The universe as a group field theory condensate

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Review: arXiv:1602.08104 (with L. Sindoni)
Toy model: arXiv:1712.07266 (with E. Adjei, W. Wieland)
Inhomogeneities: arXiv:1709.01095 (with D. Oriti), arXiv:1811.10639

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Introduction

The standard picture of modern cosmology suggests that the Universe started at the Big Bang – a singularity of the spacetime manifold where densities, energies, etc. diverge and conventional physics breaks down. The singularity suggests that *quantum gravity is needed to account for the beginning of the Universe*.

(There are other ways of avoiding the singularity, such as (classically) modifying gravity or introducing exotic matter, but generic inflationary spacetimes are singular in the past [Borde, Guth, Vilenkin 2003].)

Quantum gravity should account for the **initial conditions** for the early universe, in particular show the emergence of a regime in which usual quantum field theory (QFT) on curved spacetime techniques apply.

Quantum cosmology models built to justify the usual (Bunch-Davies) initial conditions remain controversial (e.g., [Feldbrugge, Lehners, Turok 2017]).

Introduction

Attempts to extend the usual formalism of QFT on curved spacetime by using input from quantum gravity include:

- Emergent *string gas* cosmology [Brandenberger, Vafa 1989]: geometric radiation phase preceded by a high-temperature "Hagedorn" phase of strings;
- Hořava-Lifshitz bounce scenarios [Brandenberger 2009];
- attempts to embed inflationary models into string theory.

In this talk, I will focus on the **group field theory** (GFT) approach to quantum gravity. I will explore the proposal that a macroscopic universe arises from a Bose-Einstein condensation of discrete combinatorial "building blocks" of geometry.

Outline

Matrix models

Archetypal example of "emergence" of continuum geometry from combinatorics: matrix models for two-dimensional (Riemannian) geometry.

Define an action for a Hermitian $N \times N$ matrix M by

$$S(M) = \frac{1}{2}\mathrm{Tr}(M^2) - \frac{g}{\sqrt{N}}\mathrm{Tr}(M^3) \equiv \frac{1}{2}M_{ij}M_{ji} - \frac{g}{\sqrt{N}}M_{ij}M_{jk}M_{ki}$$

and define a partition function

$$Z = \int dM\ e^{-S(M)} = \sum_{\Gamma} g^{V_\Gamma} N^{\chi} =: \sum_{\Delta} e^{\frac{4\pi}{G}\chi(\Delta) - \frac{\Lambda}{G}A(\Delta)}\,,$$

expanded in Feynman graphs (vacuum bubbles) Γ or dual triangulated 2-manifolds Δ. This gives a sum over discrete geometries, in which $\chi = 2$ (spheres) dominate in the continuum limit $N \to \infty$, weighted by the 2d Einstein-Hilbert action.

Higher dimensions

A straightforward combinatorial extension leads to **tensor models**, which admit an $\frac{1}{N}$ expansion analogous to matrix models [Gurau 2011-12]. Their continuum limit seems closer to a *branched polymer* than a manifoldlike phase [Gurau, Ryan 2013].

In 2d, a crucial property of the Einstein-Hilbert action with $\Lambda = 0$ is that it is a topological invariant because of the Gauss-Bonnet theorem

$$\int_M \mathrm{d}^2x \, \sqrt{g} \, R = 4\pi\chi(M) \, .$$

This is longer true in higher (in particular 4) dimensions, which motivates extending matrix models by adding local degrees of freedom to the combinatorial structure given by matrix models.

In **group field theory** these are group elements corresponding to parallel transports of a gravitational connection (Yang-Mills-like variables).

Group field theory [More details on the formalism: Oriti 1310.7786]

A **group field theory** (GFT) is defined in terms of a quantum field living on an abstract group manifold, usually a complex scalar

$$\varphi : \mathrm{SU}(2)^4 \to \mathbb{C}\,, \quad (g_1, \ldots, g_4) \mapsto \varphi(g_1, \ldots, g_4) \equiv \varphi(g_I)$$

Compared to matrix models, the finite index set has been replaced by a continuous Lie group, whose elements correspond to possible parallel transports of a connection (here, taken to be an $\mathrm{SU}(2)$ Ashtekar connection for gravity).
A GFT action then takes the general form

$$S[\varphi, \bar{\varphi}] = \int \mathrm{d}^4 g\; \bar{\varphi}(g_I)\mathcal{K}\varphi(g_I) + \mathcal{V}[\varphi, \bar{\varphi}]$$

where $\mathcal{V}$ is *combinatorially nonlocal*, e.g.

$$\int \mathrm{d}^{10} g\, \varphi(g_1, g_2, g_3, g_4)\varphi(g_4, g_5, g_6, g_7)\varphi(g_7, g_3, g_8, g_9)\varphi(g_9, g_6, g_2, g_{10})\varphi(g_{10}, g_8, g_5, g_1)$$

Group field theory

Feynman expansion of the partition function of a GFT now results (formally) in a sum over discrete 4d spacetime geometries,

$$Z = \int \mathcal{D}\varphi\, \mathcal{D}\bar{\varphi}\; e^{-S[\varphi,\bar{\varphi}]} = \sum_{\Gamma} \frac{\prod_i \lambda_i^{n_i}}{\mathrm{sym}(\Gamma)} Z[\Gamma] \;\text{“}= \sum_{\Gamma} w[\Gamma] \int \mathrm{d}g_{\Gamma}\; e^{-S[g_{\Gamma}]}\;\text{”}$$

where $S[\varphi,\bar{\varphi}]$ is the GFT action, λ_i are the coupling constants for the interaction and the second equality is the expansion (around $\varphi = 0$) in Feynman graphs Γ.

Each Γ, associated with a Feynman amplitude $Z[\Gamma]$, forms a two-complex that can be interpreted as a discrete spacetime (without boundary), or a *spin foam*. $Z[\Gamma]$ is a *spin foam amplitude*, the corresponding discrete path integral.

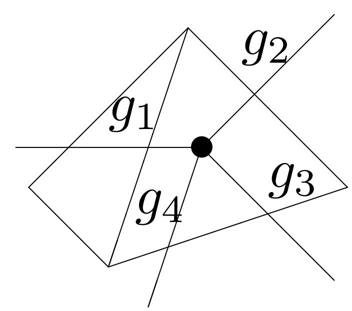
The GFT path integral generates a sum over all such two-complexes with weights determined by the couplings.

Group Field Theory – Canonical Formalism

The *canonical* formalism for GFT is closely related to loop quantum gravity. Starting from a 'no-space' Fock vacuum $|\emptyset\rangle$ that corresponds to a degenerate geometry and satisfies $\hat{\varphi}(g_I)|\emptyset\rangle = 0$, one defines

$$\begin{aligned} [\hat{\varphi}(g_I), \hat{\varphi}(g'_I)] = [\hat{\varphi}^\dagger(g_I), \hat{\varphi}^\dagger(g'_I)] &= 0\,, \\ [\hat{\varphi}(g_I), \hat{\varphi}^\dagger(g'_I)] &= \mathbf{1}(g_I, g'_I) \end{aligned}$$

where $\mathbf{1}(g_I, g'_I) := \int \mathrm{d}h\ \prod_I \delta(g'_I h g_I^{-1})$.
$\hat{\varphi}$ and $\hat{\varphi}^\dagger$ can then be expanded in annihilation and creation operators;

$$\hat{\varphi}^\dagger(g_1, g_2, g_3, g_4)|\emptyset\rangle = |$$  $$\rangle$$

corresponds to a tetrahedron, or "building block" of geometry. The GFT Fock space consists of general geometries built of many such building blocks.

Outline

A group field theory condensate

Continuum geometry cannot be found in a perturbative phase around the Fock vacuum $\varphi = 0$; instead, one looks for a transition to a phase in which many excitations are present, and a macroscopic continuum geometry can emerge.

Such a phase, a *group field theory condensate*, is characterised by a nonvanishing expectation value for the GFT field operator

$$\langle \hat{\varphi}(g_I) \rangle =: \sigma(g_I) \neq 0 \,.$$

In a **mean-field approximation**, all correlation functions can then be expressed in terms of σ, with no correlations between quanta:

$$\langle \hat{\varphi}^\dagger(g_I) \hat{\varphi}(g'_I) \rangle = \bar{\sigma}(g_I)\sigma(g'_I) \,, \quad \text{etc.}$$

Mean-field coherent states, which satisfy the mean-field condition, describe weakly interacting Bose condensates. In GFT, they describe a homogeneous universe in which spatial points decouple [SG, Oriti, Sindoni 2013].

A group field theory condensate

The mean field $\sigma(g_I)$ itself satisfies the analogue of the Gross-Pitaevskii equation, i.e., the classical GFT equation of motion. This equation is in general nonlinear and nonlocal, but often can be simplified by assuming an "isotropic form"

$$\sigma(g_I) = \sum_j \sigma_j \, \mathrm{D}^j(g_I)$$

where $\mathrm{D}^j(g_I)$ is a convolution of representation matrices in the j representation. The equation of motion, e.g., for the EPRL spin foam model, becomes

$$-B_j \sigma_j + w_j \bar{\sigma}_j^4 = 0$$

and its solution would correspond to a static ground state. Dynamics is defined with respect to a matter clock field ϕ, which becomes an additional argument of the GFT field; the equation of motion is then [Oriti, Sindoni, Wilson-Ewing 2016]

$$A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) + w_j \bar{\sigma}_j^4(\phi) = 0 \, .$$

Group field cosmology [Oriti, Sindoni, Wilson-Ewing 2016; SG 2016]

For time-dependent condensate mean field states, the total 3-volume at "scalar time" ϕ is given by

$$V(\phi) = \sum_j V_j |\sigma_j(\phi)|^2 .$$

In the simplest case, where there is only a single j component and GFT interactions can be neglected, we have

$$\begin{aligned} A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) &= 0 \\ \Rightarrow \quad V(\phi) &\stackrel{\phi\to\pm\infty}{\rightarrow} |\alpha^\pm|^2 \exp\left(\pm 2\sqrt{\frac{B_j}{A_j}}\phi\right) . \end{aligned}$$

The GFT condensate solution for $V(\phi)$ corresponds to a bounce which interpolates between the classical contracting and expanding solutions

$$V(\phi) = V_0 \exp(\pm\sqrt{12\pi G}\phi) .$$

Toy model for GFT condensates

A simple model for GFT condensate cosmology [Adjei, SG, Wieland 2017] consists of creation and annihilation operators $\hat{a}^\dagger$ and $\hat{a}$ for a single GFT mode, with dynamics given by a *squeezing* Hamiltonian

$$\hat{\mathcal{H}} = \frac{\mathrm{i}}{2}\lambda\left(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}\right) .$$

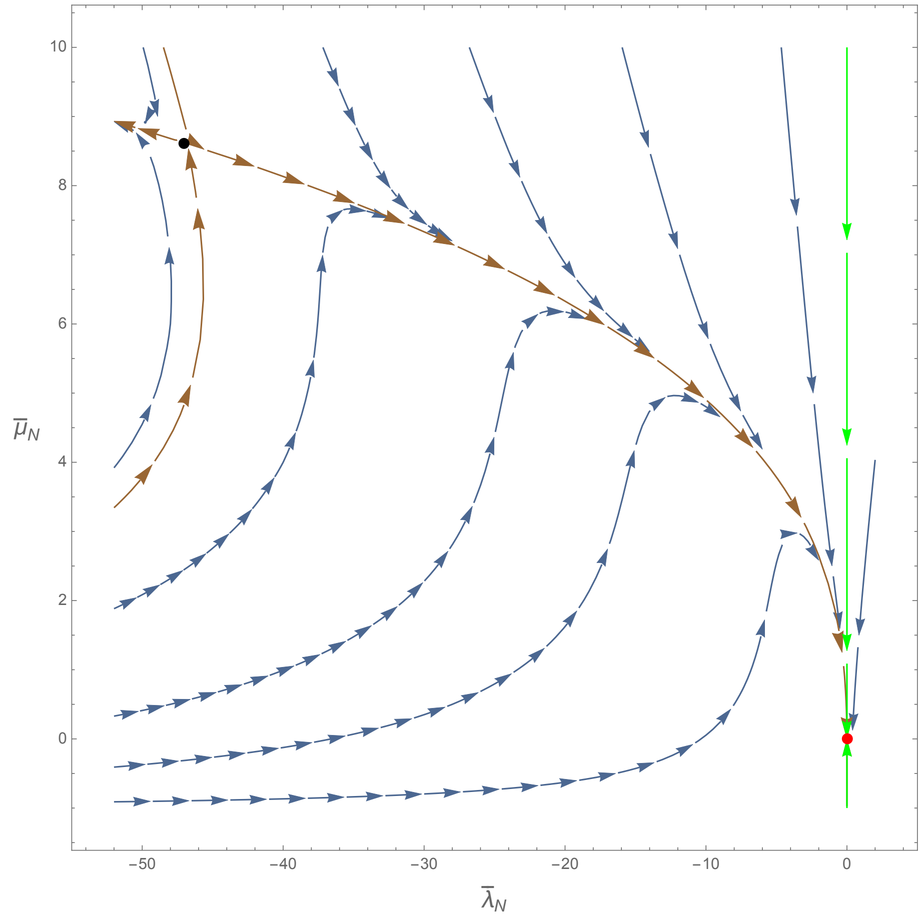
As is well known in quantum optics, the number of quanta in a squeezed state $|\phi\rangle \equiv \exp(-\mathrm{i}\phi\hat{\mathcal{H}})|0\rangle$ is

$$\langle\phi|\hat{a}^\dagger\hat{a}|\phi\rangle = \sinh^2(\lambda\phi) \sim \frac{1}{4}e^{2|\lambda\phi|}$$

which is exactly a classical "bounce" solution for $\lambda = \sqrt{3\pi G}$. By applying a Legendre transform to the full (free) GFT action one can show that the GFT Hamiltonian is also of this squeezing type [Wilson-Ewing 2018].

Renormalisation flow analysis

Ongoing studies look for phase transitions to a condensate-type phase, with an interacting fixed point, in GFT models: from [Ben Geloun, Martini, Oriti 2016]



Outline

Towards cosmological phenomenology

We already saw how the classical flat FLRW solution for a universe with a massless scalar field,

$$\begin{aligned} \left(\frac{V'(\phi)}{V(\phi)}\right)^2 &= 9\left(\frac{a'(\phi)}{a(\phi)}\right)^2 = 24\pi G N^2 \rho = 12\pi G \\ \Rightarrow V(\phi) &= V_0 \exp(\pm\sqrt{12\pi G}\phi)\,, \end{aligned}$$

can be recovered from the free dynamics of a GFT condensate, with a bounce connecting between contracting and expanding classical solutions. This bounce closely resembles that of loop quantum cosmology (LQC) for the same matter.

GFT interactions can be included; these become relevant at later times/larger volumes. For instance, a φ^6 interaction term corresponds to a cosmological constant in the effective Friedmann equation, which when chosen to be negative leads to a recollapse of the universe [de Cesare, Pithis, Sakellariadou 2016].

Towards cosmological phenomenology [SG, Oriti 2017; SG 2018]

In order to describe a more realistic universe, we extend the formalism from exactly homogeneous to slightly inhomogeneous universes. This is possible in a GFT formalism for quantum gravity coupled to *four* free, massless scalar fields, whose values now serve as "relational" spacetime coordinates.

These scalars form a harmonic coordinate system: the scalar fields satisfy the free Klein-Gordon equation $\Box\phi^I = 0$.

The mean field equation of motion now becomes (with $I = 1, \ldots, 4$)

$$A_j \frac{\partial^2 \sigma_j(\phi^I)}{\partial(\phi^0)^2} - B_j \sigma_j(\phi^I) + C_j \Delta_\phi \sigma_j(\phi^I) + w_j \bar{\sigma}_j^4(\phi^I) = 0\,.$$

In this more general setting, a spatially homogeneous mean field is now one that depends only on ϕ^0.

Towards cosmological phenomenology

There is now a local 3-volume element $V(\phi^I)$ for each spacetime point. One can compute fluctuations in this observable through the 2-point function

$$\langle\delta\hat{V}(\phi^I)\delta\hat{V}(\phi'^I)\rangle\,,\quad \delta\hat{V}(\phi^I)=\hat{V}(\phi^I)-\langle\hat{V}(\phi^I)\rangle\,,$$

finding a small, non-zero power spectrum due to vacuum fluctuations, just as for the usual formalism in inflation!

This can be converted into the observationally relevant power spectrum of the gauge-invariant curvature perturbation variable ζ, where one finds

$$\Delta^2_\zeta(k)\sim 0.01\left(\frac{m_{\rm Pl}V}{j^{3/2}M^4}\right)k^3\,.$$

Spectral index $n_s=4$ consistent with semiclassical QFT on curved spacetime!

Summary

- Fundamental challenge for theoretical cosmology to explain the emergence of a semiclassical universe described by QFT on curved spacetime; addressing it might explain initial conditions, or constrain possible early universe scenarios.

- **Group field theories** are a generalisation of matrix models in which there are additional group-theoretic degrees of freedom associated to discrete geometry. Proposal that macroscopic universe corresponds to a *condensate* in GFT.

- Simplest free dynamics for the condensate mean field correctly reproduces flat FLRW universe with massless scalar matter, resolving the classical singularity by a bounce. Interactions modify the resulting cosmological dynamics.

- New proposal for generating inhomogeneities through vacuum fluctuations in GFT, rather than in a separate scalar field. Consistency with usual semiclassical treatment shown in one example. Needs to be generalised!

Thank you!