A study on early-warning signals for transitions to desertification

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Summary

The identification of reliable early-warning signals for critical transitions is crucial in ecosystems such as arid or semi-arid ones strongly exposed to desertification risks.

Seminal works in this field related vegetation patchiness in arid ecosystems to an approaching transition to desertification.

We performed an analysis of indicators associated with vegetation pattern, and applied results of percolation theory in order to assess the state of a system at risk and indentify early warning signals for desertification. For a detailed description see:

R. Corrado, A.M.Cherubini, C.Pennetta, *Time Fluctuations of Vegetation Patterns and Early Warning Signals of Desertification Transition*, submitted

Drylands: regions where water scarcity limits vegetation growth

Aridity Index (AI): mean annual precipitation/ potential annual evaporation

Dry subumid lands: $0.5 < AI < 0.65$

Semiarid lands: $0.2 < AI < 0.5$

DRYLANDS: $AI < 0.65$

Arid lands: $0.05 < AI < 0.2$

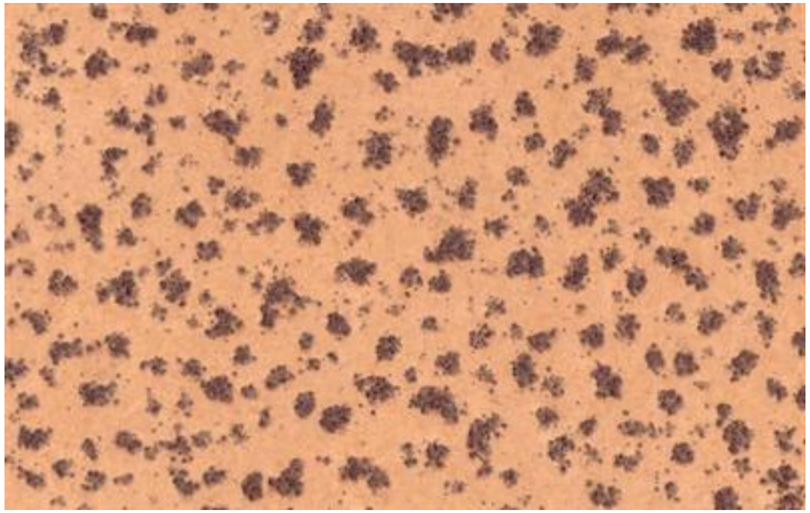
Hyper-arid: $AI < 0.05$

Drylands cover about 41% of the Earth land area

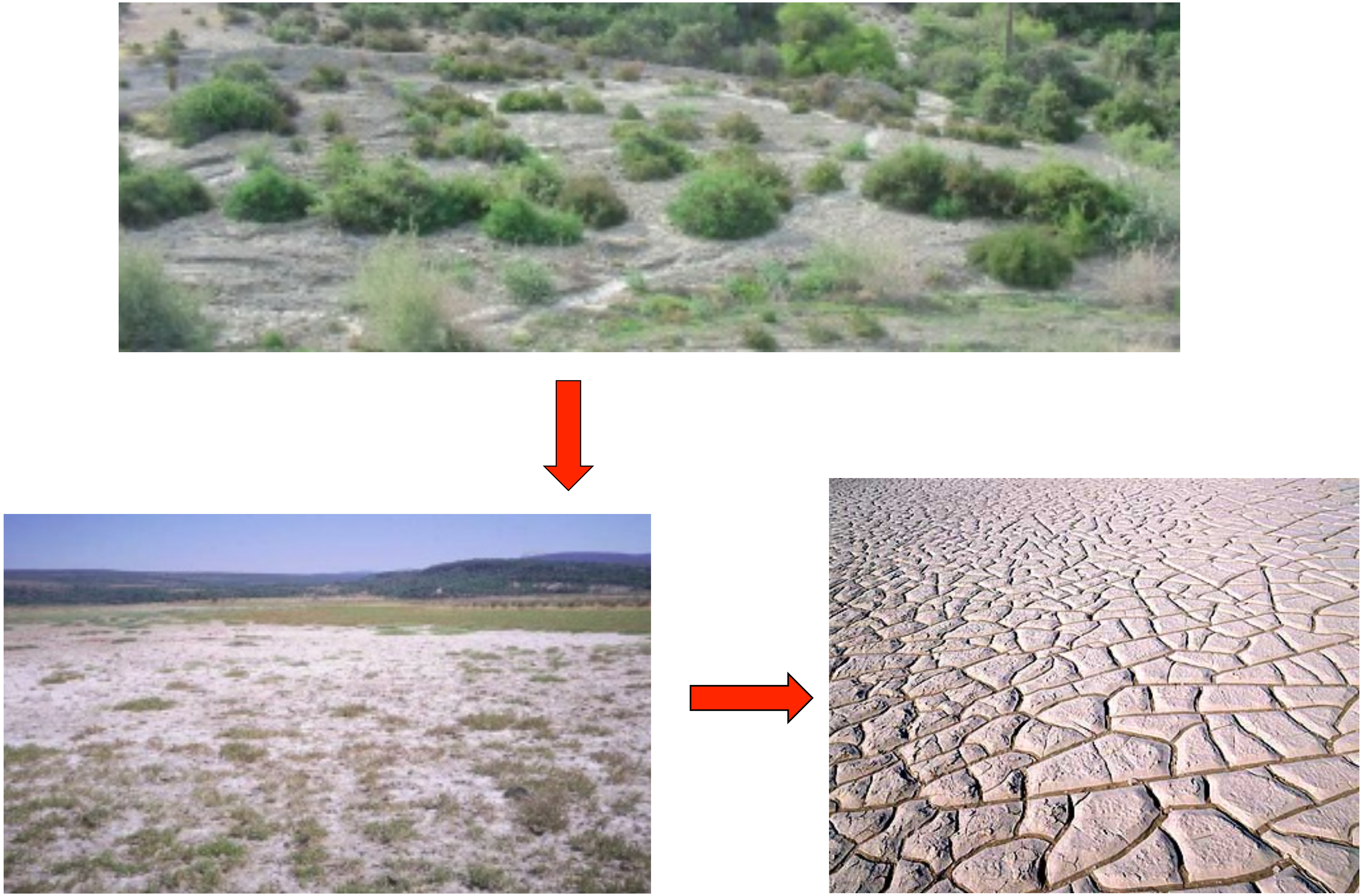
- Vegetation cover $\leq$ 60%
- Mosaic of vegetation patches and bare soil

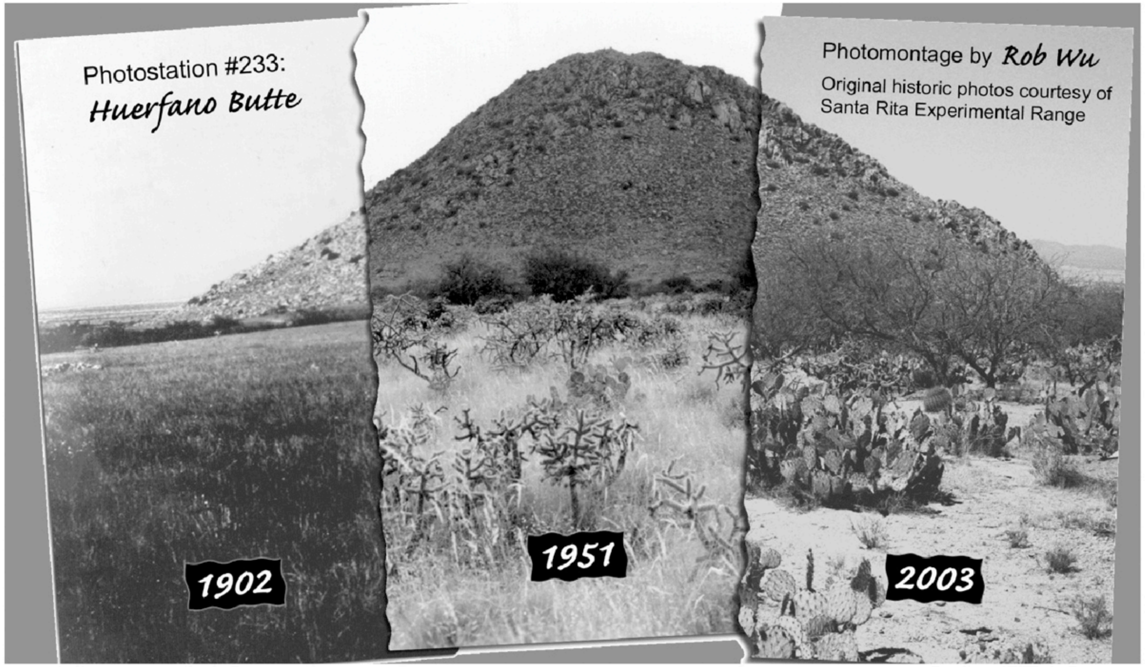
(Millennium Ecosystem Assessment 2005, World Resources Inst. Washington DC)

Typical vegetation patchiness observed in arid landscapes near desertification.



Desertification transition in arid or semi-arid ecosystems





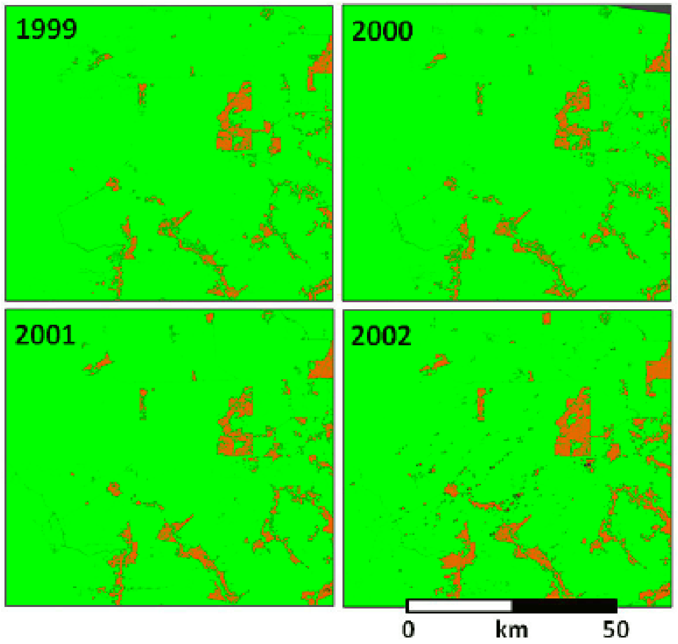
CCSP SAP 4.3[243]

The photo series shows the progression from arid grassland to desert (desertification) over a 100-year period. The change is the result of grazing management and reduced rainfall in the Southwest. [250,252,253]

from: *2009 Report on Global Climate Change Impacts in the US*
http://nca2009.globalchange.gov/desertification-arid-grassland-near-tucson-arizona-1902-2003

Several technologies are available for a fast and efficient measure of the vegetation patchiness, such as satellite remote sensing, automate CLASlite monitoring technique…

http://claslite.carnegiescience.edu/en/



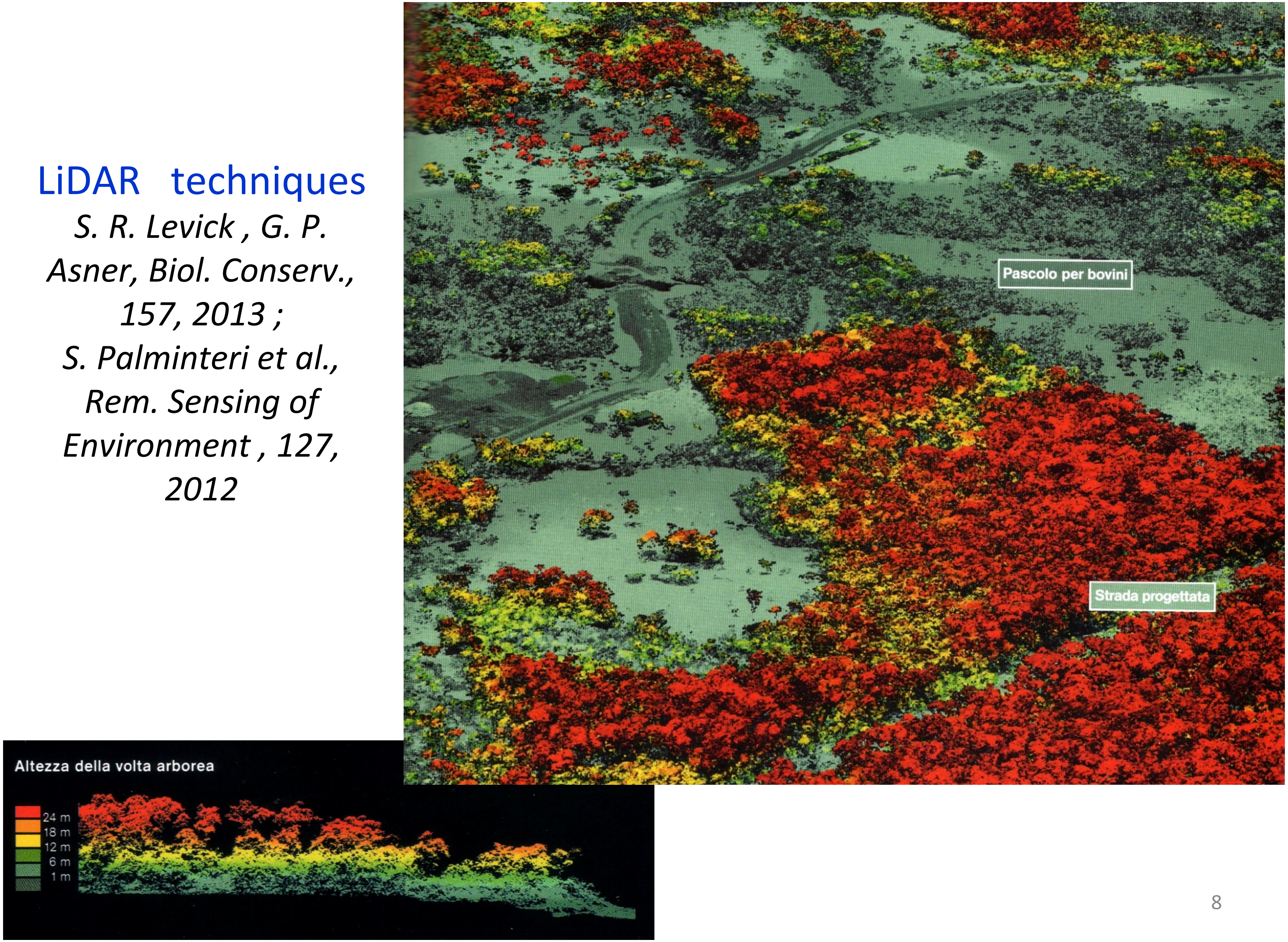
Automate CLASlite analysis of forest (green) and non-forest (orange) cover over 3000 Km^2 of the eastern Brazilian Amazon

G. P. Asner et al. ; J. Of App. Remote Sensing, 3, 033543, 2009.

LiDAR techniques

S. R. Levick , G. P. Asner, Biol. Conserv., 157, 2013 ;

S. Palminteri et al., Rem. Sensing of Environment , 127, 2012



A warning signal for desertification: deviation from power-law in the patch-size distributions.

In [K07], the authors investigated the influence of external stresses on the spatial organization of the vegetation, by combining modeling and field data from three grazed Mediterranean arid ecosystems in Spain, Greece, Morocco.

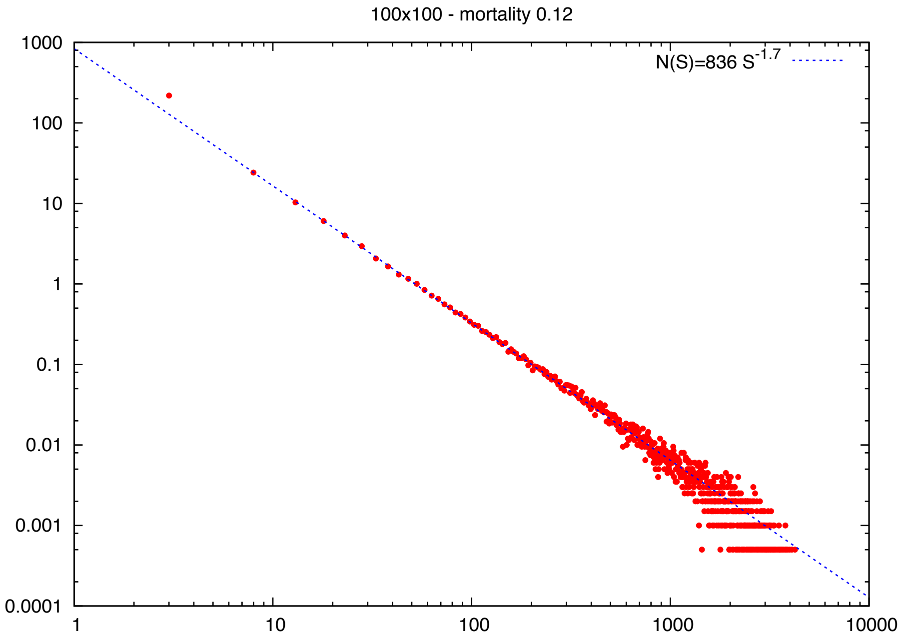
The model accounts for several ecological mechanisms and describes different ecological landscapes by a number of parameters, in particular the *mortality* m , associated with the external stress (in this case grazing pressure).

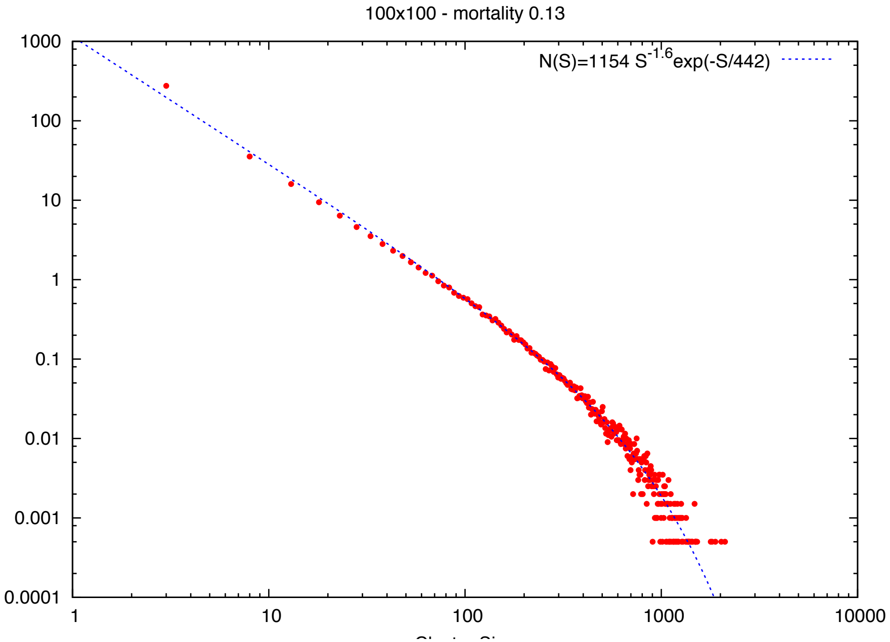
A relevant result is that, for increasing m, the vegetation patch-size distribution significantly deviates from a power law, showing an exponential cut-off for m greater than a value $m_{k.}$ This was proposed as an early warning signal.

[K07] S. Kéfi, M. Rietkerk, C.L. Alados, Y. Pueyo, V. P. Papanastasis, A. ElAich, P. C. de Ruiter, *Nature*, **449** (213), 2007.

For further discussion:
A. Manor, N.M. Shnerb, *Phys. Rev. Lett*, **101** (268104), 2008.
A. Manor, N.M. Shnerb, *Phys. Rev. Lett*, **103** (030610), 2009.

Deviation from power law in patch-size distribution





$m=0.11$

$m_k=0.13$

- m_k depends on various parameters;
- in this example $m_k \sim 0.13$ with desertification transition at $m_c \sim 0.18$

We will show that the onset of a desertification process in a arid ecosystem can be very effectively described in a percolation framework. This also allows the definition of new early indicators.

In particular, this approach provides a simple and general explanation for the behaviour of patch-size distribution for increasing external stress described in [K07].

Moreover, we analysed the time fluctuation properties of the largest vegetation cluster under external stress of increasing strength, thus identifying other potential early indicators for desertification.

Model

In order to describe the ecosystem dynamics, we performed numerical simulations on the model for arid ecosystems proposed in [K07] and [K07bis].

A three states stochastic cellular automaton (CA) on a square lattice (N × N) with periodic boundary conditions. A cell can be:

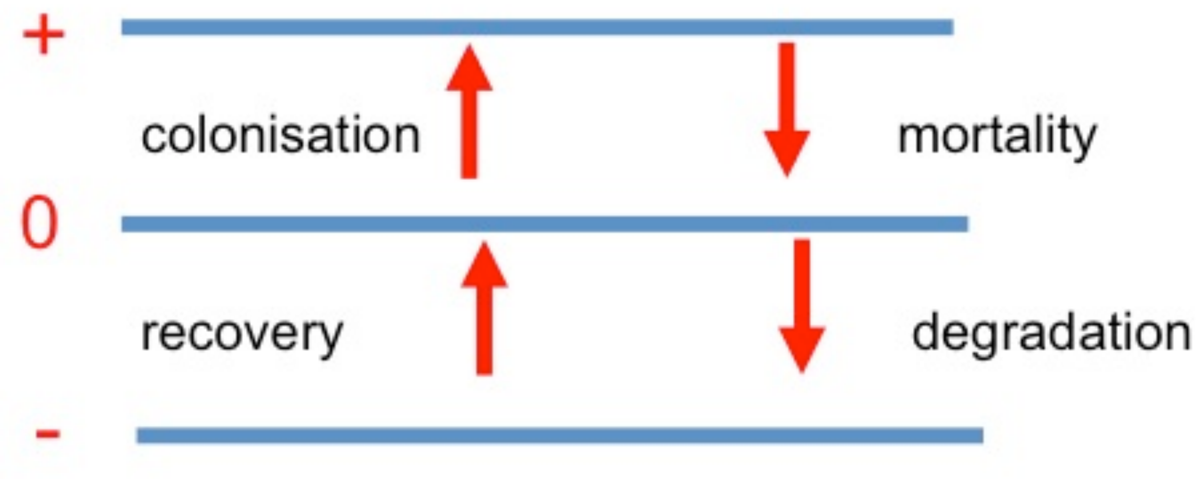
+ occupied by vegetation;

0 dead, empty of vegetation but colonisable ;

- degraded, empty, not directly colonisable.

A degraded site has to go through state 0 to reach state +

[K07bis] S. Kéfi, M. Rietkerk, M. van Baalen, M. Loreau, *Theor. Popul. Biol.* **71** (267), 2007



ρ_+ , ρ_0 , ρ_- fractions of cells in the states + , 0 , -

$q_{j|i}$ prob. of finding a *j* cell in a neighbourood of a *i* cell

Transition rates and parameters

Colonisation

An unoccupied cell can be colonised by vegetation with a rate w_{0+} depending on seed production and dispersion, competition for resources etc.

$$w_{0\,+}=[\delta\,\rho_{+}+(1-\delta)\,q_{+|0}]\,(b-c\,\rho_{+})$$

δ fraction of seed globally dispersed

b intrinsic properties of the cell: soil quality, seed production and survival probability, germination and survival probability of plants.

c global competition for resources

Mortality

Mortality rate m is taken as independent from density and depending from external factors

$$w_{+\,0} = m$$

Degradation

Empty cells can be degraded by rain/wind erosion, etc

$$w_{0\,-} = d$$

Recovery

Regeneration of a degraded cell depends on rain, soil type etc. and is faster if there is vegetation nearby

$$w_{-\,0} = r + f\,q_{+|-}$$

r spontaneous regenerative rate,

f facilitation, local cohoperative effects

- Short and long-range interactions between cells of the lattice

- In applications, the choice of the time unit depends on the ecosystem and the time-scale characterising the plants

- Though not directly relevant for the present discussion, we mention that using pair approximation (and other rules, see [DLM200]) this model can be translated into a system of ODEs for the densities ρ_i and pair densities ρ_{ij}

[DLM200]
U. Dieckmann, R. Law, J.Metz eds., The Geometry of Ecological Interactions,
Cambridge University Press, 2000

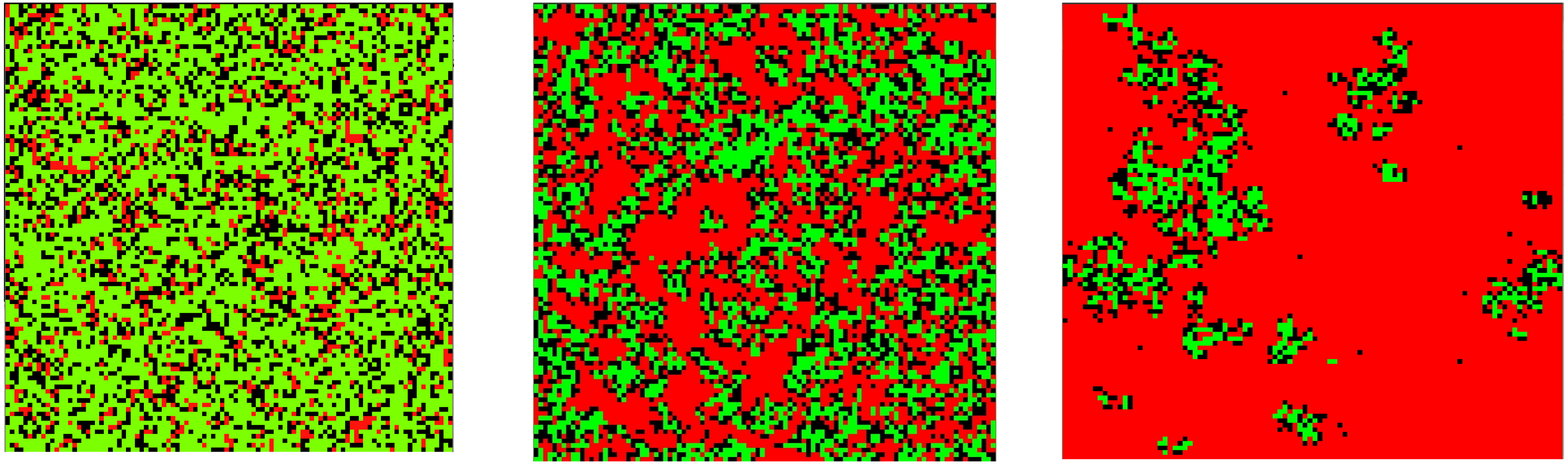
Time series statistical analysis for the densities, size of maximum cluster for different populations, biggest cluster size concerned the stationary region of the series, after relaxation of the initial transient.

We considered lattices of different sizes: N =50, 100, 150 and several values of the mortality parameter m

The results here were obtained by fixing the other parameters at following values:
b = 0.6, c = 0.3, d = 0.2, δ = 0.1, f = 0.9, r = 0.0004

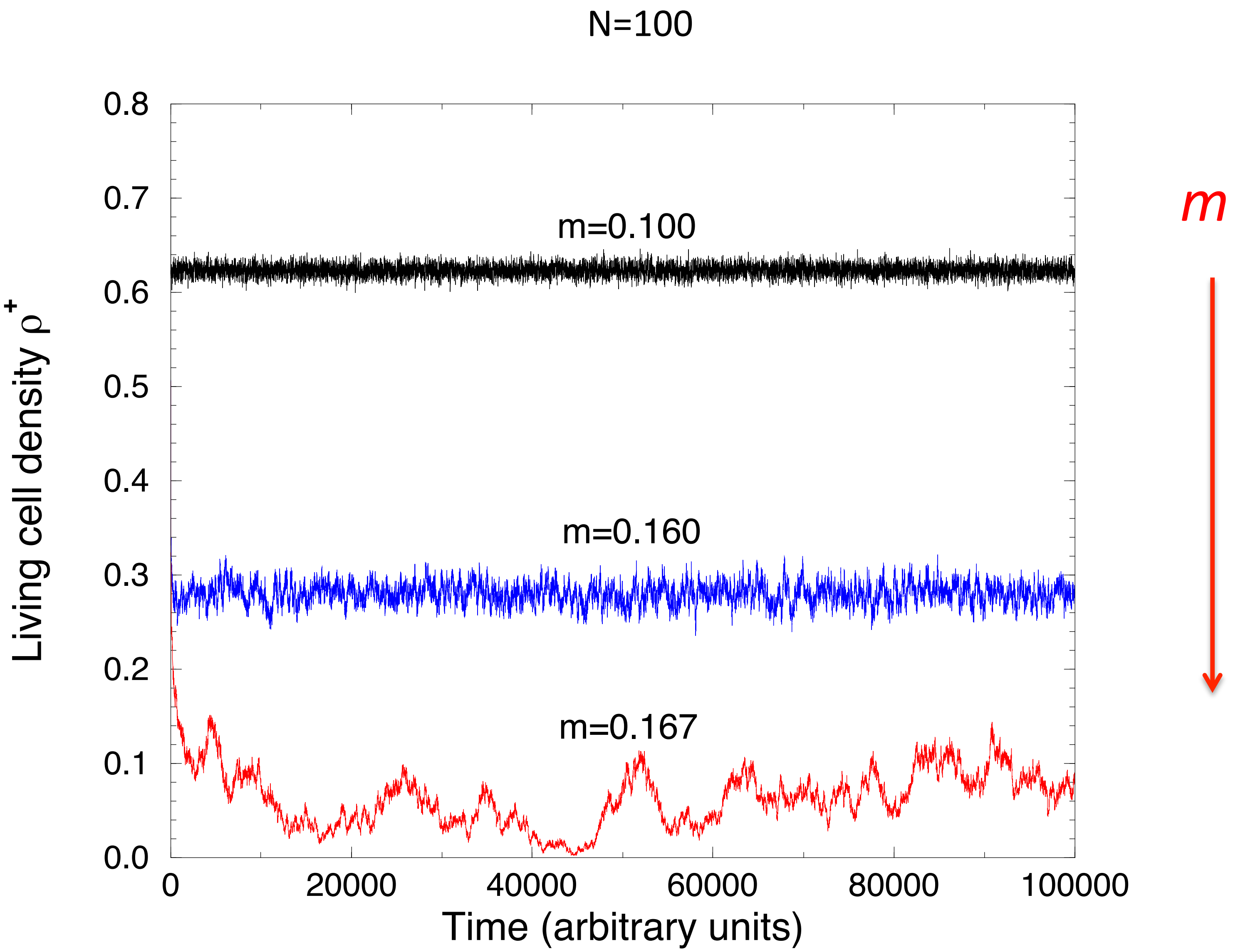
Snapshots of the CA for different *m*

living cells 🟩 dead cells ⬛ degraded cells 🟥



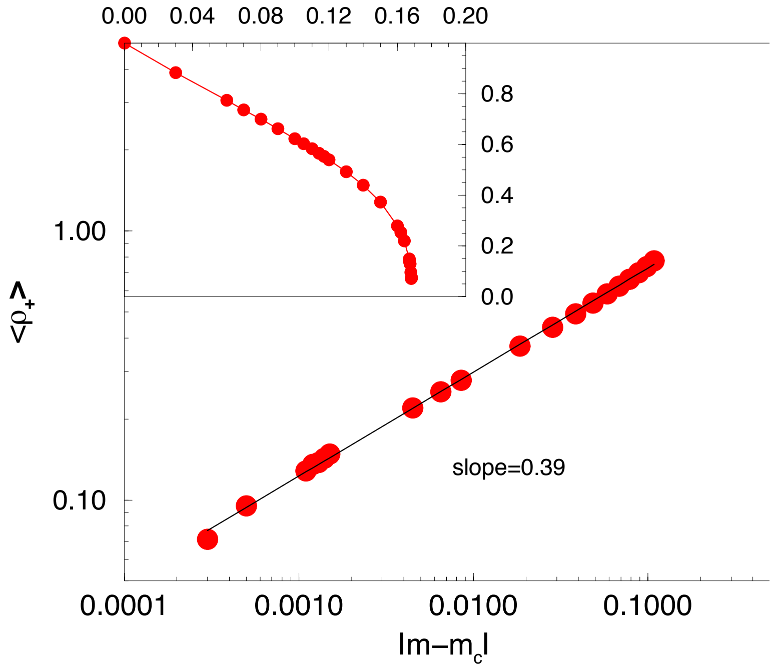
m=0.10 m=0.16 m=0.1680

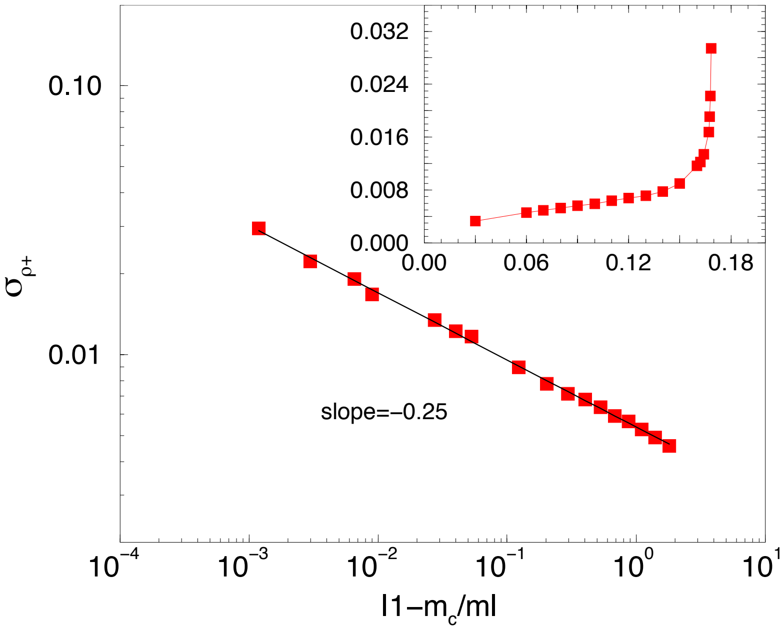
Vegetation density vs. time, increasing *m*



Identification of the transition to desertification

There exists a value m_c above which the steady state corresponds to $<\rho_+>=0$





$$<\rho_+>\approx |m-m_c|^{\alpha}$$

$$\sigma_+ \approx \left|1-\frac{m_c}{m}\right|^{\beta}, \qquad \beta<0$$

In the insets linear plots of $<\rho_+>$ and $\sigma_{\rho+}$ vs. m

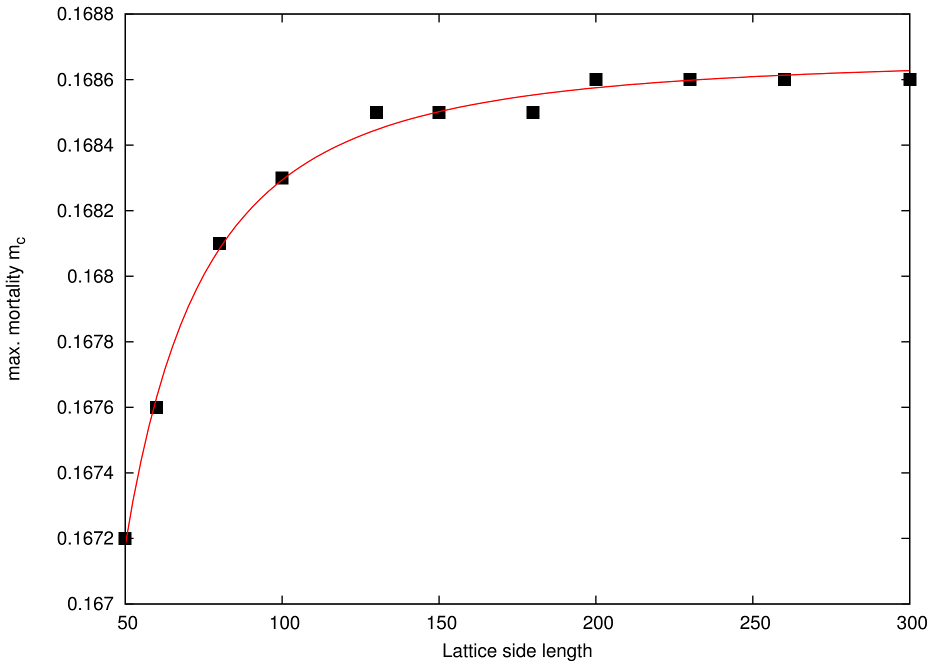
Exponents α and β are independent of system size; they depend on the other parameters.

m_c is associated with a critical transition (extinction of vegetation, full desertification).

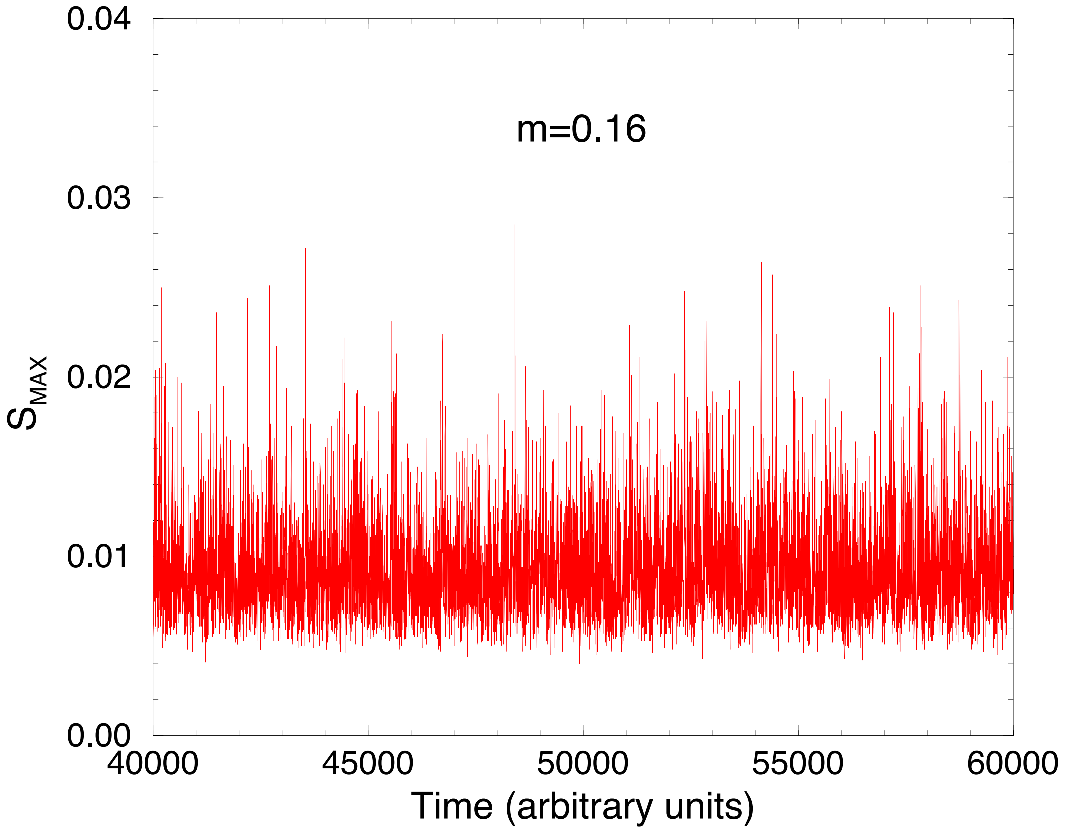
The value depends ‘weakly’ on N: here N=100, $m_c \sim 0.1685$

$$m_c \approx m_c^{\infty}\left(1-\frac{k}{N^2+k}\right);$$

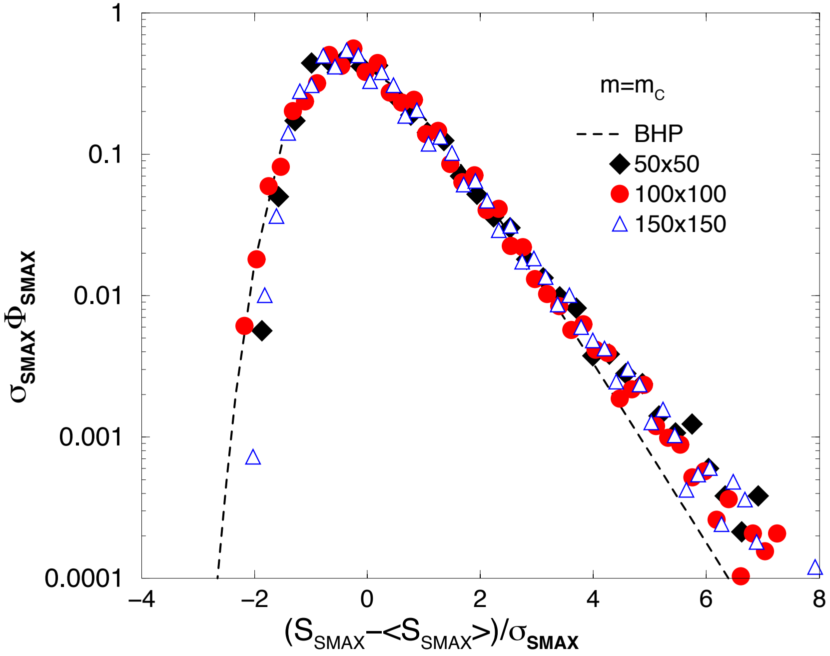
$$m_c^{\infty} = 0.1686 \pm 10^{-4}$$



Behaviour of the maximum vegetation cluster size S_{max} for m approaching m_c



Facilitation mechanism allows the survival of small clusters of vegetation at high values of *m*, frequently connected into a bigger one. This explains the asymmetry the fluctuations.



$m = m_c$

Normalised PDF of time fluctuations of biggest vegetation cluster size, for different system sizes.

(Φ_{smax} PDF of ΔS_{max} ; the product $\sigma_{smax}\Phi_{smax}$ is plotted vs. normalized fluctuation $\Delta S_{max}/\sigma_{smax}$)

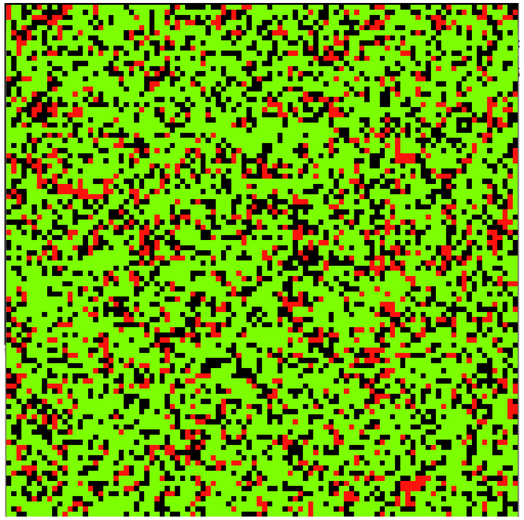
The dashed curve is a generalized Gumbel, characterising fluctuations of systems of different nature close to criticality (see [C04], [Br08])

[C04] M. Clusel, J. Y. Fortin and P. C. W. Holdsworth, *Phys. Rev. E* **70**, 2004
[Br08] S. T. Bramwell, P. C. W. Holdsworth and J. F. Pinton, *Nature* **396** (552), 2008,

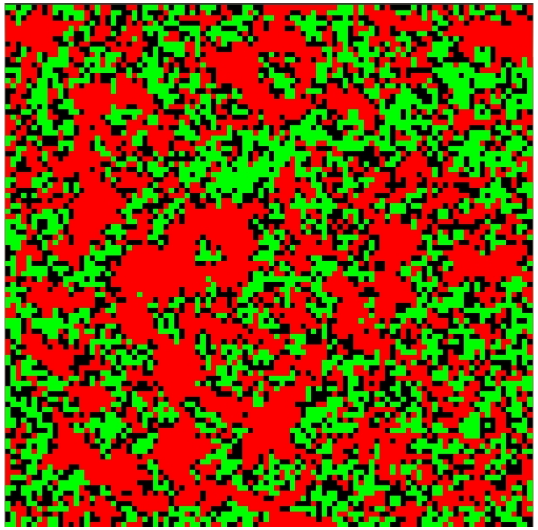
Looking at the CA for increasing values of *m* one can see the percolation of degraded and dead cells (*non-living*) in contrast with the living ones.

We applied results in percolation theory to perform an analysis for the 2-phases system of living and non-living cells.

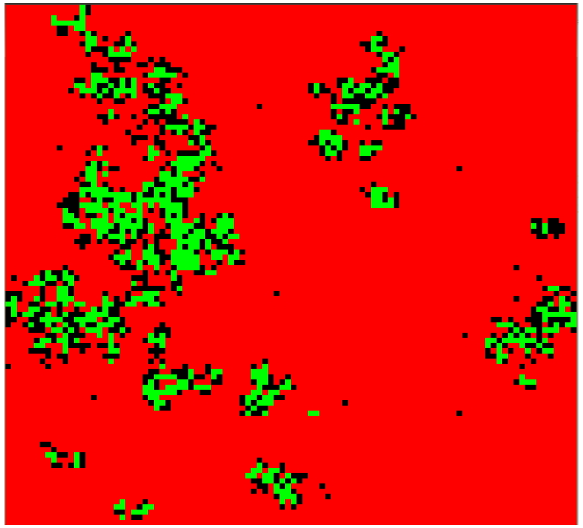
In this framework, for increasing values of *m*, we will be able to identify, well before the extinction, two intervals of values corresponding to the percolation transition of the living cells (onset of the fragmentation of vegetation) and, later, to the percolation transition of NL cells (onset of desertification).



m=0.10



m=0.16

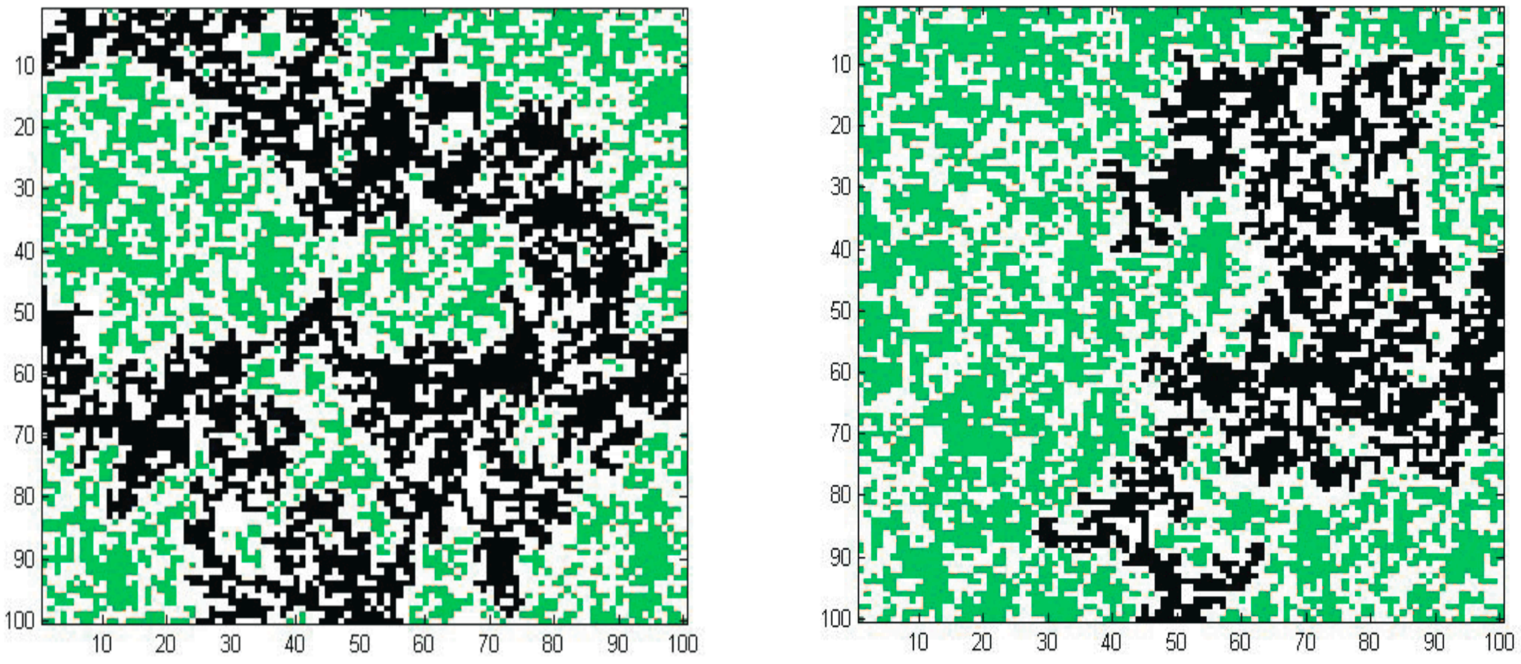


m=0.1680

Two examples of percolating clusters of living cells.

Here white cells are non-living (dead and degraded), the other ones are living cells.

Black colour denotes the spanning clusters: one connects simultaneously both vertical and horizontal boundaries (left), the other connects horizontal boundaries (right).



Different spanning criteria

We considered four different spanning criteria, characterising different stages in the percolation transitions

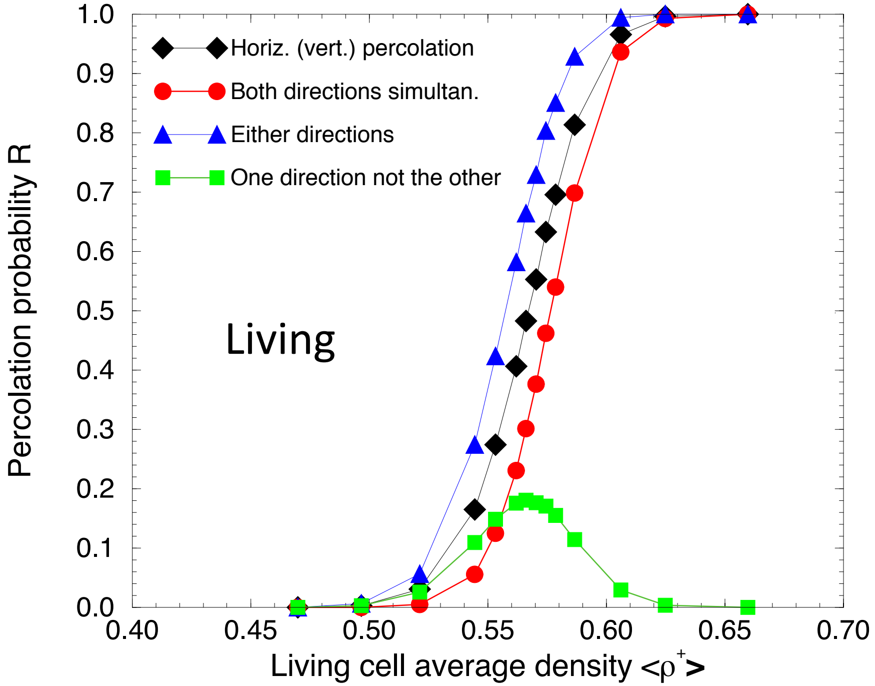
- Both directions simultaneously ●
- Horizontal (vertical) direction ◆
- Either direction **Δ**
- Only one direction 🟩

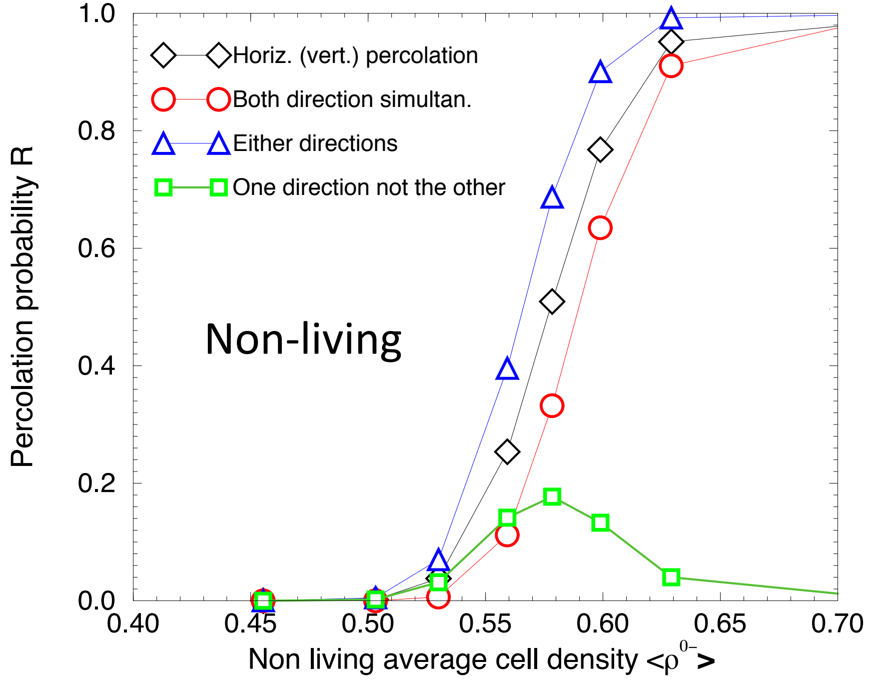
According to different criteria, we computed four different percolation probabilities R_{b+}, R_{h+} , R_{1+} ,R_{e+} for a cluster of living cells (and correspondig ones for non-living clusters). The probabilities are linearly related, only two independent ones (e.g. R_b and R_h). They depend on N, we dropped the index here because we refer only to N=100.

[NZ2000] M. E. J. Newman, R. M. Ziff, *Phys. Rev. Lett.* **85**, 2000
[NZ2002] M. E. J. Newman, R. M. Ziff, *Phys. Rev. E.* **66**, 2002
[HA1996] J.P. Hovi, A. Aharony, *Phys. Rev. E* **53**, (1996)

Percolation probabilities for living and non-living cells

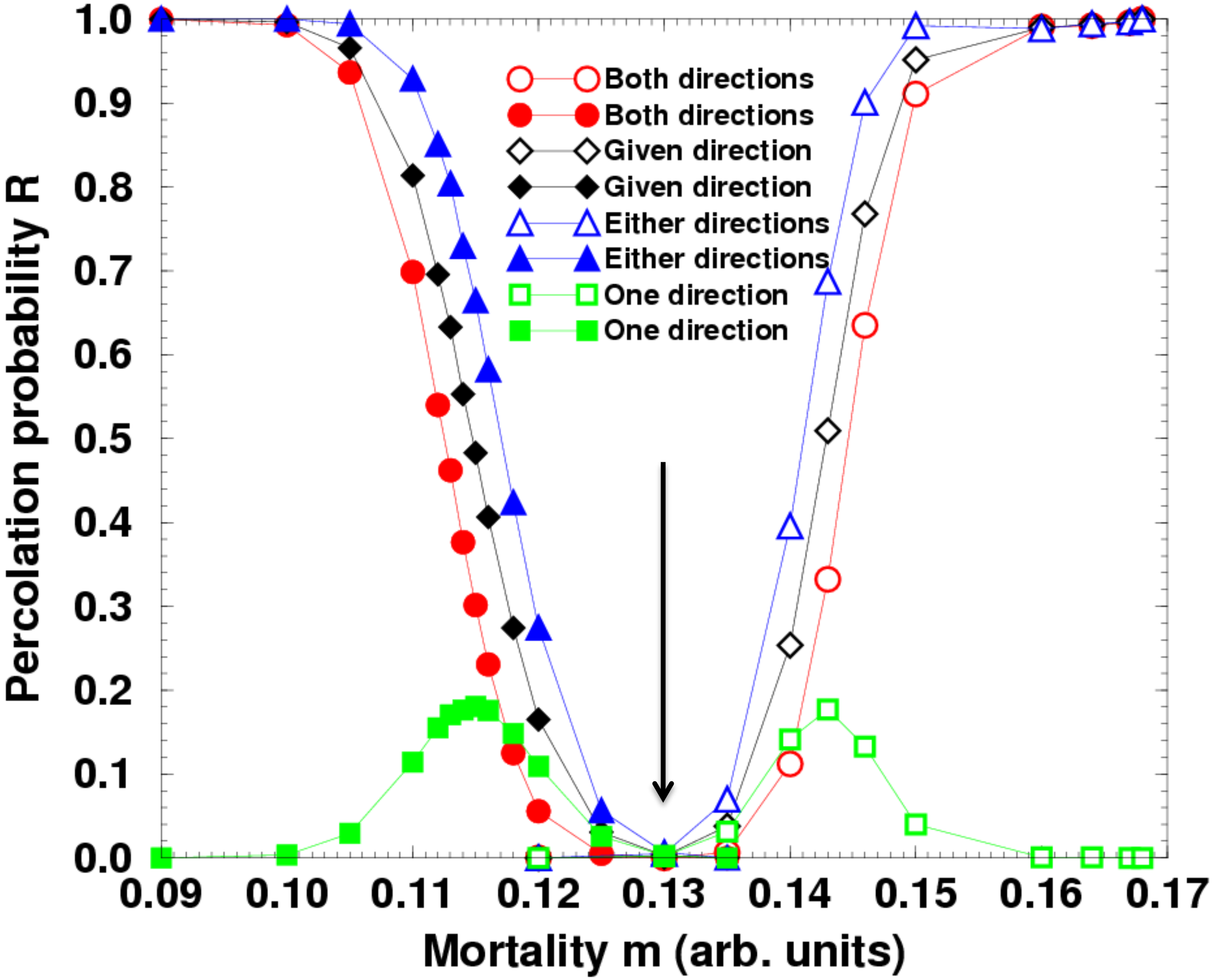
- Both directions simultaneously ●
- Horizontal (vertical) ◆
- Either direction Δ
- Only one direction 🟩





Percolation probabilities vs. mortality

living (filled symbols) and non-living cells

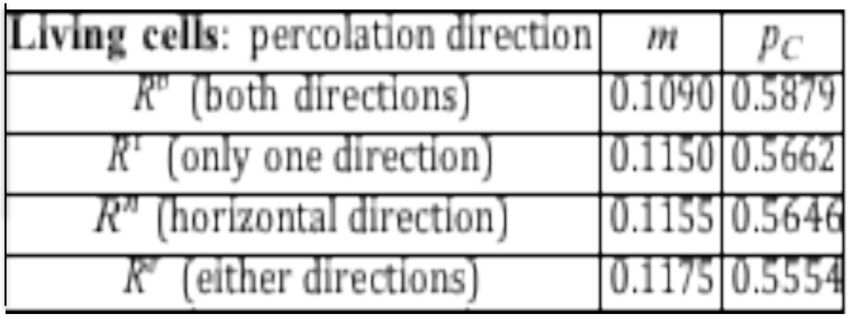


- The previous figure highlights a value for m at which neither phase percolates. It corresponds to the system's maximum fragmentation: no prevailing phase, neither gives rise to an incipient cluster spanning through the lattice.

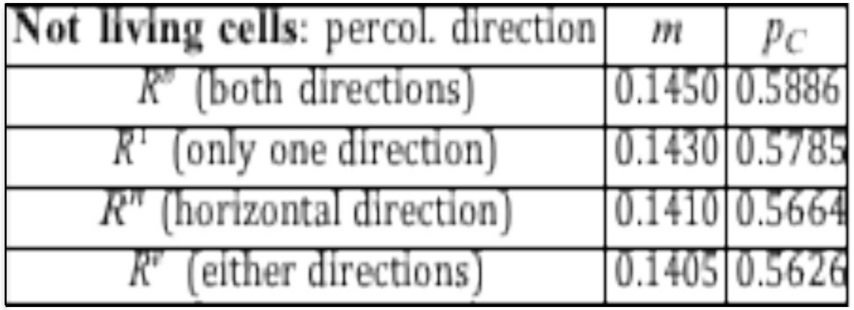
- This point corresponds to the threshold m_k , identified in [K07] as early warning, at which an exponential cut-off emerges in the vegetation patch-size distribution. This behaviour is simply explained in a percolation framework: in fact, the cluster size distribution follows a power law only at the percolation threshold and decays exponentially with a power-law factor under or above threshold.
- The value of m_k depends on parameters. In this case (N=100, parameters as defined) $m_k \sim 0.132$

Thresholds

Percolation thresholds (see [NZ2002]) p_c for each criterium and corresponding values of *m* (± 0.0002)*:*



Living cells: percolation direction	m	p_C
R^b (both directions)	0.1090	0.5879
R^1 (only one direction)	0.1150	0.5662
R^h (horizontal direction)	0.1155	0.5646
R^e (either directions)	0.1175	0.5554



Not living cells: percol. direction	m	p_C
R^b (both directions)	0.1450	0.5886
R^1 (only one direction)	0.1430	0.5785
R^h (horizontal direction)	0.1410	0.5664
R^e (either directions)	0.1405	0.5626

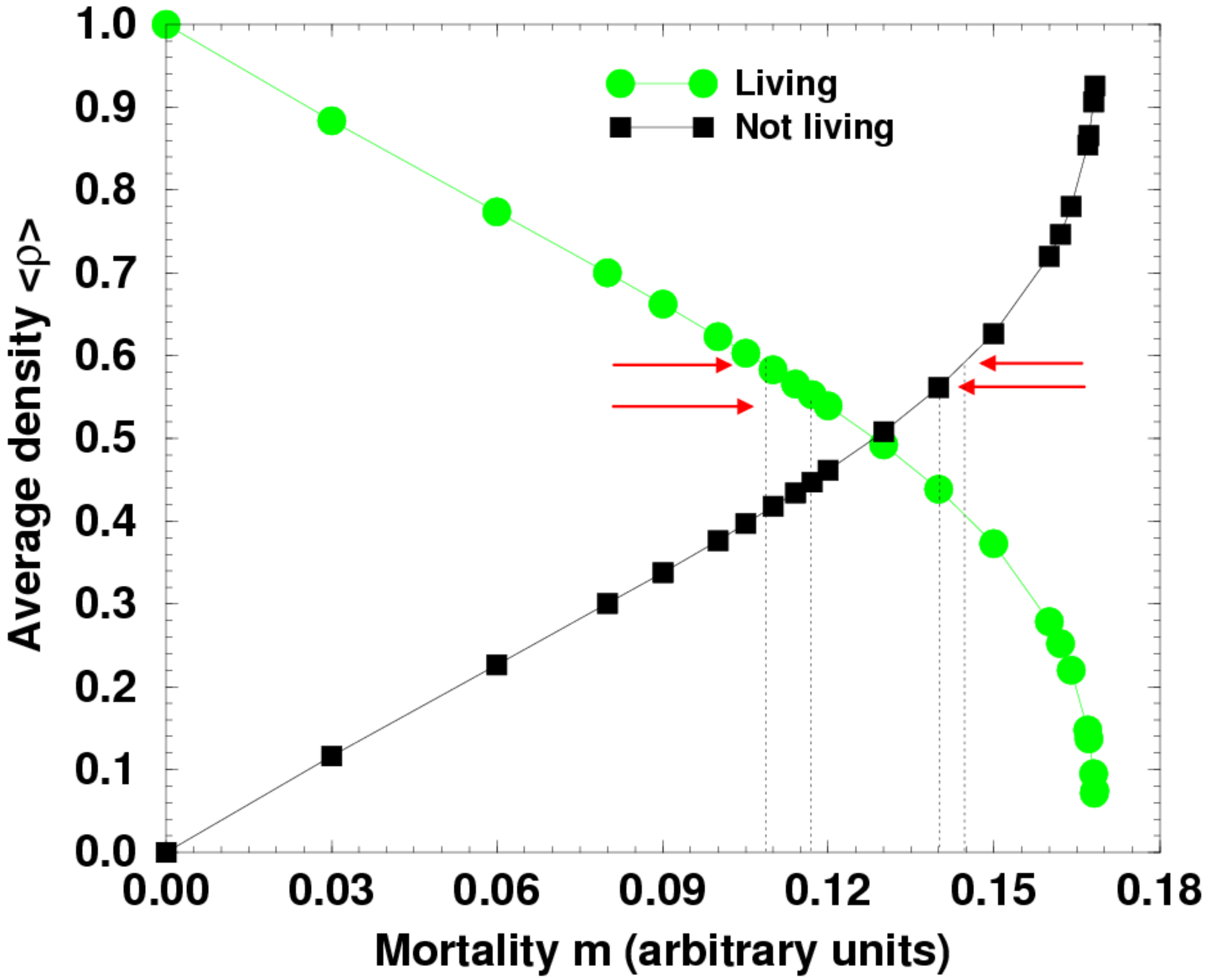
$$p_c = 1 - \int_0^1 R(\rho)\, d\rho$$

[NZ2002] M. E. J. Newman, R. M. Ziff, *Phys. Rev. E* **66**, 2002

Fragmentation and *desertification* transitions

For each phase (living, non-living) the percolation thresholds define an interval of values of *m* (or corresponding average densities) in which the system progressively breaks its connectivity. This is a general character for percolation externally driven.

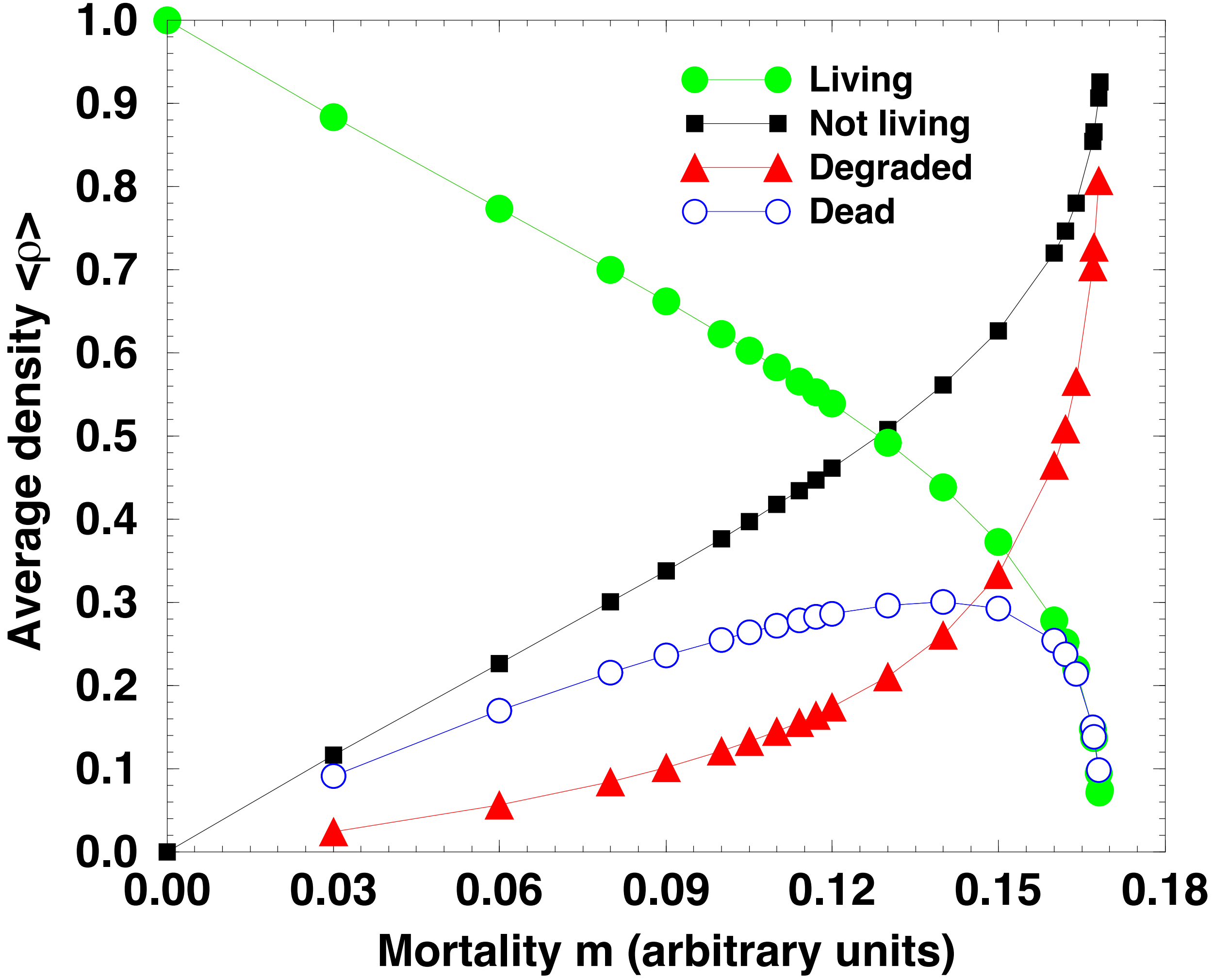
The different spanning criteria, and related percolation thresholds, provide a tool for assessing the increasing degradation of the system. Before reaching the extinction the systems goes through *fragmentation* and *desertification* transitions.



Average densities of living and non-living cells vs. m. The vertical lines highlight the intervals of *m* corresponding, from left, to the fragmentation and desertification transition. m_k is at the densities crossing.

- Fragmentation Interval ~ 0.11- 0.117
- Desertification interval ~ 0.14- 0.145

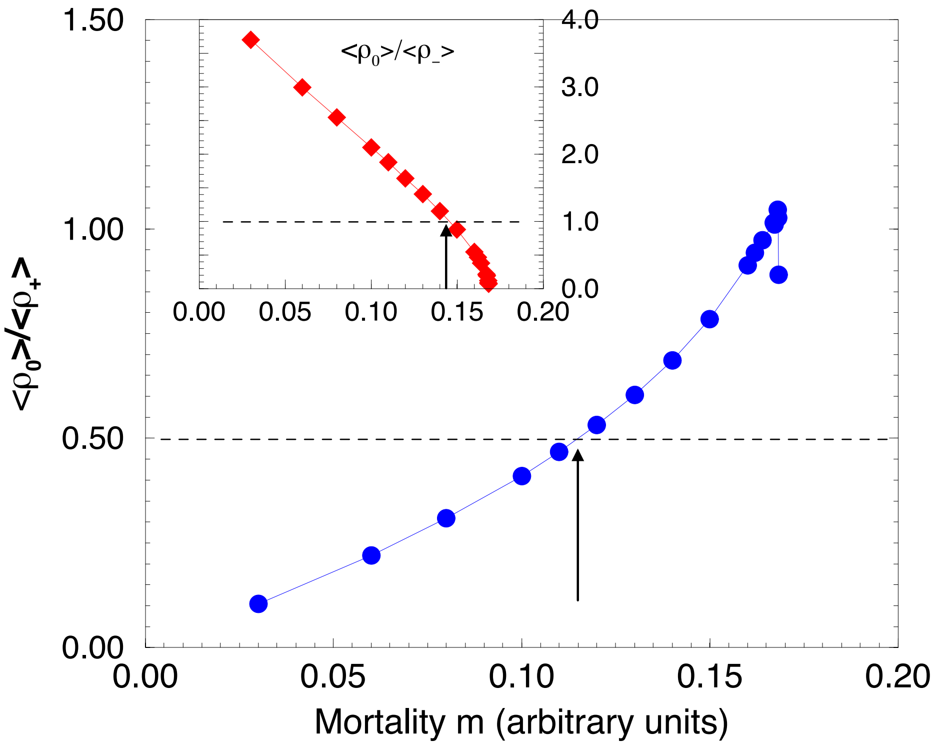
The distinction into living/non-living is useful for the percolation approach. The dead or degraded cells play different roles in the ecosystem and obey to different rules. Here we detail their different behaviours.



The dead cell density $<\rho_0>$ increases in the fragmentation transition,
has a maximum during the desertification transition then decreases.

This is explained by the role that the dead cells play in this model, since they are always on the boundary between clusters of living and degraded cells. As it is typical in a two-phase lattice where one phase is close to a percolation transition, clusters have a fractal geometry, so cells in the boundary of the incipient spanning cluster became a significant fraction of the total number of that population.

Increase in $<\rho_0>$ starts at the onset of fragmentation, it provides therefore a very early indicator for that transition

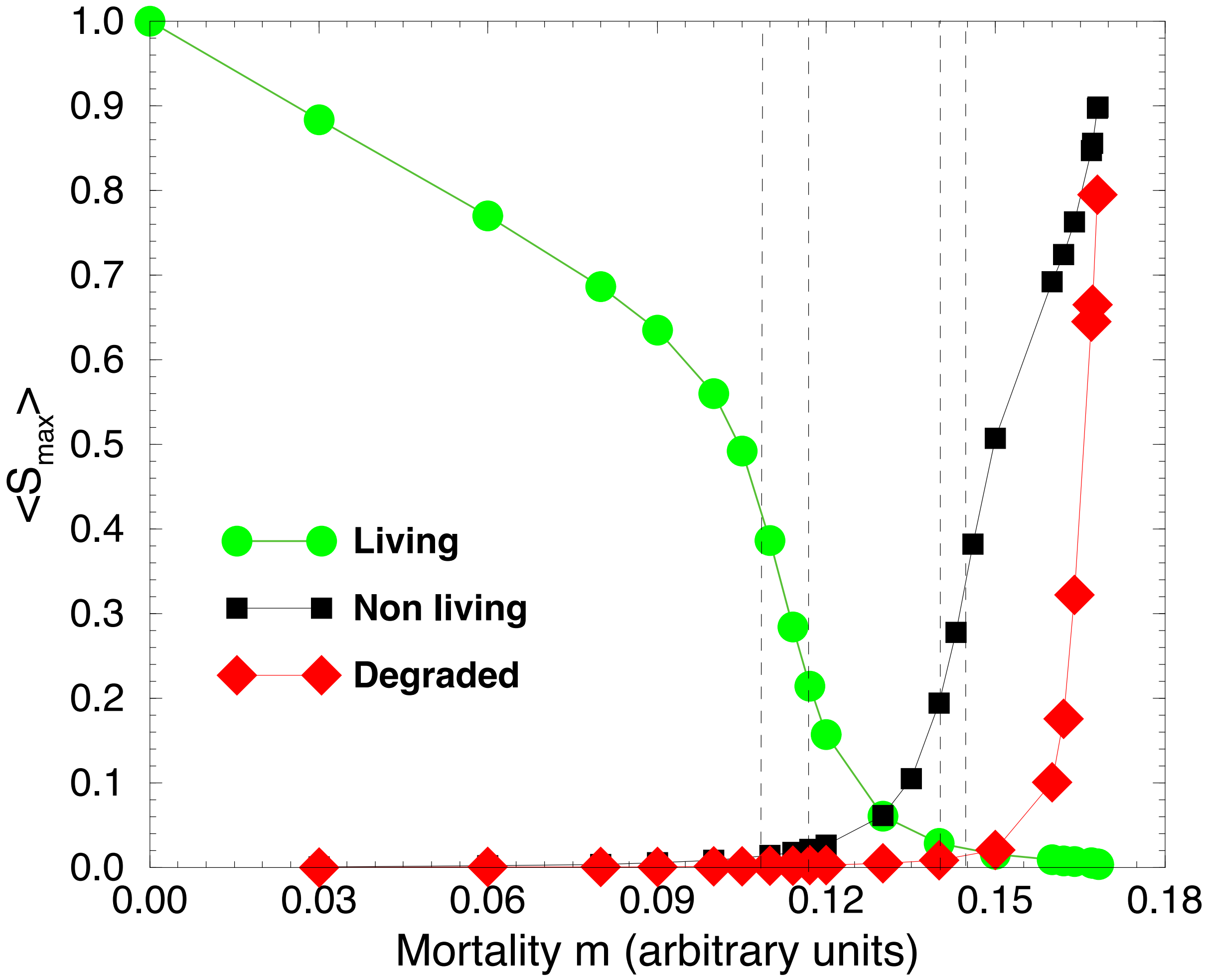


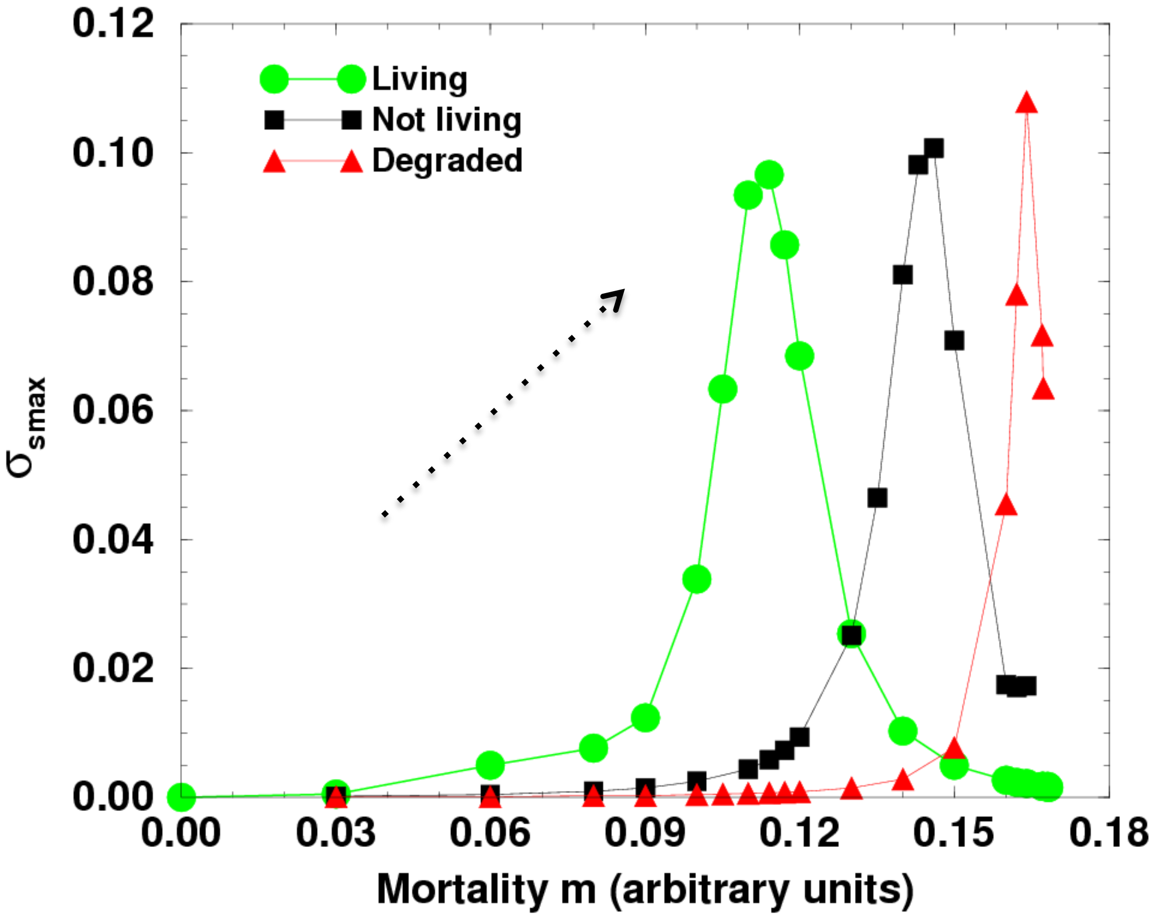
$\frac{\langle \rho_0 \rangle}{\langle \rho_+ \rangle}$ and $\frac{\langle \rho_0 \rangle}{\langle \rho_- \rangle}$ (blue and red), vs. m

Average dead cells density is half the vegetation one in the fragmentation interval.

Average dead cells density is equal to the degraded cells density in the desertification interval: it significantly contributes to the spanning cluster.

Maximum cluster size vs. *m* for different populations





For any population, maximum values for standard deviations of maximum clusters sizes are attained in the corresponding percolation intervals.

We can use the increase in σ_{SMAX} (in particular for living cells) as an early indicator.

Insight in the behaviour of fluctuations of S_{max} can be provided by results in [Ba00] for uncorrelated percolation.

Roughly:

$S_{max} \sim s_{\xi} \log N$, s_{ξ} is the characteristic cluster size at the crossover.

$\sigma_{SMAX} \sim s_{\xi} \Pi(N)$, Π periodic function of N.

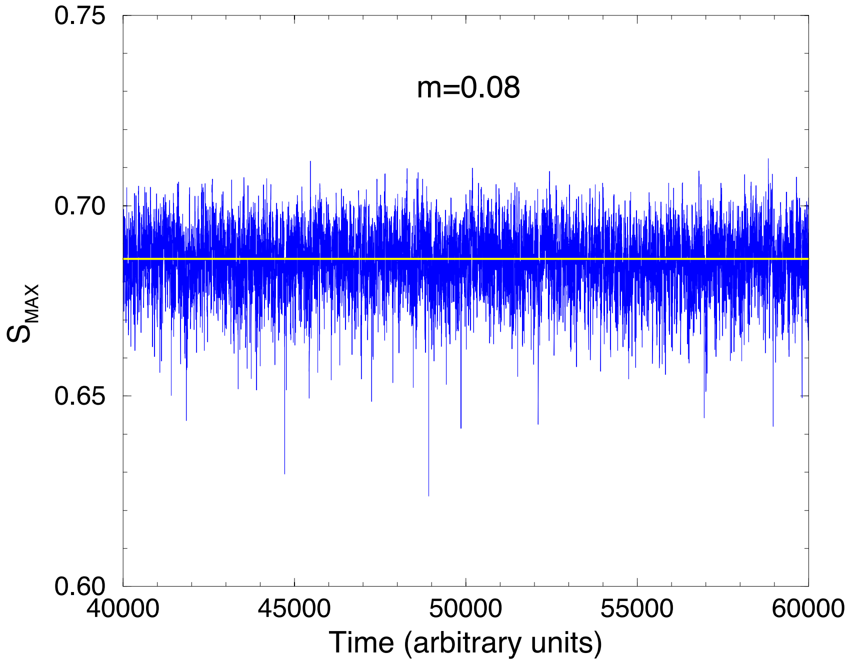
The individual values (e.g. for s_{ξ}) and the length of the transition intervals depend on N, but maxima for σ_{SMAX} for different populations are always attained close to the related threshold values, i.e. respectively in the fragmentation and desertification transitions and at the extinction.

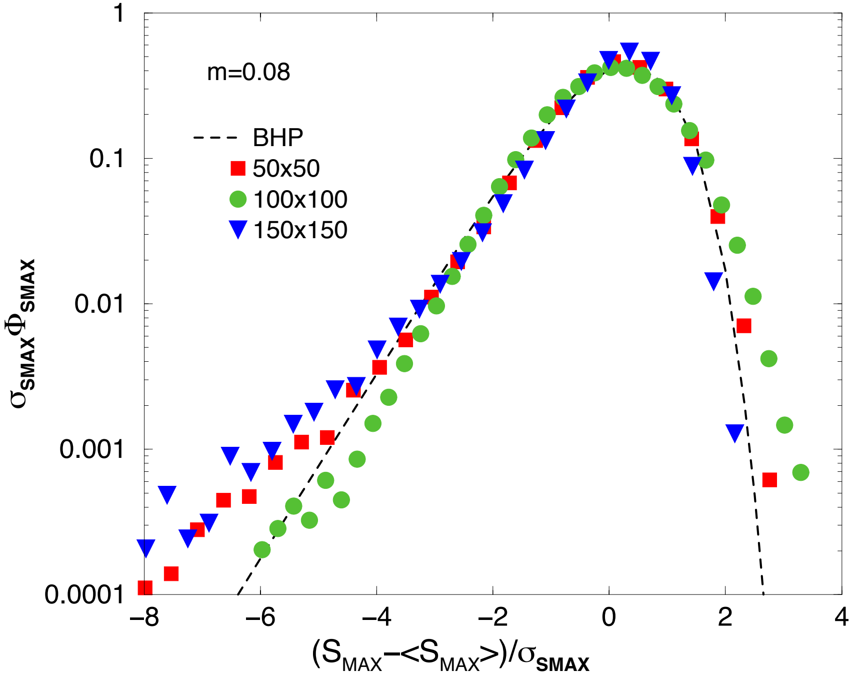
[Ba00] M. Z. Bazant, *Phys. Rev. E* **62**, 2000

Asimmetry in max. vegetation cluster fluctuations for small *m*

Relatively large clusters of living cells are frequently disconnected from biggest cluster, to which they are linked by narrow bottle necks: hence a pronounced asymmetry in the fluctuations.

Strong asymmetry between average return times of negative and positive extreme values of ΔS_{max} can also be verified.

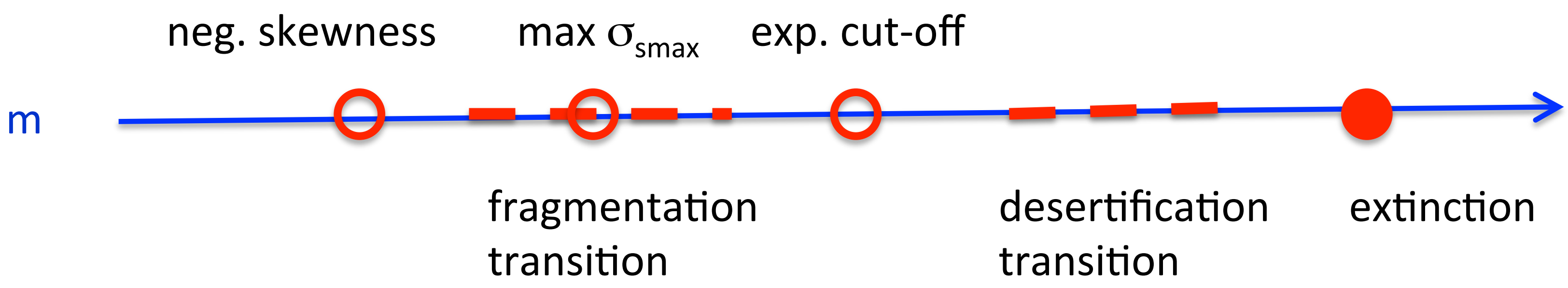




Normalized PDF for time fluctuations of vegetation biggest cluster size for different system sizes.

A left tail appears very early, before the minimum of percolation thresholds.
The distributions are generalized Gumbel (see [C04], [Br08],[Ba00])

The non-gaussianity provides an early indicator for the fragmentation transition, i.e. a very early indicator for the full desertification transition.



neg. skewness

max σ_{smax}

exp. cut-off

m

fragmentation transition

desertification transition

extinction

Conclusions

The application of a percolation framework to analyse ecosystems subject to desertification risk provides a detailed description of the system behaviour.

Before the extinction of the vegetation and full degradation of the soil, the system undergoes a fragmentation and a desertification transition. The increasing degradation can be closely followed through the intervals defined by different percolation criteria.

In the fragmentation transition the system is still 'fully working', while the desertification transition is marked by a significant degradation. Dead cells density provides indicators for stages in these transitions.

The analysis of time fluctuations of vegetation patterns, and in particular of time series for the biggest cluster size of vegetated and non vegetated cells, is another tool for the understanding of the system evolution. Indicators for transitions are provided by the maxima of σ_{SMAX} for the fluctuations of biggest cluster size of living/non-living, always attained in the framentation/desertification interval.

The negative skewness in the PDF for the fluctuations of the vegetation biggest cluster is an early indicator for the onset of the fragmentation.

Since the direct observation in nature of the time fluctuations of the vegetation patterns can be easily performed, this analysis could help to assess the degradation processes in ecosystems.