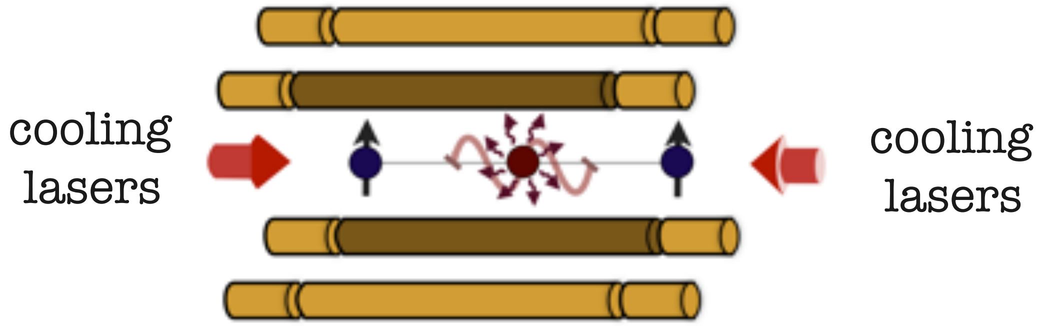
Quantum Heat Transport Toolbox



Phonon reservoirs & batteries

1. Continuous laser cooling red ions (τ-species)

$$\mathscr{D}_{\rm v}^{\ell_\tau}(\rho)=\Gamma^{+}_{\ell_\tau}(a^{\dagger}_{\ell_\tau}\rho a_{\ell_\tau}-a_{\ell_\tau}a^{\dagger}_{\ell_\tau}\rho)+ \\ +\Gamma^{-}_{\ell_\tau}(a_{\ell_\tau}\rho a^{\dagger}_{\ell_\tau}-a^{\dagger}_{\ell_\tau}a_{\ell_\tau}\rho)+\text{H.c.}$$

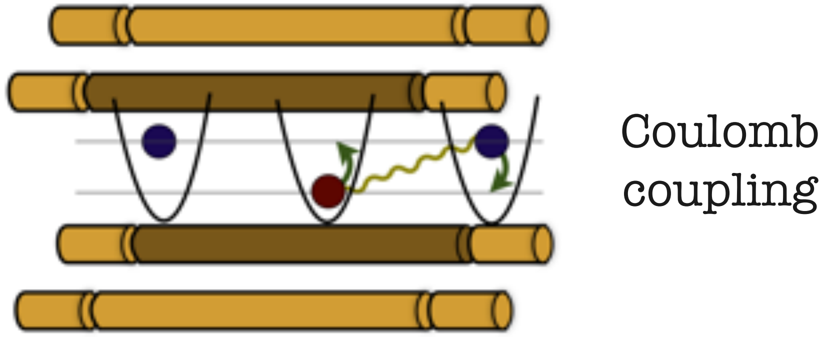


2. Exchange of vibrational excitations

$$H_{\rm vt}=\sum_{i_\sigma\neq\ell_\tau}J_{i_\sigma\ell_\tau}(a^{\dagger}_{i_\sigma}+a_{i_\sigma})(a^{\dagger}_{\ell_\tau}+a_{\ell_\tau})+\text{H.c.},$$

effective dipolar coupling

D. Porras, et al., PRL **93,** 263602 (2004).
K. R. Brown, et al., Nature **471,** 196 (2011).
M. Harlander, et al., Nature **471,** 200 (2011).



3. Effective damping of the ion crystal

$$|J_{i_\sigma\ell_\tau}|\gg\text{Re}\{\Gamma^{-}_{l_\tau}-\Gamma^{+}_{l_\tau}\}\qquad\tilde{\mathscr{D}}(\rho)=\sum_{n}\{\Gamma^{+}_{n}(b^{\dagger}_{n}\rho b_{n}-b_{n}b^{\dagger}_{n}\rho)+\Gamma^{-}_{n}(b_{n}\rho b^{\dagger}_{n}-b^{\dagger}_{n}b_{n}\rho)\}+\text{H.c.}$$

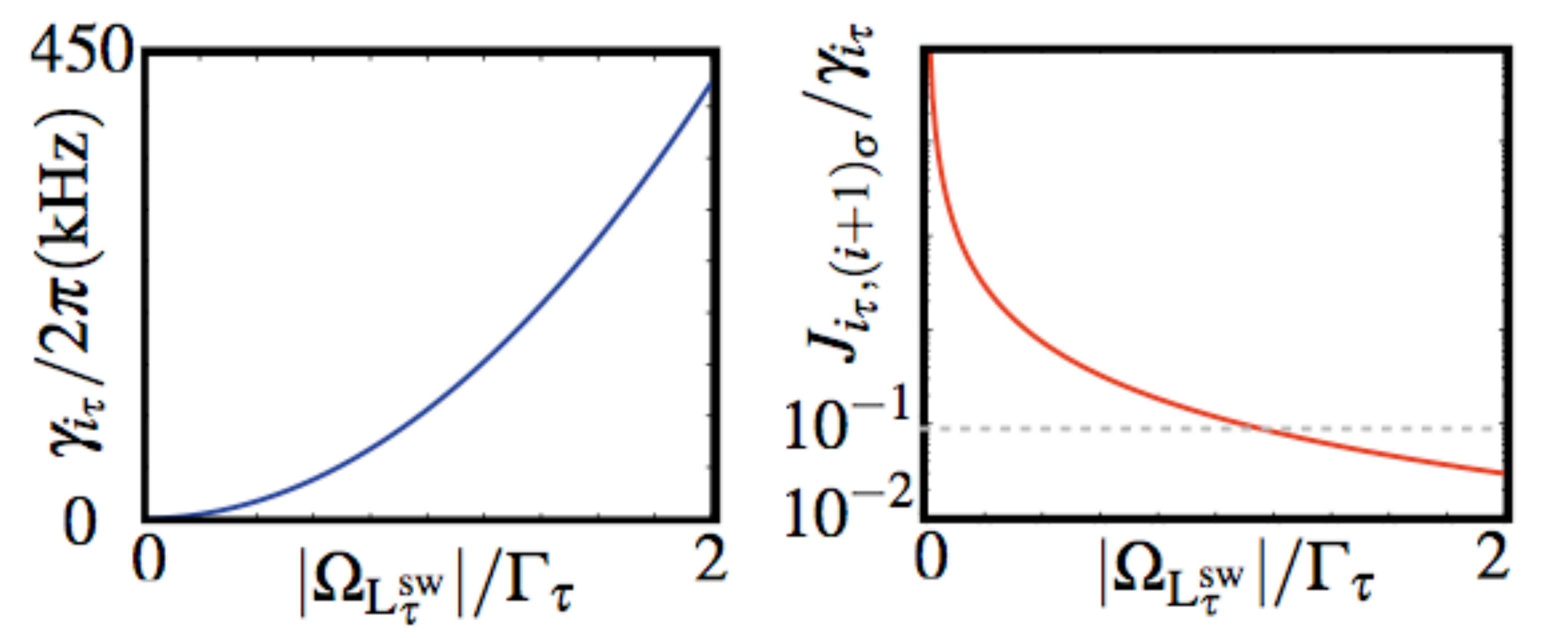
Cooling to the **normal modes** of the ion crystal

(Th) D. Kielpinski, et al., PRA **61,** 032310 (2000).
G. Morigi, et al., EPJD **13,** 261 (2001).
(Exp) H. Rohde, et al. JOB **3,** S3 (2001).
M.D. Barrett, et al., PRA **68,** 042302 (2003).

What happens if this condition is not fulfilled ?

If the **cooling is much stronger than the phonon tunneling,** the laser cooled species reaches the steady state (thermal state) very fast

$$|J_{i_\sigma \ell_\tau}| \ll \mathrm{Re}\{\Gamma^-_{l_\tau} - \Gamma^+_{l_\tau}\}$$



If the **cooling is much stronger than the phonon tunneling,** the laser cooled species reaches the steady state (thermal state) very fast

$$|J_{i_\sigma \ell_\tau}| \ll \mathrm{Re}\{\Gamma^-_{l_\tau} - \Gamma^+_{l_\tau}\} \qquad \rho(t) \to \mu^{\mathrm{th}}_{1_\tau} \otimes \rho_{\mathrm{bulk}}(t) \otimes \mu^{\mathrm{th}}_{N_\tau}$$

The **edge ions act as phonon reservoirs/battery** for the bulk ions.

- remain in the steady state while the lasers are switched on (i.e. a thermal bath should be unaffected by its coupling to the system)
- absorve/emit phonons from/into the bulk in their effort to equilibrate
- If $\bar{n}_{1_\tau} > \bar{n}_{N_\tau}$, we have a phonon battery that sets a **heat current** across the chain

The **bulk phonons thermalise on a slower timescale by virtual tunneling in/out** of the edges

Renormalized tunneling for the bulk ions

$$\dot{\rho} = -\mathrm{i}[H_{\mathrm{rtb}}, \rho] + \mathscr{D}_{\mathrm{bulk}}(\rho),$$

Collective dissipation via edge-bulk tunneling

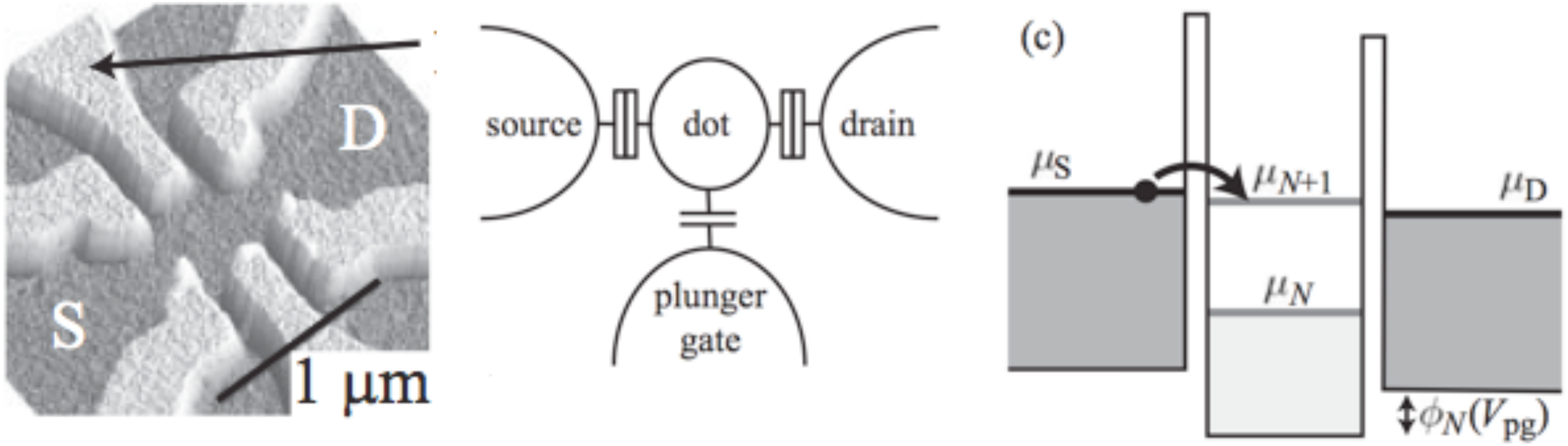
$$H_{\mathrm{rtb}} = \sum_{i_\sigma} \tilde{\omega}_{i_\sigma} a^\dagger_{i_\sigma} a_{i_\sigma} + \sum_{i_\sigma \neq j_\sigma} \tilde{J}_{i_\sigma j_\sigma} a^\dagger_{i_\sigma} a_{j_\sigma},$$

$$\mathscr{D}_{\mathrm{bulk}}(\bullet) = \sum_{i_\sigma j_\sigma} \tilde{\Lambda}^+_{i_\sigma j_\sigma} (a^\dagger_{j_\sigma} \bullet a_{i_\sigma} - a_{i_\sigma} a^\dagger_{j_\sigma} \bullet) + \tilde{\Lambda}^-_{i_\sigma j_\sigma} (a_{j_\sigma} \bullet a^\dagger_{i_\sigma} - a^\dagger_{i_\sigma} a_{j_\sigma} \bullet) + \mathrm{H.c.},$$

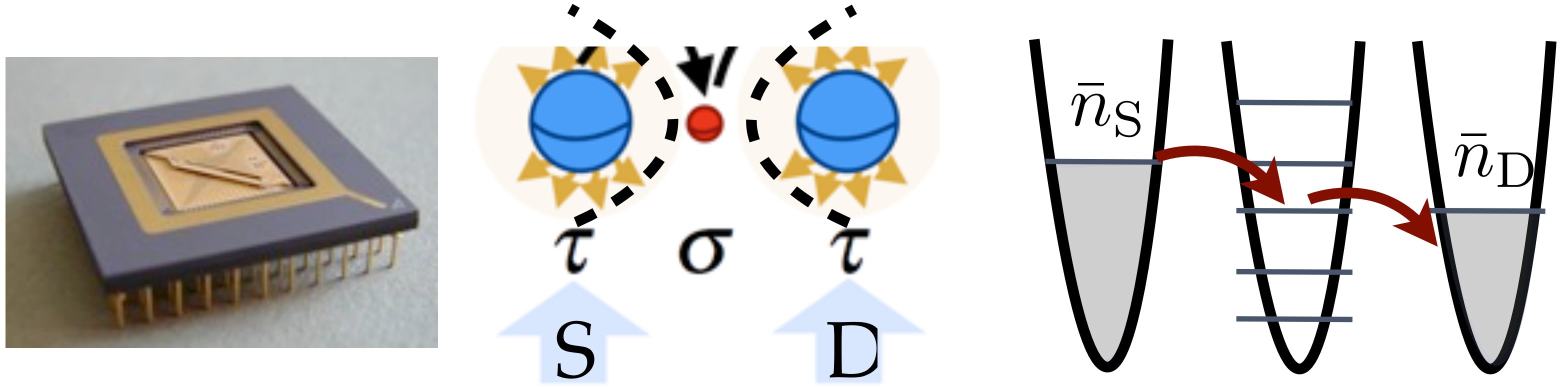
Thermal quantum dot

To test the validity of our theory, we consider the simplest possible situation

electronic quantum dot



Trapped-ion heat analogue

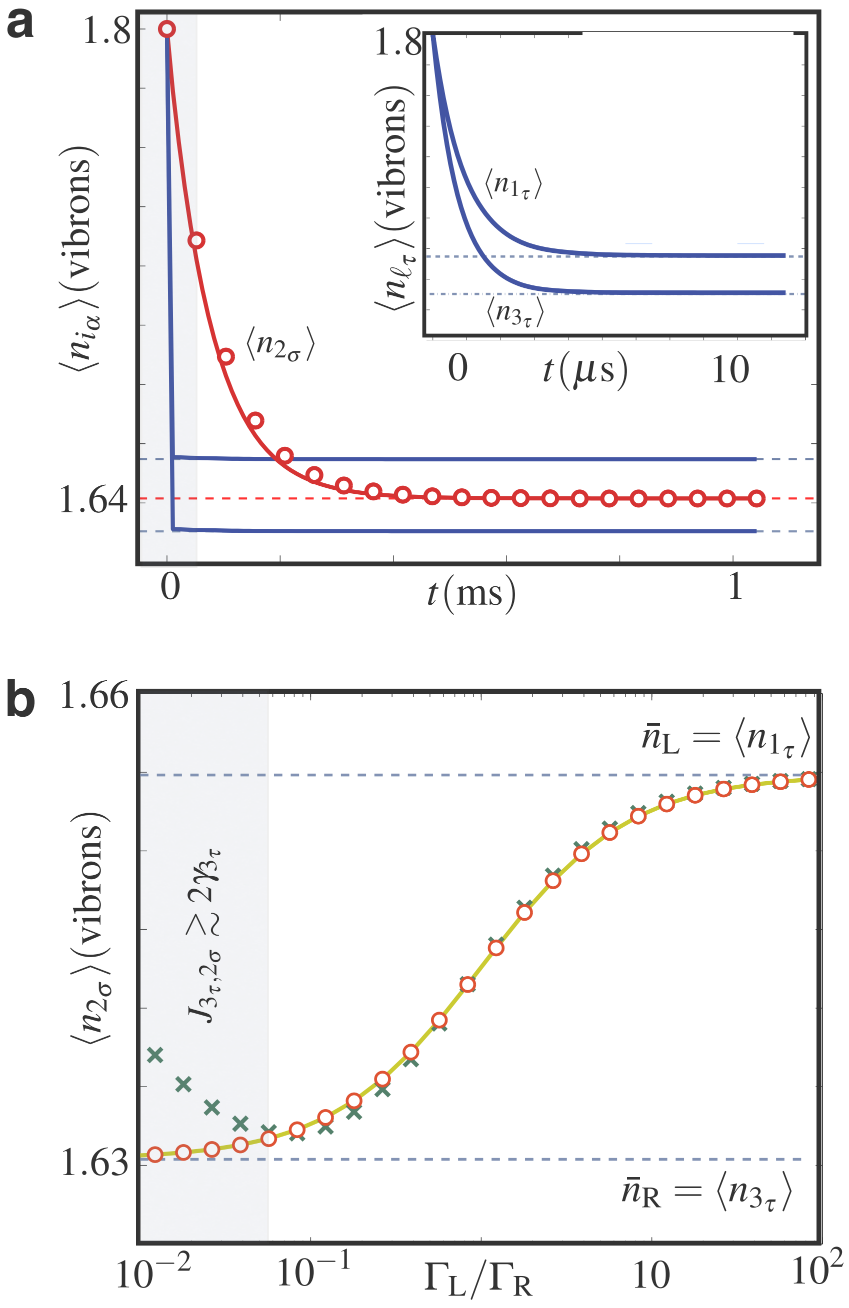


Analytic expression for the non-equilibrium steady-state heat transport

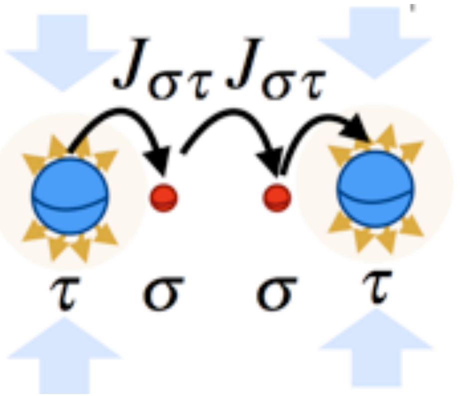
$$\langle n_{i_{\mathrm{bulk}}}\rangle_{\mathrm{ss}} = \frac{\Gamma_{\mathrm{L}}\bar{n}_{\mathrm{L}} + \Gamma_{\mathrm{R}}\bar{n}_{\mathrm{R}}}{\Gamma_{\mathrm{L}} + \Gamma_{\mathrm{R}}}$$

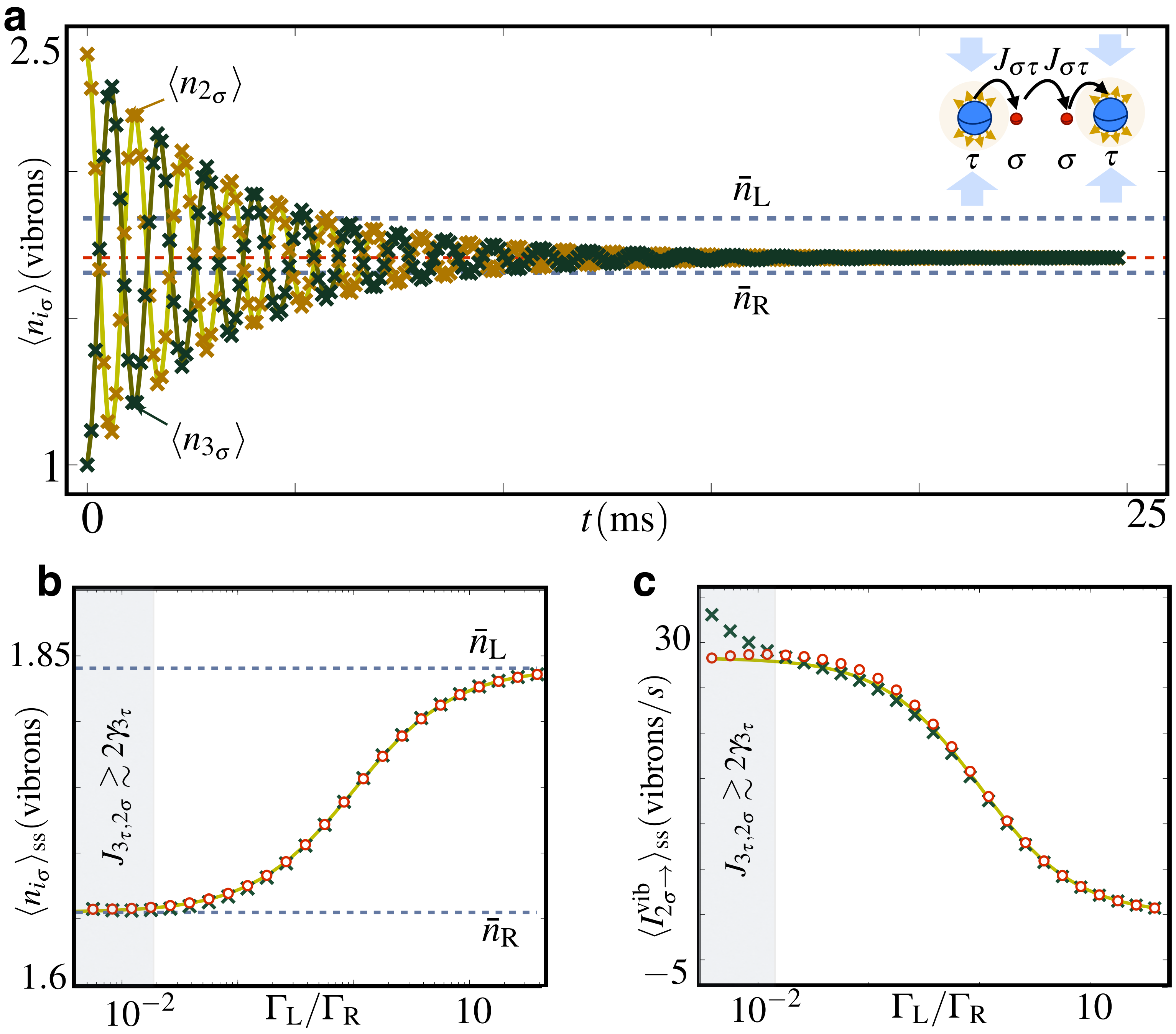
$$\langle I^{\mathrm{vib}}_{i_{\mathrm{bulk}}}\rangle_{\mathrm{ss}} = \frac{\Gamma_{\mathrm{L}}\Gamma_{\mathrm{R}}}{\Gamma_{\mathrm{L}} + \Gamma_{\mathrm{R}}}(\bar{n}_{\mathrm{L}} - \bar{n}_{\mathrm{R}}),$$

Thermal-QD



Thermal-DQD





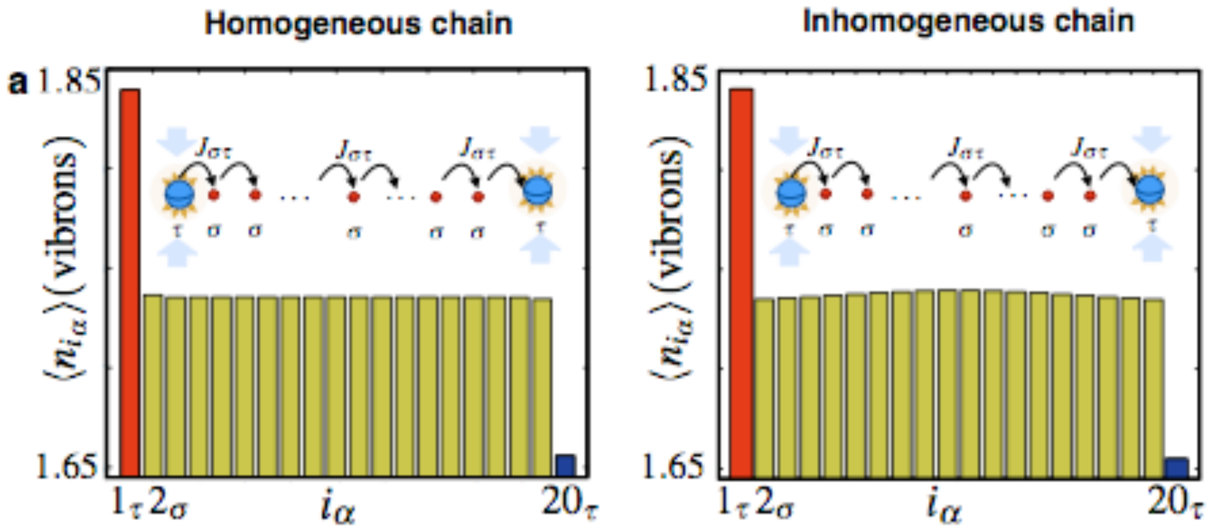
Thermal quantum wire & emergence of Fourier's law

Fourier's law is violated in microscopic systems (e.g. harmonic chain)

Z. Rieder, et al., JMP **8**, 1073 (1967).

This violation is consistent with our trapped-ion Master eq. formalism

$$\langle n_{i_{\rm bulk}}\rangle_{\rm ss} = \frac{\Gamma_{\rm L}\bar{n}_{\rm L}+\Gamma_{\rm R}\bar{n}_{\rm R}}{\Gamma_{\rm L}+\Gamma_{\rm R}} \qquad \forall i_{\rm bulk}$$



Fourier's law emerges when leaving the regime of purelly ballistic transport

1. **dephasing**

   A. Asadian, et al., PRE **87,** 012109 (2013).

   by noisy modulation of the trap freq

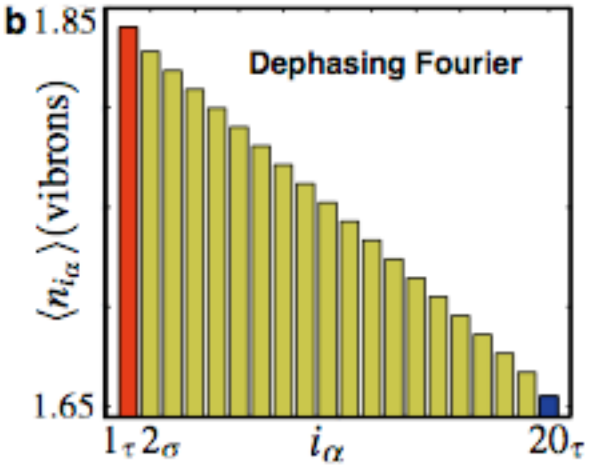
   C. J. Myatt, et al., Nature **403,** 269 (2000).

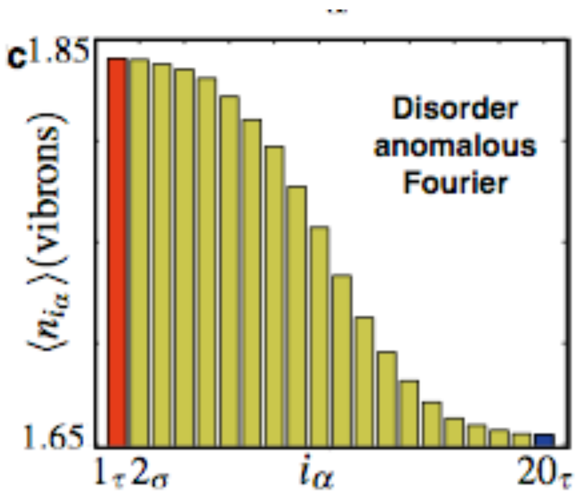
2. **disorder**

   R. J. Rubin, et al, JMP. **12,** 1686 (1971)

   induced by spin-motion coupling

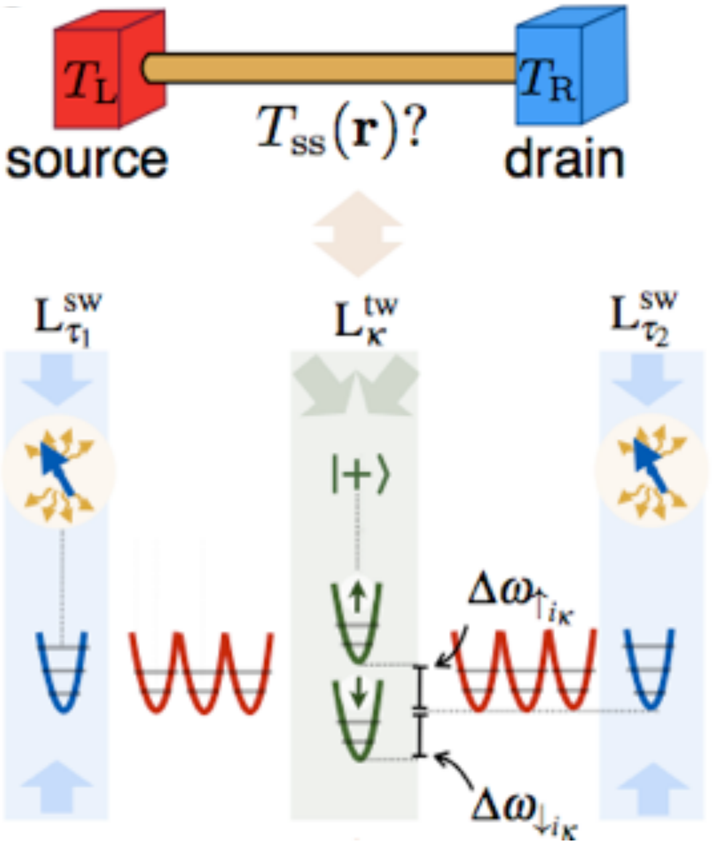
   A. Bermudez, et al., NJP **12,** 123016 (2010).





Ramsey probe of the temperature & heat current, and quantum heat switch

We map the information of a **phononic observable** O_{i_κ} onto the spins by introducing a **spin-phonon perturbation**



$$\tilde{H}^O_{\rm sv} = \sum_{i_\kappa} \tfrac{1}{2} \lambda_O O_{i_\kappa} \sigma^z_{i_\kappa} \qquad |\lambda_O| \ll J_{i_\sigma j_\sigma}, \Gamma^{\pm}_{i_\sigma j_\sigma},$$

non-invasive probe

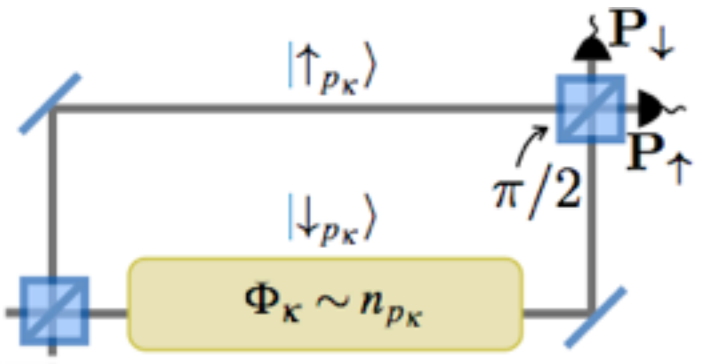
The bulk phonons reach the steady-state, while the spin coherence captures

Expectation value **Power spectrum of fluctuations**

$$\langle \tilde{\sigma}^x_{i_\kappa}(t)\rangle = \cos(\lambda_O \langle O_{i_\kappa}\rangle_{\rm ss} t) \mathrm{e}^{-\lambda_O^2 \mathrm{Re}\{S_{O_{i_\kappa} O_{i_\kappa}}(0)\} t}.$$

$$S_{O_{i_\alpha} O_{i_\alpha}}(\omega) = \int_0^\infty \mathrm{d}t \langle \tilde{O}_{i_\alpha}(t) \tilde{O}_{i_\alpha}(0)\rangle_{\rm ss} \mathrm{e}^{-\mathrm{i}\omega t}, \quad \tilde{O}_{i_\alpha} = O_{i_\alpha} - \langle O_{i_\alpha}\rangle_{\rm ss},$$

1. Initial $\pi/2$-pulse to the probe spin $\langle \sigma^x_{i_\kappa}(0)\rangle = 1$
2. Let evolve under $\tilde{H}^O_{\rm sv}$ for a time t
3. Perform a final $\pi/2$ -pulse & measure the spin-dependent fluorescence



quantum dot probing phase of BEC M. Bruderer, et al., NJP **8,** 87 (2006).

SC qubit probing full-counting statistics of electronic currents G.B. Lesovik, et al., PRL. **96,**106801 (2006).

For the phonon distribution (**temperature**)

$$\tilde{H}^{O}_{\rm sv} = \sum_{i_\kappa} \tfrac{1}{2}\lambda_O b^\dagger_{i_\kappa} b_{i_\kappa} \sigma^z_{i_\kappa}$$

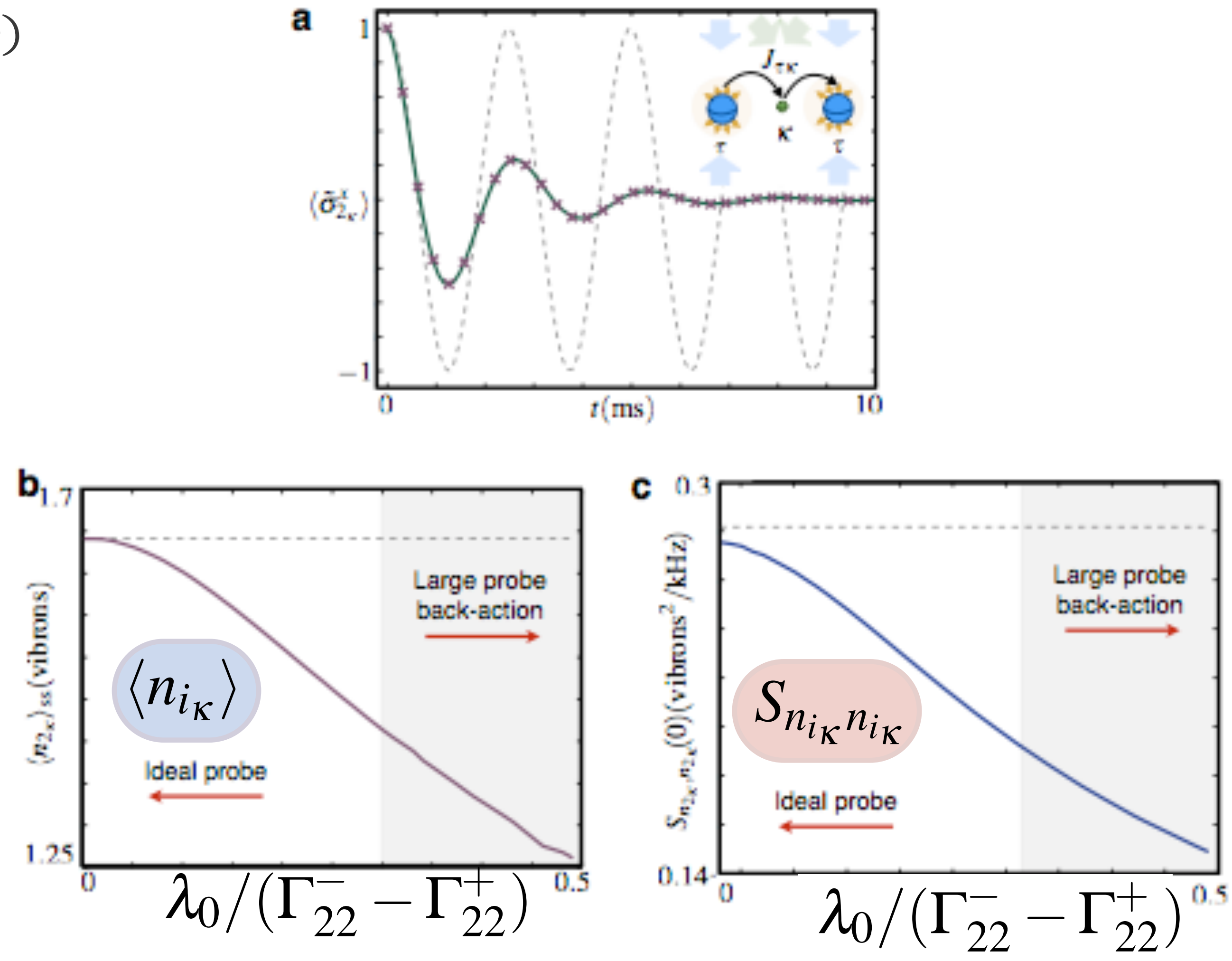
can be obtained from

e.g. state-dependent force where the two Raman lasers have the same frequency

$$\omega_{\rm L_1} = \omega_{\rm L_2} \qquad \lambda_O = |\Omega_{\rm Raman}|\eta^2_{\rm L}$$

$\langle \sigma^x_{i_\kappa}(t) \rangle$

- oscillations $\langle n_{i_\kappa} \rangle_{\rm ss}$
- damping $S_{n_{i_\kappa} n_{i_\kappa}}(0)$



Fluctuations contain lots of useful information about the transport (**full counting statistics**)

Fluctuations (i.e. **Fano factor**) distinguish quantum statistics of carriers

$$\mathscr{F} = \frac{\langle O^2_{i_\kappa} \rangle - \langle O_{i_\kappa} \rangle^2}{\langle O_{i_\kappa} \rangle}$$

in contrast to expectation values, which coincide for fermions & bosons

	Fermions	Bosons	Classical
Occupation	1 – ⟨n⟩	1 + ⟨n⟩	⟨n⟩
Current*	1 – ½nL	1 + ½nL	1

* nR = 0 and symmetric coupling

M. Esposito, et al., RMP **81,** 1665 (2009).

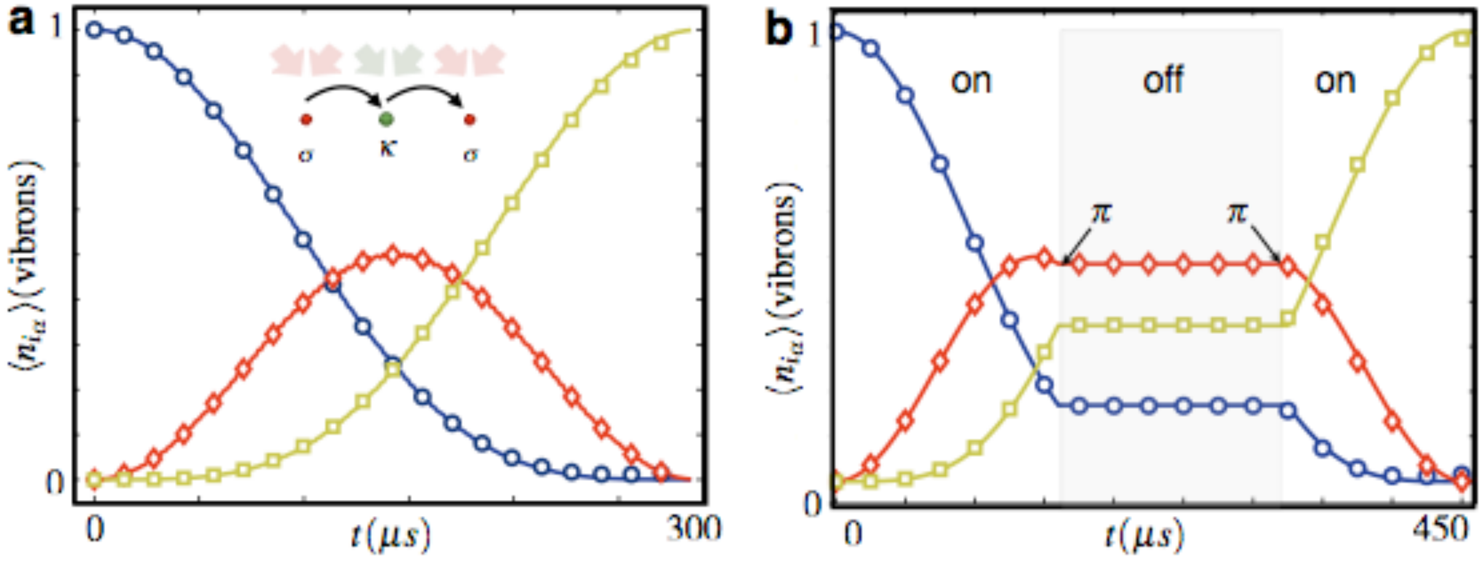
Using ideas of **photon-assisttted tunneling** of phonons A. Bermudez, et al., PRL **107,** 150501 (2011).

↪ phonon tunneling against a trap-frequency mismatch assisted by a periodic driving

- We can build a **single-spin heat switch** $H_{\text{PAT}} \approx -\sum_{i_\sigma < p_\kappa} 2\tilde{J}_{i_\sigma p_\kappa}\mathfrak{J}_1\left(\zeta_\kappa(1+\sigma^z_{p_\kappa})\right)a^\dagger_{i_\sigma}a_{p_\kappa} + \text{H.c.}$

$$\mathfrak{J}_1\left(\zeta_\kappa(1+\sigma^z_{p_\kappa})\right)|\uparrow_{p_\kappa}\rangle \neq 0$$

$$\mathfrak{J}_1\left(\zeta_\kappa(1+\sigma^z_{p_\kappa})\right)|\downarrow_{p_\kappa}\rangle = 0$$



Controlling the spin state (e.g. by microwaves), we can switch on/off the heat current.

- We can construct a Ramsey probe of the **heat current** by using a bichromatic driving

$$I^{\text{vib}}_{i_\alpha\rightarrow} = -\text{i}\sum_\beta \sum_{j_\beta > i_\alpha} J_{i_\alpha j_\beta} a^\dagger_{j_\beta} a_{i_\sigma} + \text{H.c.}, \qquad H^{\text{PAT}}_{\text{L}\kappa\text{R}} = -\text{i}2\mathfrak{J}_1(\pi)\sum_{i_\sigma}(\tilde{J}_{i_\sigma p_\kappa} a^\dagger_{i_\sigma} a_{p_\kappa} + \text{H.c.}),$$

we need one frequency to acquire a complex phase

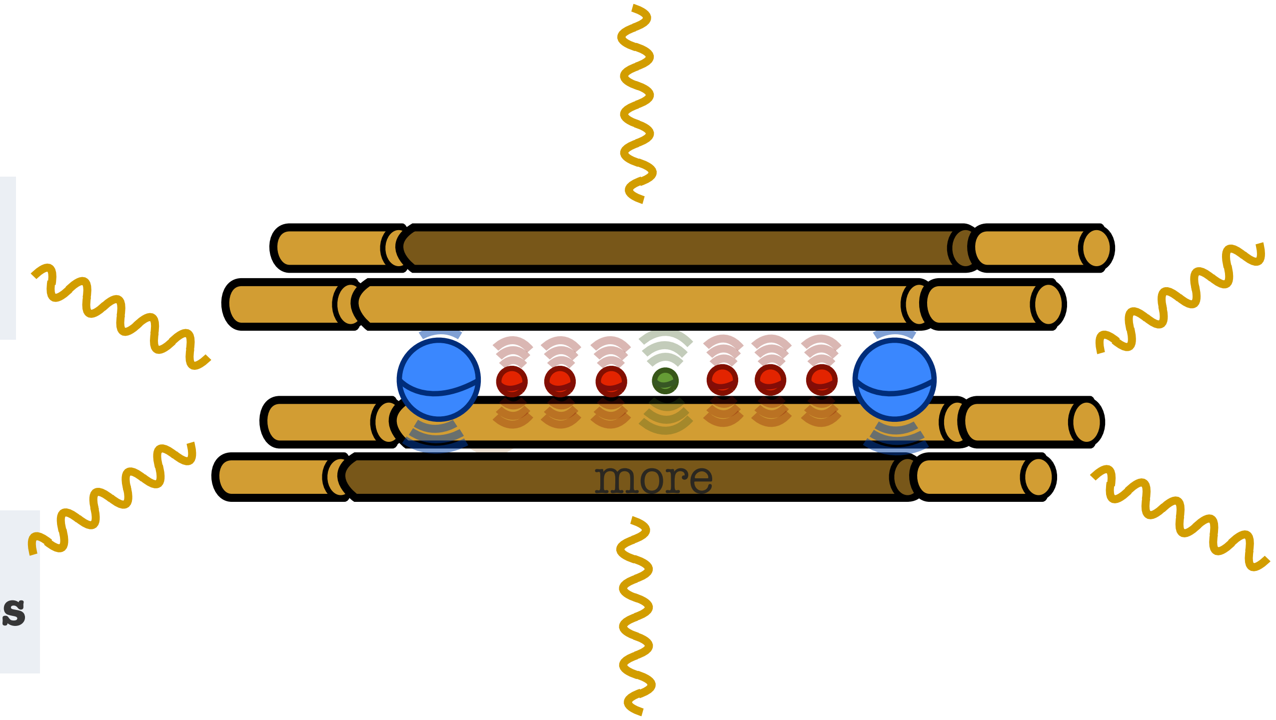
&

a second frequency to engineer the spin-phonon coupling

$$\tilde{H}^O_{\text{sv}} = \sum_{i_\kappa} \tfrac{1}{2}\lambda_O(I_{i_\kappa\rightarrow} + I_{i_\kappa\leftarrow})\sigma^z_{i_\kappa}$$

Conclusions & Outlook

We can use always-on sympathetic cooling to mimic the effect of reservoirs & study quantum transport problems with ion traps



heat reservoirs

single-atom heat switch

temperature & current probes

bosonic quantum wires

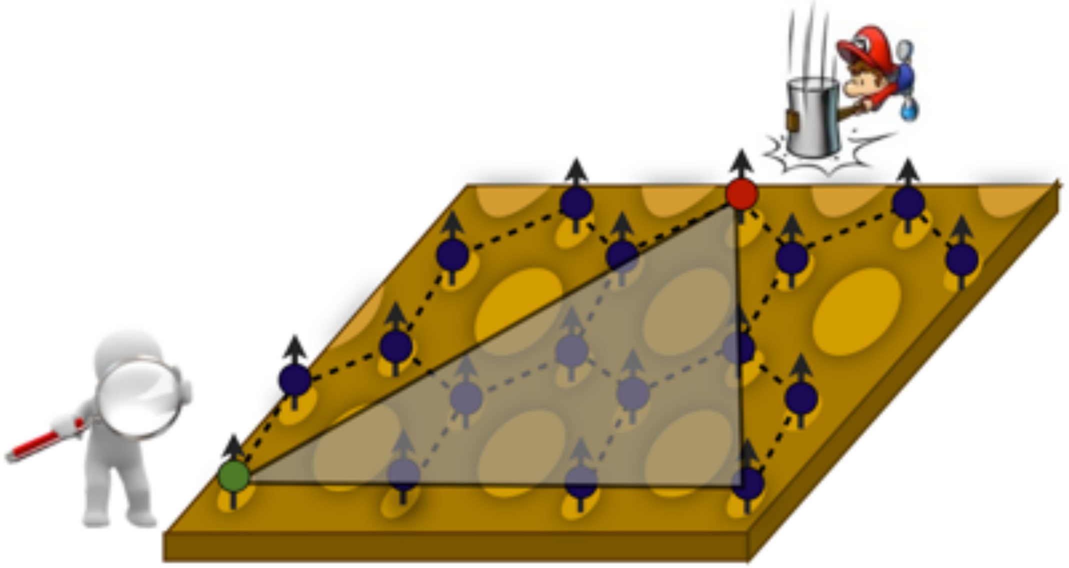
bosonic quantum dots

phonon battery

A. Bermudez, M. Bruderer & M. Plenio
Phys. Rev. Lett. **111,** 040601 (2013).

Part II

Quantum Transport of Correlations



II.a. Lieb-Robinson bounds & causality

In Collaboration with **J. Juenemann, A. Cadarso, D. Pérez-García & J.J. García-Ripoll**
arXiv:1307.1992 (2013).

Motivation

Emergent collective phenomena in many-body systems

↪ The laws that govern the microscopic constituents are not valid at all length scales

P. W. Anderson, Science **177,** 393 (1972).

❧ **Equilibrium**: New physical laws emerge and lead to exotic phenomena

↪ There is a wealth of known effects

e.g. Emerging Topological field theories (Quantum Hall effect)
Emerging relativistic field theories (quantum phase transitions)

❧ **Nonequilibrium**: New physical laws emerge dynamically

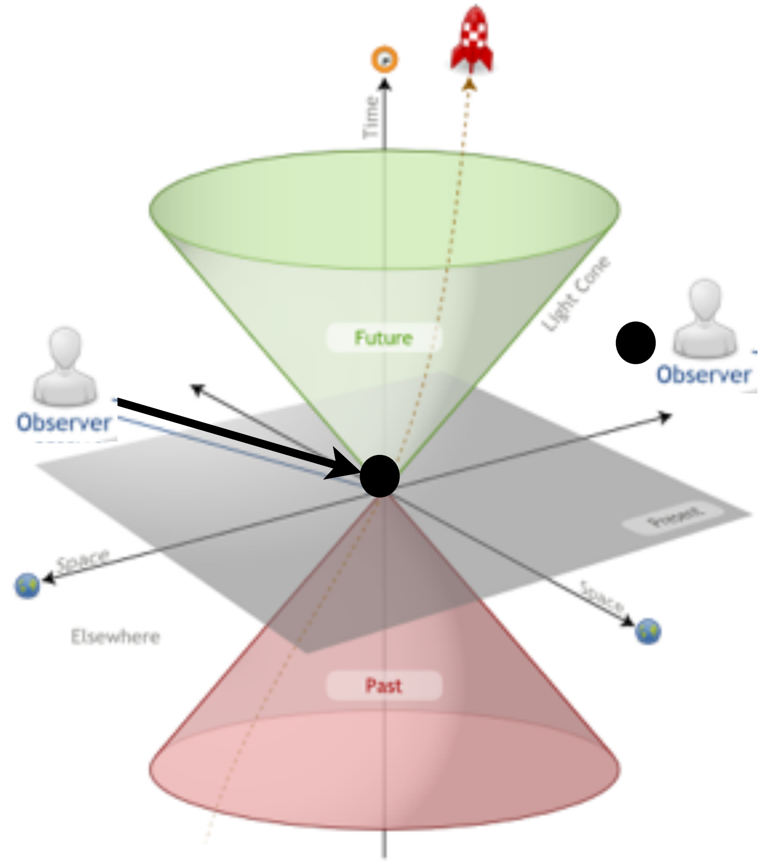
⚠ There are not as many known effects

e.g. Emerging **causality** (e.g. **Lieb-Robinson bounds**)

↓ recent review , M. Kliesch, et al., arXiv:1306.0716 (2013)

How a measurement/perturbation can affect subsequent measurements in distant regions of the system $\langle [O_j(t), O_k(0)] \rangle$

Causality in RQFT: The built-in Lorentz invariance, together with the propagation of particles & anti-particles, guarantees that subsequent **measurements by space-like separated observers have no correlation.**



Causality in non-relativistic (NR) systems :

- NR single-particle systems violate causality
- Causality can emerge dynamically in NR many-body quantum lattice models

"folk knowledge": At low energies, correlations are carried by quasiparticles (fermions), or collective excitations (bosons), with a maximum speed that defines an **effective "light cone"**

The striking thing is that there are rigorous proofs of this emerging causality regardless of energy, initial state, etc.

$$||[O_j(t), O_k(0)]|| \leq C\mathrm{e}^{(d_{jk} - v_{\mathrm{LR}} t)},$$

e.g. spin models

E.H. Lieb, et al., CMP **28**, 251 (1972)

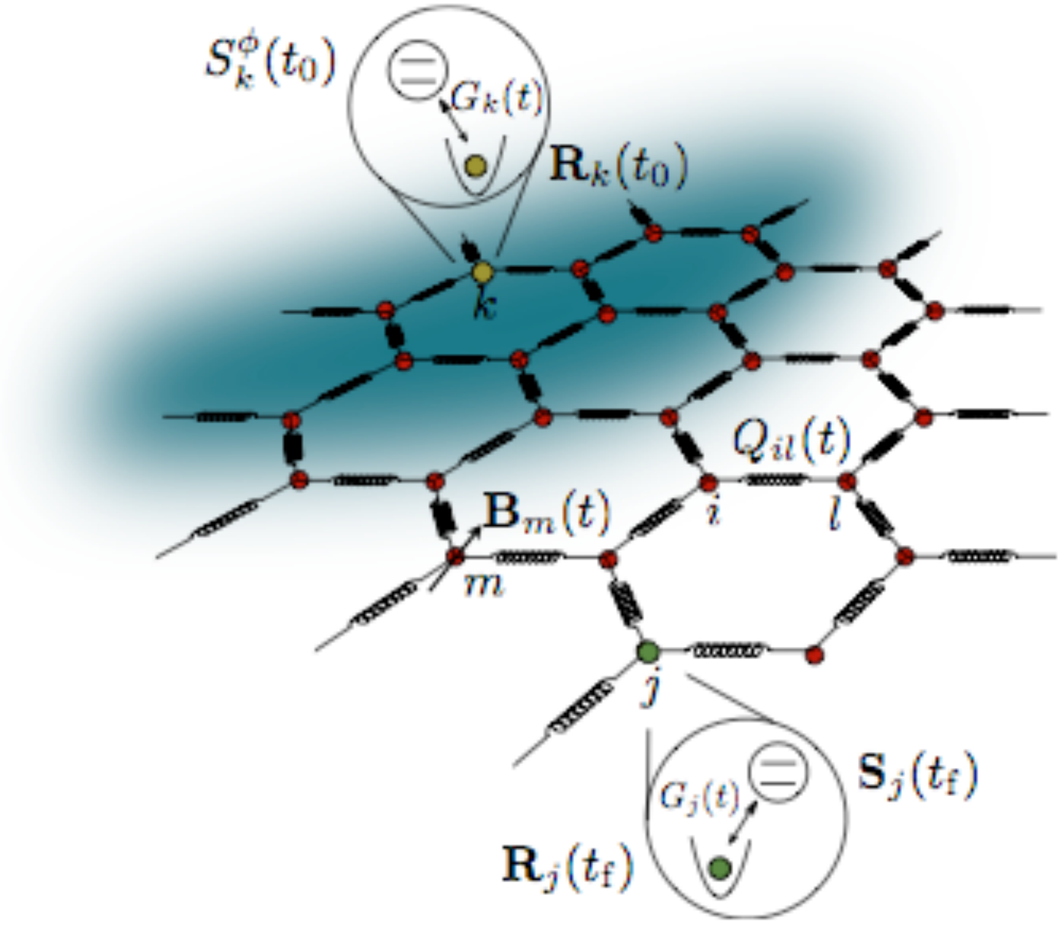
e.g. bosonic models

M. Cramer, et al.,QI&MBS (2008)

Can we find a scheme to **probe Lieb-Robinson bounds in trapped-ion crystals**?

Ion crystals as spin-boson lattice media

Ion crystals are a versatile platform for the sudy of a variety of **spin-boson lattice models**



Spins ⟷ long-lived atomic levels

e.g. spin-1/2 $\boldsymbol{S}_i = \frac{1}{2}(\sigma_i^x, \sigma_i^y, \sigma_i^z)$

Bosons ⟷ local transverse vibrations

$$\boldsymbol{R}_i \propto ((a_i + a_i^\dagger), \mathrm{i}(a_i^\dagger - a_i))$$

We want to study the emergence of causality in the retarded spin-spin correlations

$$\langle [\mathbf{S}_j(t), S_k^\phi(0)] \rangle$$

under archetype Hamiltonians

$$H = \frac{1}{2}\sum_{i,j} \boldsymbol{R}_i^T Q_{ij}(t) \boldsymbol{R}_j + \sum_i \boldsymbol{B}_i(t)^T \boldsymbol{S}_i + \sum_i \boldsymbol{R}_i^T G_i(t) \boldsymbol{S}_i.$$

Kinetic energy & Coulomb couplings

single-qubit drivings

spin-dependent forces

(review sd forces) K-A Brickman Soderberg, et al.,, RPP **73**, 036401(2010)

Deriving a new Lieb-Robinson bound (LRB)

To derive the LRB, one starts by calculating the Heisenberg equations of motion for the correlators

$$\left.\begin{aligned}\mathbf{Z}_{jk}(t) &= [\mathbf{S}_j(t), S_k^{\phi}(0)]\\ \mathbf{C}_{jk}(t) &= [\mathbf{R}_j(t), S_k^{\phi}(0)]\end{aligned}\right\} \xrightarrow{\text{Heisenberg}}$$

system of coupled ordinary differential equations

The spin-boson correlator $\mathbf{C}_{jk}(t)$ can be integrated formally in terms of the **bosonic propagator**

$$\mathbf{R}(t) = W(t,t_0)\mathbf{R}(t_0) + \cdots$$

phonons fulfill a LRB themselves

$$\frac{d}{dt}W(t,t_0) = Q(t)W(t,t_0),$$

propagation of the phonons due to dipolar Coulomb couplings

$$\|W_{jk}(t,0)\| \leq \frac{1+a_0}{a_0}\frac{e^{v_{\rm LR}t}}{(1+d_{jk})^3},$$

LR speed

geometric factor

dipolar range

By feeding the formal solution of $\mathbf{C}_{jk}(t)$ into the spin-spin correlator $\mathbf{Z}_{jk}(t)$, & after performing some local unitaries, one finds a **Dyson-type integral equation.** One solves it iteratively by resumming all terms of a recurrence formula

$$\|[\mathbf{S}_j(t), S_k^{\phi}(0)]\| \leq 2\frac{e^{v_{\rm LR}t}}{a_0(1+d_{jk})^3}\left(\mathrm{e}^{\frac{2g^2(1+a_0)}{v_{\rm LR}}t} - 1\right).$$

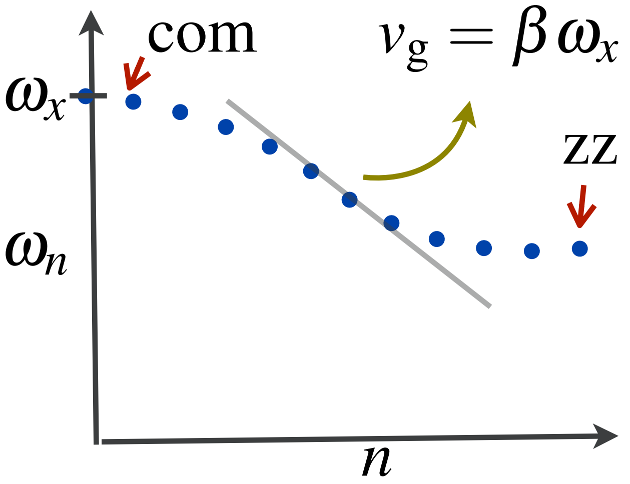
Implications of the Lieb-Robinson bound

The LRB tells us that, under any linear spin-phonon coupling & any initial state, spin-spin correlations will never build up faster than

$$\|[\mathbf{S}_j(t),S_k^\phi(0)]\|\leq 2\overbrace{\frac{e^{v_{\rm LR}t}}{a_0(1+d_{jk})^3}}^{\textbf{bosonic propagation speed}}\overbrace{\left(\mathrm{e}^{\frac{2g^2(1+a_0)}{v_{\rm LR}}t}-1\right)}^{\textbf{spin-phonon adiabatic correction}}.$$

The ultimate propagation speed of spin-spin correations is given by the **phonons maximal group velocity** (faster than spin-spin models)

The propagation speed also depends on how efficiently **spins excite & subsequently absorbe a propagating phonon**



trap	$v_{\rm LR} = 8\beta\omega_x$
1D rf	100kHz
Penning	10kHz
surface	1kHz

$$\beta = (e^2/4\pi\varepsilon_0 d_0)/(m\omega_x^2 d_0^2)$$

$$g \ll v_{\rm LR} \Rightarrow \|[\mathbf{S}_j(t),S_k^\phi(0)]\|\approx 0$$

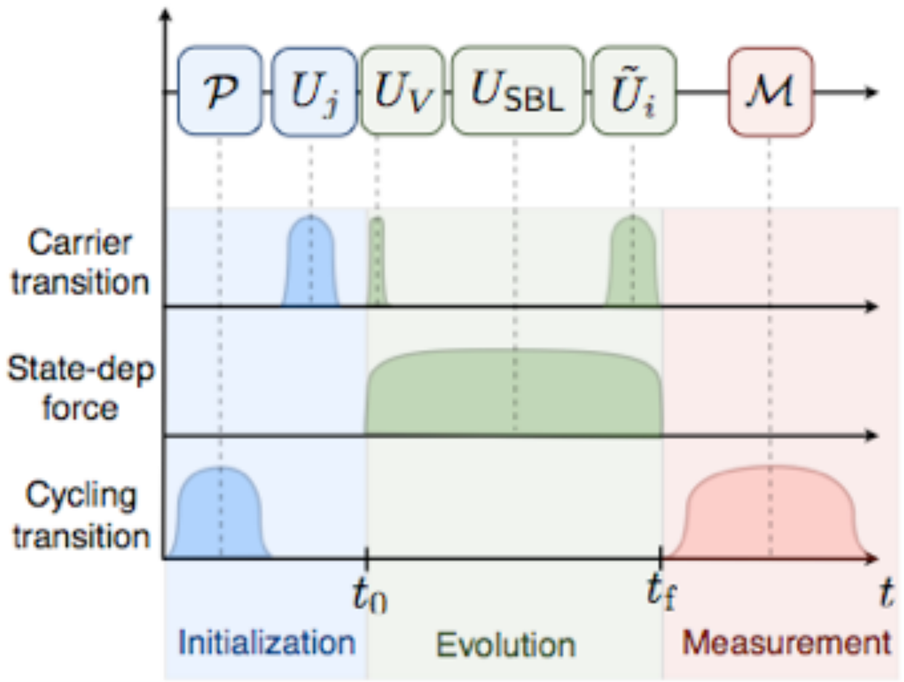
maximal spin-phonon coupling

adiabatic/type argument

No matter how fast the phonons propagate, the spins are so “slow” that cannot efficiently excite/absorbe the phonons to get correlated

Probing the Lieb-Robinson bound

We map the retarded spin correlation functions to the spin-dependent fluorescence spectrum



CCD images give access to the correlation build-up

$$\langle[\sigma_j^\alpha(t), \sigma_k^\beta(0)]\rangle$$

U_V perturbation at t=0 proportional to σ_k^β

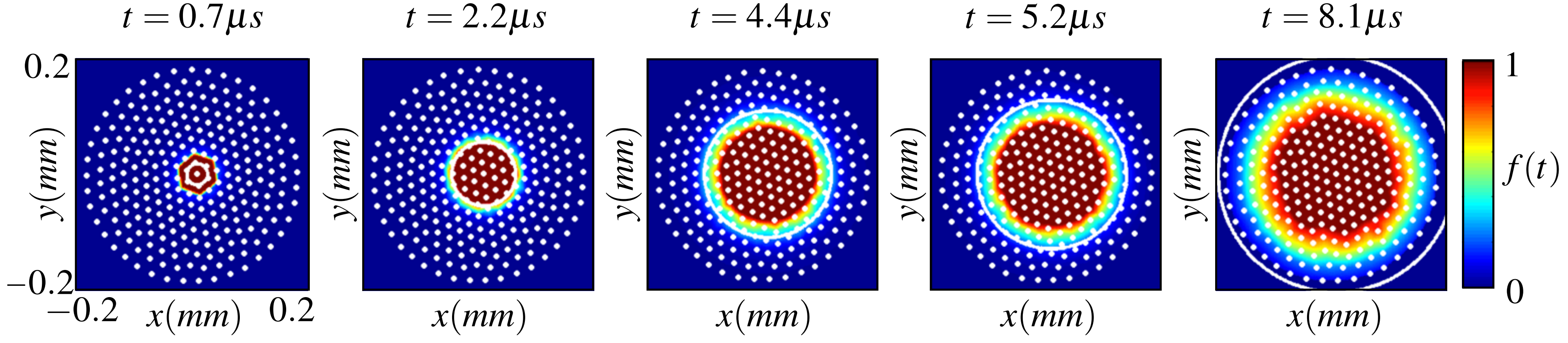
$\tilde{U}_i$ single qubit gate at t=tf that rotates $\sigma_j^z \rightarrow \sigma_j^\alpha$

linear-response-theory-like calculation

$$\langle\sigma_i^z(t_{\mathrm{f}})\rangle_{\mathrm{pert}} = \langle\sigma_i^\alpha(t_{\mathrm{f}})\rangle_{\mathrm{unpert}} - \mathrm{i}\lambda_{\mathrm{B}}\langle[\sigma_i^\alpha(t_{\mathrm{f}}), \sigma_j^\beta(t_0)]\rangle_{\mathrm{unpert}}.$$

similar ideas to probe retarded correlation functions M. Knap, et al., arXiv:1307.0006 (2013).

e.g. buildup of correlations in the impulsive regime for a Penning trap


$t = 0.7\mu s$ $t = 2.2\mu s$ $t = 4.4\mu s$ $t = 5.2\mu s$ $t = 8.1\mu s$