Lectures

Andrea Braides

Local minimization, variational evolution and Gamma-convergence

Gamma-convergence is designed for the treatment of global minimization problems and in general does not describe neither the behaviour of local minimization problems nor of variational evolution. We will discuss some issues linked to those notions in which, nevertheless, Gamma-convergence may be of interest.

Geoffrey Burton

Water Waves with Vorticity and Rearrangements

We introduce a variational principle governing steady spatially periodic waves on the surface of an incompressible inviscid fluid in the presence of vorticity. A novel feature is that all the prescribed data are time-invariants of the corresponding unsteady problem. The necessary conditions for a minimum are shown to imply weak versions of the steady incompressible Euler equations and the Bernoulli condition at the free surface. Existence of a minimiser representing a non-parallel flow can be proved under certain extra assumptions about the surface. This is joint work with John Toland.

José A. Carrillo

Stability and Pattern formation in Nonlocal Interaction Models

I will review some recent results for first and second order models of swarming in terms of patterns, stationary states, and qualitative properties. I will discuss the stability of these patterns for the continuum and discrete particle cases. These non-local models appear in collective behavior for animals, control engineering, and molecular structures among others. We first concentrate in the spatial shape of these patterns and the dynamics when inertia terms are neglected. The mathematical question behind consists in finding properties about local minimizers of the total interaction energy. Concerning 2nd order models, we will discuss particular properties of two patterns: flocks and mills. We will discuss the stability of these patterns in the discrete case. In both cases, we will describe the properties obtained for the continuum limits.

Camillo De Lellis

From Nash to Onsager: funny coincidences across differential geometry and the theory of turbulence

The incompressible Euler equations were derived more than 250 years ago by Euler to describe the motion of an inviscid incompressible fluid. It is known since the pioneering works of Scheffer and Shnirelman that there are nontrivial distributional solutions to these equations which are compactly supported in space and time. If they were to model the motion of a real fluid, we would see it suddenly start moving after staying at rest for a while, without any action by an external force. A celebrated theorem by Nash and Kuiper shows the existence of C^1 isometric embeddings of a fixed flat rectangle in arbitrarily small balls of the threedimensional space. You should therefore be able to put a fairly large piece of paper in a pocket of your jacket without folding it or crumpling it. In a first joint work with Laszlo Szekelyhidi we pointed out that these two counterintuitive facts share many similarities. This has become even more apparent in some recent results of ours, which prove the existence of Hoelder continuous solutions that dissipate the kinetic energy. Our theorem might be regarded as a first step towards a conjecture of Lars Onsager, which in his 1949 paper about the theory of turbulence asserted the existence of such solutions for any Hoelder exponent up to $\frac{1}{3}$. Currently the best result in this direction, $\frac{1}{5}$, has been reached by Phil Isett.

Adriana Garroni

Γ-*convergence analysis of discrete topological singularities: metastability and dynamics*

The main mechanism for crystal plasticity is the formation and motion of a special class of defects, the dislocations. These are topological defects in the crystalline structure that can be identified with lines on which energy concentrates.

I will consider a discrete model for straight screw dislocations, that turns out to be very close to the model of vortices in spins systems. We exploit the relation with the Ginzburg Landau theory for vortices in superconductors and we give an asymptotic expansion of the discrete energy (in terms of Γ-convergence).

As a consequence of this asymptotic analysis we obtain the existence of many metastable conffigurations with given distributions of dislocations and hence a pinning effect for the associated dynamics. A notion of discrete dynamics associated to a crystalline dissipation will be also discussed.

The results are obtained in collaboration with R. Alicandro, L. De Luca and M. Ponsiglione.

Jan Kristensen

Rank-one convexity and integral estimates

It is known that many interesting questions about sharp integral estimates involving derivatives of mappings can be reformulated as questions about quasiconvexity properties of associated integrands. In this talk we discuss the strong convexity properties of one–homogeneous rank–one convex integrands and some of their consequences for integral estimates. The talk is based on joint work with Bernd Kirchheim (Leipzig).

Rosario Mingione

TBA

Mariapia Palombaro

A multi-scale analysis of dislocations in nanowire heterostructures

Dislocations represent an important class of defects in crystalline solids and their presence influences the behavior of materials in many ways. For example, in semiconductor electronics dislocations play a crucial role in the development of nanowire heterostructures, which can be defined as the combination of two or more materials within the same nanowire structure. Indeed, a large lattice mismatch between the materials of interest may result in poor quality interfaces with high misfit dislocation density. We present a rigorous mathematical analysis showing that, for a given mismatch, formation of dislocations is energetically more favorable than purely elastic deformations when the radius of the cross-section is sufficiently large.

The analysis is performed in the framework of Γ-convergence both in the continuous and in the discrete setting.

Thomas Schmidt

TBA

Jey Sivaloganathan

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Dimitrios Tsagkarogiannis

Young-Gibbs measures for the Ising model

The value of the magnetisation at a macroscopic point of a magnetic material when observed at a mesoscopic scale, can be realised either as a homogeneous state or as a fine mixture of the two pure phases of the system. This has been mathematically described with the use of Young measures. On the other hand, from an atomistic point of view at finite temperature, each such pure phase is described via an extremal Gibbs measure and mixtures via convex combinations of the extremal ones. In this talk we connect the two descriptions deriving a macroscopic continuum mechanics theory for scalar order parameter starting from statistical mechanics. For this, we revisit a recent work by Kotecky and Luckhaus and we construct the so-called Young-Gibbs measures for the case of the nearest neighbour Ising model. This is work in progress jointly with Alessandro Montino (GSSI) and Nahuel Soprano Loto (Argentina).

Nikos Tzirakis

Smoothing for Nonlinear Dispersive PDE: The Talbot Effect

We study the evolution of the one dimensional periodic NLS equation with bounded variation data. For the linear evolution, it is known that for irrational times the solution is a continuous, nowhere differentiable fractal-like curve. For rational times the solution is a linear combination of finitely many translates of the initial data. We prove that a similar phenomenon occurs in the case of the cubic NLS equation.