Supersymmetric Lepton Flavour Violation

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Talk based on Phys. Rev. D **80** (2009) 091902 [arXiv:0904.2381],
with A. Ilakovac

Plan of the talk

- LFV in the MSSM with Singlet Neutrinos $N_{1,2,3}$

- Supersymmetric LFV in the MSSM with Low-Scale $N_{1,2,3}$

- SLFV Quantum Effects

- Numerical Estimates for $\ell \longrightarrow \ell'\gamma$, $\mu \longrightarrow e$ conversion, $\mu \longrightarrow eee$, $\tau \longrightarrow eee$ etc

- Conclusions

• LFV in the MSSM $+ N_{1,2,3}$

Leptonic part of the **superpotential**:

$$W_{\rm lept} = \mathrm{h}_e^{ij} E_{iR}^c H_{dL} \cdot L_{jL} + \mathrm{h}_\nu^{ij} N_{iR}^c H_{uL} \cdot L_{jL} + \frac{\mathrm{m}_M^{ij}}{2} N_{iR}^c N_{jR}^c$$

• LFV in the MSSM + $N_{1,2,3}$

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LFV induced after integrating out the super-heavy $N_{1,2,3}$:

[F. Borzumati, A. Masiero, PRL57 (1986) 961; J. Hisano *et al.*, PRD53 (1996) 2442.]

$$\mathcal{M}^2_{\tilde e} \;=\; \begin{pmatrix} M^2_{\tilde L} + (m_e m_e^\dagger) + D_1\,\mathbf{1} & m_e(A_e^* - \mu t_\beta\,\mathbf{1}) \\ (A_e^T - \mu^* t_\beta\,\mathbf{1}) m_e^\dagger & M^2_{\tilde e} + (m_e^\dagger m_e) + D_2\,\mathbf{1} \end{pmatrix},$$

$$(\Delta M^2_{\tilde L})^{\rm RG}_{ij} \;\approx\; -\frac{1}{8\pi^2}\,(3m_0^2 + A_0^2)\,(\mathrm{h}_\nu^\dagger \mathrm{h}_\nu)_{ij}\,\ln\frac{M_X}{m_N}\,, \qquad \mathrm{m}_M \;\approx\; m_N\,\mathbf{1}$$

$$(\Delta A_e)^{\rm RG}_{ij} \;\approx\; -\frac{3}{8\pi^2}\,A_0\,\mathrm{h}_e(\mathrm{h}_\nu^\dagger \mathrm{h}_\nu)_{ij}\,\ln\frac{M_X}{m_N}\,.$$

LFV induced by the flavour structure of the soft SUSY-breaking sector

Numerical Example for soft LFV from Hisano etal.:

$M_R = 2 \times 10^{13}$ GeV, $\mathbf{h}_\nu \sim 1$:

$$\boldsymbol{B(\mu \to e\gamma) \;\sim\; 2 \cdot 10^{-13} \times (\mathrm{h}_\nu^\dagger \mathrm{h}_\nu)^2_{\mu e} \left(\frac{\tan\beta}{3}\right)^2 \underbrace{\left(\frac{\ln(M_{\mathrm{GUT}}/m_N)}{\ln(M_{\mathrm{GUT}}/M_R)}\right)^2}_{:\sim\; 10}}$$

$$\boldsymbol{\sim\; 2 \cdot 10^{-12} \times (\mathrm{h}_\nu^\dagger \mathrm{h}_\nu)^2_{\mu e} \left(\frac{\tan\beta}{3}\right)^2 \,, \quad \text{for } m_N = 2 \text{ TeV}}$$

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$$\boldsymbol{\sim 2 \cdot 10^{-12} \times (\mathrm{h}_\nu^\dagger \mathrm{h}_\nu)^2_{\mu e} \left(\frac{\tan\beta}{3}\right)^2}, \quad \text{for } \boldsymbol{m_N = 2} \text{ TeV}$$

Soft LFV is more important at large $\tan\beta$

$$\boldsymbol{\frac{B(\mu \to eee)}{B(\mu \to e\gamma)} \sim 7 \times 10^{-3}}$$

Photonic charged lepton decays dominate in soft LFV

• SLFV in the MSSM with Low-Scale $N_{1,2,3}$

– Small Neutrino Masses from Flavour Symmetries:

$$M_\nu \;=\; \begin{pmatrix} 0 & \mathrm{m}_D^T \\ \mathrm{m}_D & \mathrm{m}_M \end{pmatrix}, \qquad \mathrm{m}_D \;=\; \frac{\mathrm{h}_\nu\, v_u}{\sqrt{2}}$$

Light and Heavy Neutrino Mass Matrices:

$$\mathrm{m}_\nu^{\mathrm{light}} \;\approx\; -\,\mathrm{m}_D^T\,\mathrm{m}_M^{-1}\,\mathrm{m}_D\,, \qquad \mathrm{m}_M^{\mathrm{heavy}} \;\approx\; m_N\,\mathbf{1}$$

Light-to-Heavy Neutrino Mixing: $\mathrm{m}_D^\dagger \mathrm{m}_M^{-1} \;\approx\; \mathrm{m}_D^\dagger / m_N$

Define LFV parameters:

$$\Omega_{\ell\ell'} \;\equiv\; (\mathrm{m}_D^\dagger \mathrm{m}_M^{-1} \mathrm{m}_M^{-1\dagger} \mathrm{m}_D)_{\ell\ell'} \;\approx\; \frac{v_u^2}{2m_N^2} (\mathrm{h}_\nu^\dagger \mathrm{h}_\nu)_{\ell\ell'}$$

$\Omega_{\ell\ell'}$ unconstrained from $\mathrm{m}_\nu^{\mathrm{light}}$ in Non-Seesaw Models

– The Non-Seesaw Paradigm

[A.P., PRL95 (2005) 081602 [hep-ph/0408103];
based on A.P., ZPC55 (1992) 275;
D. Wyler, L. Wolfenstein, NPB218 (1983) 205;
R.N. Mohapatra, J.W.F. Valle, PRD34 (1986) 1642.]

Break SO(3) and U(1)$_l$ flavour symmetries:

$$\mathrm{SO(3)} \longrightarrow \mathrm{SO(2)} \simeq \mathrm{U(1)}_l \xrightarrow{\sim \varepsilon_{e,\mu,\tau}} \mathrm{I}$$

U$_l$(1)-broken Yukawa sector:

$$\mathrm{m}_D^T = \frac{v_u}{\sqrt{2}} \begin{pmatrix} \varepsilon_e & a\, e^{-i\pi/4} & a\, e^{i\pi/4} \\ \varepsilon_\mu & b\, e^{-i\pi/4} & b\, e^{i\pi/4} \\ \varepsilon_\tau & c\, e^{-i\pi/4} & c\, e^{i\pi/4} \end{pmatrix},$$

where a, b, c are unconstrained from $\mathrm{m}_\nu^{\mathrm{light}}$, but only $|\varepsilon_{e,\mu,\tau}|$.

If $|\varepsilon_{e,\mu,\tau}| \sim 10^{-6}$–$10^{-7}$

$$\Longrightarrow \quad \mathrm{m}_\nu^{\mathrm{light}} \sim \frac{\varepsilon_\ell \varepsilon_{\ell'}\, v_u^2}{2m_N} \sim 0.1 \text{ eV} \quad \Longrightarrow \quad m_N \sim 100 - 500 \text{ GeV}$$

$\Longrightarrow$ **3 nearly degenerate heavy Majorana neutrinos.**

Light neutrino-mass spectrum:

[A.P., T. Underwood, PRD72 (2005) 113001.]

$$m_\nu^{\rm light} \;=\; \frac{v_u^2}{2m_N}\begin{pmatrix} \frac{\Delta m_N}{m_N}\,a^2 - \varepsilon_e^2 & \frac{\Delta m_N}{m_N}\,ab - \varepsilon_e\varepsilon_\mu & \frac{\Delta m_N}{m_N}\,ac - \varepsilon_e\varepsilon_\tau \\ \frac{\Delta m_N}{m_N}\,ab - \varepsilon_e\varepsilon_\mu & \frac{\Delta m_N}{m_N}\,b^2 - \varepsilon_\mu^2 & \frac{\Delta m_N}{m_N}\,bc - \varepsilon_\mu\varepsilon_\tau \\ \frac{\Delta m_N}{m_N}\,ac - \varepsilon_e\varepsilon_\tau & \frac{\Delta m_N}{m_N}\,bc - \varepsilon_\mu\varepsilon_\tau & \frac{\Delta m_N}{m_N}\,c^2 - \varepsilon_\tau^2 \end{pmatrix} ,$$

where

$$\Delta m_N \;=\; 2(\Delta m_M)_{23} + i[(\Delta m_M)_{33} - (\Delta m_M)_{22}], \qquad \frac{b}{a} \;=\; \frac{19}{50} ,$$

and (in $\sim 10^{-7}$ units)

$$\sqrt{\frac{\Delta m_N}{m_N}}\,a \;=\; 2\,, \quad \varepsilon_e \;=\; 2 + \frac{21}{250}\,, \quad \varepsilon_\mu \;=\; \frac{13}{50}\,, \quad \varepsilon_\tau \;=\; -\frac{49}{128}\,.$$

Prediction: inverted mass hierarchy, $m_{\nu_3} < m_{\nu_1} < m_{\nu_2}$, with

$$m^2_{\nu_2} - m^2_{\nu_1} \;=\; 7.54\times 10^{-5}\ {\rm eV}^2\,, \qquad m^2_{\nu_1} - m^2_{\nu_3} \;=\; 2.45\times 10^{-3}\ {\rm eV}^2\,,$$

$$\sin^2\theta_{12} \;=\; 0.362\,, \qquad \sin^2\theta_{23} \;=\; 0.341\,, \qquad \sin^2\theta_{13} \;=\; 0.047\,.$$

– **Scalar-Neutrino Mass Matrix:**

[e.g. F. Deppisch and J.W.F. Valle, PRD72 (2005) 036001]

$$\mathcal{M}^2_{\tilde\nu} = \begin{pmatrix} H_1 & M & 0 & N \\ M^\dagger & H_2 & N^T & 0 \\ 0 & N^* & H_1^T & M^* \\ N^\dagger & 0 & M^T & H_2^T \end{pmatrix},$$

$$\begin{aligned} H_1 &= m^2_{\tilde L} + (\frac{1}{2} M_Z^2 c_{2\beta} \mathbf{1}) + (\mathbf{m}_D \mathbf{m}_D^\dagger) \\ H_2 &= m^2_{\tilde\nu} + (\mathbf{m}_D^\dagger \mathbf{m}_D) + (\mathbf{m}_M^\dagger \mathbf{m}_M) \\ M &= \mathbf{m}_D\,(A_\nu - \mu\,/t_\beta) \\ N &= \mathbf{m}_D\,\mathbf{m}_M^\dagger \end{aligned}$$

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- **Model assumptions to determine the significance of SLFV:**

[A. Ilakovac and A.P., PRD80 (2009) 091902]

– N-$\tilde{N}$ **sector nearly supersymmetric, if** $m_N \gg M_{\mathrm{SUSY}}$

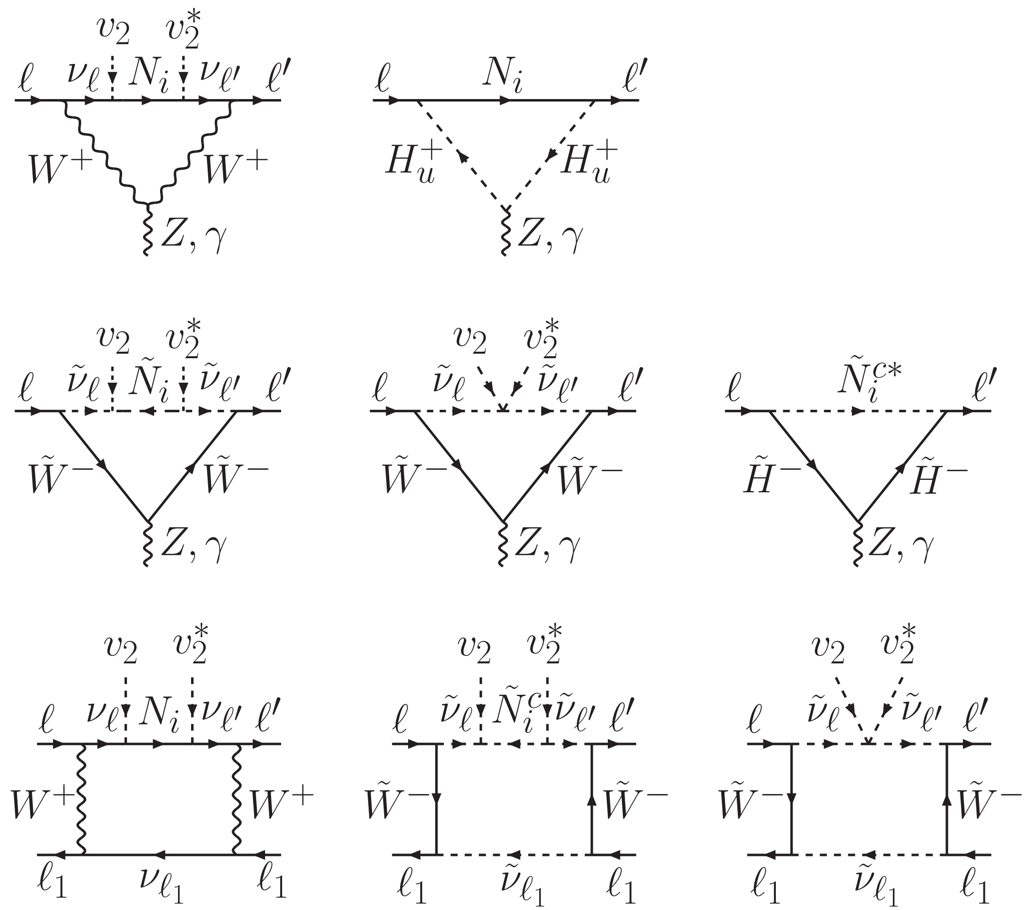
– $\mu \ll m_N$

– $\tilde{M}_L^2$, $\tilde{M}_e^2$, A_e **diagonal at** m_N

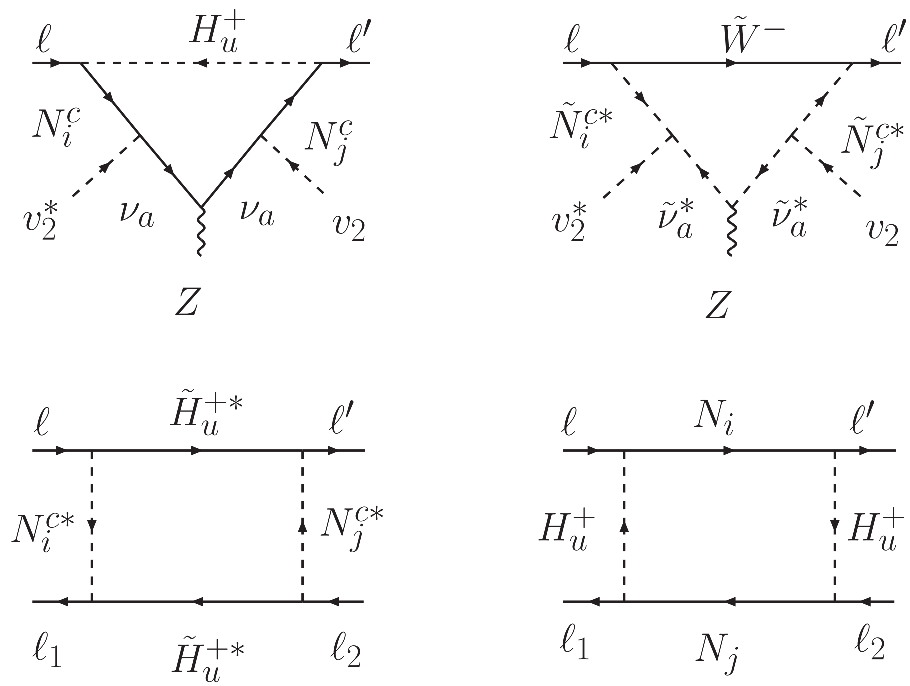
• SLFV Quantum Effects

Dominant terms to lowest order in g_w and v_u:

– Two neutrino Yukawa couplings $\propto\ \mathbf{h}_\nu^2$



– Four neutrino Yukawa couplings $\propto\ \mathbf{h}_{\boldsymbol{\nu}}^4$



SLFV Transition Amplitudes:

$$\mathcal{T}_\mu^{\ell\ell'\gamma} = \frac{e\alpha_w}{8\pi M_W^2}\,\bar{\ell}'\,\left[F_\gamma^{\ell\ell'}(q^2\gamma_\mu - \not{q}q_\mu)P_L + G_\gamma^{\ell\ell'} i\sigma_{\mu\nu}q^\nu m_\ell P_R\right]\ell$$

$$\mathcal{T}_\mu^{\ell\ell' Z} = \frac{g_w\alpha_w}{8\pi c_W}\,\bar{\ell}'\gamma_\mu P_L\ell\; F_Z^{\ell'\ell}$$

$$\mathcal{T}_{box}^{\ell\ell'\ell_1\ell_2} = -\frac{\alpha_w^2}{4M_W^2}\,F_{box}^{\ell\ell'\ell_1\ell_2}\,\bar{\ell}'\gamma_\mu P_L\ell\bar{\ell}_1\gamma^\mu P_L\ell_2$$

$$\mathcal{T}_{box}^{\ell\ell' qq} = -\frac{\alpha_w^2}{4M_W^2}\,F_{box}^{\ell\ell' qq}\,\bar{\ell}'\gamma_\mu P_L\ell\bar{q}\gamma^\mu P_L q\,, \qquad q = u, d$$

Photon Form Factors:

[A. Ilakovac and A.P., PRD80 (2009) 091902]

$$
\begin{aligned}
(F_\gamma^{\ell\ell'})^N &= \frac{\Omega_{\ell\ell'}}{6s_\beta^2}\ \ln\frac{m_N^2}{M_W^2}, \\
(F_\gamma^{\ell\ell'})^{\tilde{N}} &= \frac{\Omega_{\ell\ell'}}{3s_\beta^2}\ \ln\frac{m_N^2}{\tilde{m}_h^2},
\end{aligned}
$$

$$
\begin{aligned}
(G_\gamma^{\ell\ell'})^N &= -\Omega_{\ell\ell'}\ \left(\frac{1}{6s_\beta^2}+\frac{5}{6}\right) \\
(G_\gamma^{\ell\ell'})^{\tilde{N}} &= \Omega_{\ell\ell'}\ \left(\frac{1}{6s_\beta^2}+f\right)
\end{aligned}
$$

Z-Boson and Box Form Factors:

[A. Ilakovac and A.P., NPB437 (1995) 491;
PRD80 (2009) 091902]

$$(F_Z^{\ell\ell'})^N = -\frac{3\Omega_{\ell\ell'}}{2}\ln\frac{m_N^2}{M_W^2} - \frac{\Omega_{\ell\ell'}^2}{2s_\beta^2}\frac{m_N^2}{M_W^2},$$

$$(F_Z^{\ell\ell'})^{\tilde{N}} = \frac{\Omega_{\ell\ell'}}{2}\ln\frac{m_N^2}{\tilde{m}_1^2} + \frac{\Omega_{\ell\ell'}^2}{4s_\beta^2}\frac{m_N^2}{M_W^2}\ln\frac{m_N^2}{\tilde{m}_1^2}$$

$$(F_{box}^{\ell\ell'\ell_1\ell_2})^N = -\Omega_{\ell\ell'}\delta_{\ell_2\ell_1} - \Omega_{\ell\ell_1}\delta_{\ell_2\ell'} + \frac{1}{4s_\beta^4}\left(\Omega_{\ell\ell'}\Omega_{\ell_2\ell_1} + \Omega_{\ell\ell_1}\Omega_{\ell_2\ell'}\right)\frac{m_N^2}{M_W^2}$$

$$(F_{box}^{\ell\ell'\ell_1\ell_2})^{\tilde{N}} = -\frac{M_W^2}{\tilde{m}^2}\left(\Omega_{\ell\ell'}\delta_{\ell_2\ell_1} + \Omega_{\ell\ell_1}\delta_{\ell_2\ell'}\right) + \frac{1}{4s_\beta^4}\left(\Omega_{\ell\ell'}\Omega_{\ell_2\ell_1} + \Omega_{\ell\ell_1}\Omega_{\ell_2\ell'}\right)\frac{m_N^2}{M_W^2}$$

$$(F_{box}^{\ell\ell' uu})^N = -4(F_{box}^{\ell\ell' dd})^N = 4\,\Omega_{e\mu}$$

$$(F_{box}^{\ell\ell' uu})^{\tilde{N}} = -\frac{4\tilde{m}_W^2}{\tilde{M}_Q^2}(F_{box}^{\ell\ell' dd})^{\tilde{N}} = \frac{2M_W^2\tilde{m}_W^2}{\tilde{M}_Q^4}\,\Omega_{e\mu}$$

REMARKS:

– **SUSY Limit:**

$$\tilde{m}_W^2,\ \tilde{m}_h^2,\ \tilde{m}_{1,2}^2,\ \tilde{m}_2^2,\ \tilde{m}^2 \quad \stackrel{\rm SL}{\longrightarrow} \quad M_W^2\ , \qquad t_\beta \stackrel{\rm SL}{\longrightarrow} 1\ , \qquad \mu \stackrel{\rm SL}{\longrightarrow} 0$$

– **Non-Renormalization of the SUSY Dipole Operator:**

[S. Ferrara, E. Remiddi, PLB53 (1974) 347]

$$G_\gamma^{\ell\ell'} \ = \ (G_\gamma^{\ell\ell'})^N + (G_\gamma^{\ell\ell'})^{\tilde{N}} \quad \stackrel{\rm SL}{\longrightarrow} \quad 0$$

– **Positive Interference for Box Form Factors**

– **SUSY Enhancement in Z-Boson Form Factor through** $\dfrac{m_N^2}{M_W^2} \ln \dfrac{m_N^2}{\tilde{m}_1^2}$

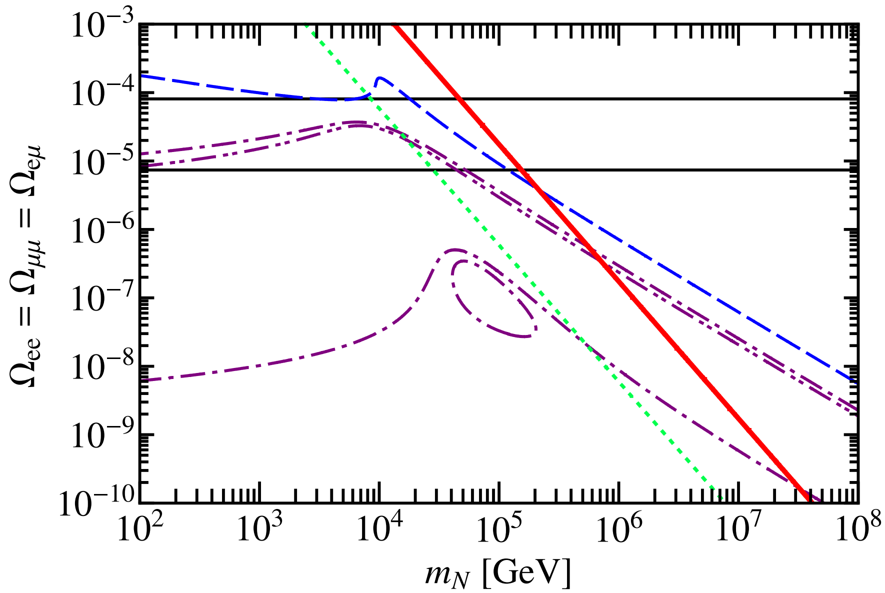
– **The Importance of the Large Quartic Yukawa-coupling Effects:**

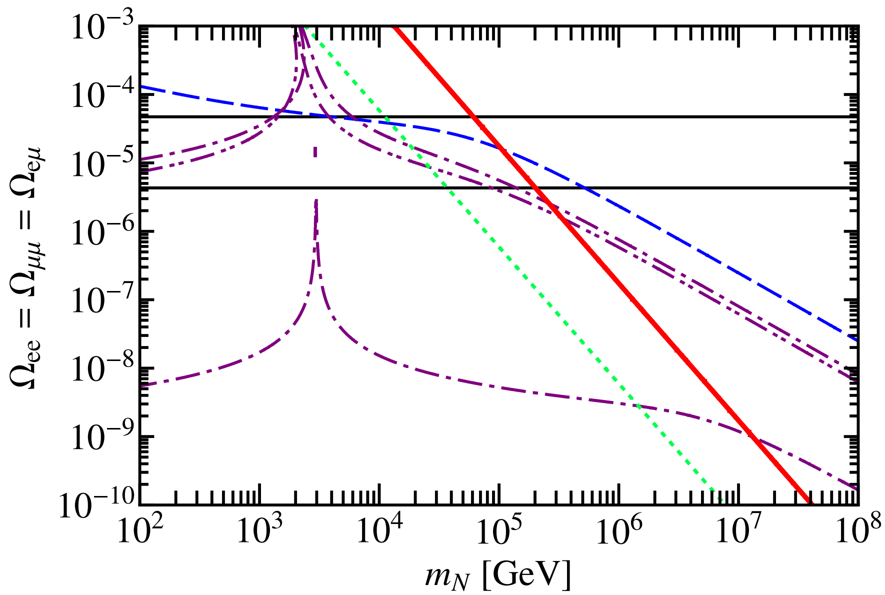
[A. Ilakovac and A.P., NPB437 (1995) 491.]

h_ν^4 **dominate when** $\mathrm{h}_\nu / g_w \gtrsim 1$ $\qquad (\Omega_{\ell\ell'} \frac{m_N^2}{M_W^2} = 2(\mathrm{h}_\nu^\dagger \mathrm{h}_\nu)_{\ell\ell'}/g_w^2)$

• Numerical Estimates

[A. Ilakovac and A.P., PRD80 (2009) 091902]





$\tan\beta = 3$

$\tilde{M}_Q = M_{\tilde{\nu}} = -\mu = 200$ GeV

$M_{\tilde{W}} = 100$ GeV

$\mathbf{\Omega}_{\mu e} = \mathbf{\Omega}_{ee} = \mathbf{\Omega}_{\mu\mu}$, other $\mathbf{\Omega}_{\ell\ell'} = 0$

Upper Bounds:

$B(\mu^- \to e^-\gamma)$	$< 1.2 \times 10^{-11}$	[1]
	$< 1 \times 10^{-13}$	[2]
$B(\mu^- \to e^-e^-e^+)$	$< 1 \times 10^{-12}$	[1]
$R^{Ti}_{\mu e}$	$< 4.3 \times 10^{-12}$	[3]
	$< 1 \times 10^{-18}$	[4]
$R^{Au}_{\mu e}$	$< 7 \times 10^{-13}$	[5]

[1] Amsler, PLB 667 (2008) 1

[2] Ritt, NPBPS 162 (2006) 279

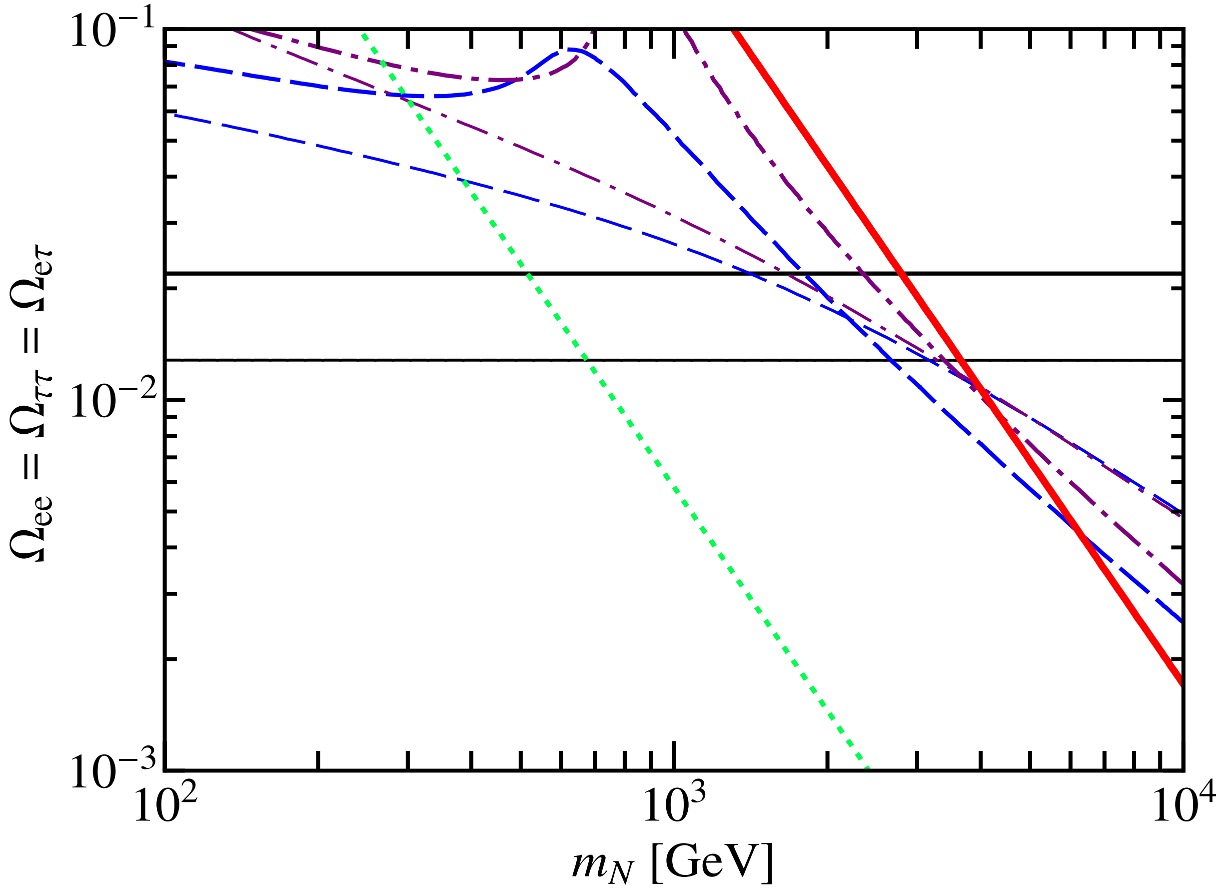
[3] Dohmen, PLB 317 (1993) 631

[4] Kuno, NPBPS 149 (2005) 376

[5] Bertl, EPJC 47 (2006) 337

SLFV in the τ Sector: $\quad \Omega_{\tau e} = \Omega_{ee} = \Omega_{\tau\tau}$, other $\Omega_{\ell\ell'} = 0$

[A. Ilakovac and A.P., PRD80 (2009) 091902]



Upper Bounds: [C. Amsler et al., PLB 667 (2008) 1]

$$
\begin{array}{lcl}
B(\tau^- \to e^-\gamma) & < & 1.1 \times 10^{-7} \\
B(\tau^- \to e^-e^-e^+) & < & 3.6 \times 10^{-8} \\
B(\tau^- \to e^-\mu^-\mu^+) & < & 3.7 \times 10^{-8}
\end{array}
$$

• Conclusions

- **SLFV is a new quantum-mechanical mechanism for LFV, within low-scale SUSY seesaw models with approximate flavour symmetries.**

- **SLFV is independent of the flavour structure of the soft SUSY sector.**

- **SLFV becomes dominant in non-photonic charged lepton decays, e.g. $\mu \to eee$, $\tau \to eee$, $\mu \to e$ conversion . . .**

- **Heavy sector of low-scale seesaw models with large ν-Yukawa couplings is potentially testable at the LHC.**

  [A.P., ZPC55 (1992) 275;
  A. Datta, M. Guchait, A.P., PRD50 (1994) 3195;
  A. Atre, T. Han, S. Pascoli, B. Zhang, JHEP0905 (2009) 030]

- **Further theoretical studies are required to explore the full range of SLFV implications.**