S&P 500 Index Returns and Implied Relative Risk Aversion

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Abstract

The monthly relative risk aversion (RRA) implied by the S&P 500 Index and option prices, has a significant explanatory power on the monthly index returns, so are the RRA lags and the yearly standard deviation of the monthly RRA. However, this is not the case when these RRA variables are used to explain VIX prices. Interestingly, lags of the index returns and VIX are all shown to have significant impact on RRA through a naïve regression analysis. I therefore hypothesize that the market's RRA at any point in time is adjusted following the previous realised returns and volatility, and in turn, poses an impact on the returns at the same time or later. I plan to further validate this result with more statistical analyses in the future.

1 Introduction

Investors' risk aversion has always been considered the fundamental element in defining the risk-return trade-off of an asset. Its theoretical role is formally established by pioneers such as von Neumann and Morgenstern (1944); Markowitz (1952); Tobin (1958); Arrow (1965); Pratt (1964). Knowing the market's risk aversion would undoubtedly aid all efforts on finding the fundamental values of financial assets (see Constantinides, 1990; Campbell, 1996; Cochrane, 2001; Restoy and Weil, 2011). Many scholars since attempt to estimate the market's risk aversion using the stock and option data (see Chou, 1988; Segal and Spivak, 1988; Ait-Sahlia and Lo, 2000; Jackwerth, 2000; Rosenberg and Engle, 2002; Quiggin and Chambers, 2003; Bliss and Panigirtzoglou, 2004; Jackwerth, 2004; Kang and Kim, 2006; Bakshi and Madan, 2006; Kang et al., 2010; Kostakis et al., 2011; Barone-Adesi et al., 2014; Duan and Zhang, 2014; Fabozzi et al., 2014; Yoon, 2017; Li, 2018; Skiadopoulos et al., 2019).

While the entire focus of the literature is on how to estimate the market's risk aversion, limited attention has been paid on taking the established estimation methods and studying the impact of the market's risk aversion on stock market returns. Perhaps one has taken it for granted that an increase in the risk aversion

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would increase the risk premium required by the market. But does this mean an increase in market's risk aversion increases the stock returns within the same period of time? Or an increase in risk aversion pushes up the market expectation and subsequently reduces the asset prices which lower down the realised returns? Also, what about the risk aversion and stock returns at different points in time? Does the change in the risk aversion from an earlier time trails a change in the stock returns for a later time period? Could the change in the risk aversion be a result from a change in stock returns even before?

With these questions in mind, I conduct several sets of regressions between the S&P 500 Index returns and relative risk aversion (RRA) implied by the Index prices and Index option prices. I adopt the most widely accepted, model-free approach by Bakshi et al. (2003); Kang et al. (2010), to estimate the relative risk aversion. I focus on S&P 500 index simply because it is one of the best indicators of the US stock market as a whole. Given the tight time constraints on conducting my analysis, I use daily data to construct monthly RRA estimates and my regressions are all based on data with a monthly frequency.

I find in all cases, a significant and positive explanatory power of RRA, some of its lags and its standard deviation (computed over yearly, rolling-windows), on the S&P 500 Index returns. This may indicate that RRA affects the stock returns not only instantly but in the future too. Interestingly, through a naïve regression analysis using index returns, VIX and their lags on RRA, we find that RRA is significantly and negatively affected by the index returns and return variations ahead of the RRA estimation. These findings potentially shed some light on the dynamics between RRA and stock returns; I therefore hypothesize that the market's RRA at any point in time is adjusted (negatively) following the previous realised returns and risks, and in turn, impacts positively on the returns at the same time or later. Of course more statistical analyses are required to test this hypothesis and I plan to do so in my future research.

Section 2 reviews the literature on the definition and estimation methods of RRA. Section 3 summarises the model-free approach we take to estimate RRA, as well as a minor modification I propose on the approach. Section 4 presents three major characteristics I observe on RRA movements over time. Section 5 presents my regression models, results and discussions about the relationship between RRA, S&P 500 Index returns, VIX and their lags. Section 6 summaries a few ideas that can form the immediate direction of my future research. Section 7 concludes. I estimate RRA and run the regression analyses using Python codes which are available upon request.

2 Literature Review

The relative risk aversion (RRA) is defined as the risk preference of a decision maker and it is imported as an argument to the utility function used to describe the investor. Risk aversion is a cornerstone of the economic theory, and it is based on the assumption that investment decisions are characterized by a strong variability, that doubts the existence of a concave utility function through its whole range, where Markowitz (1952) suggests that people are represented by utility functions that are constituted by both a concave and a convex segment.

The RRA of the representative investor, has long been studied from many practitioners. Markowitz (1952) and Tobin (1958) use the game theory and economic behavior described from von Neumann and

Morgenstern (1944), to reproduce the risk tolerance that will contribute to the portfolio selection. Risk aversion has a lead role in portfolio management, as it describes the risk attitude of the investor. However, although the definition of RRA stays the same, the market degree of relative risk aversion differs from the individual investor risk attitude used in practice. Arrow (1965) and Pratt (1964), described a measure for RRA by employing a utility function, which provides a useful insight into people's risk tolerance. This measure defines the RRA as a parameter that is multiplied by the fraction of the second derivative to the first derivative of the utility function that suits the representative investor.

The estimation of the RRA depends on the nature of the risk tolerance itself. There are studies that examine the constant RRA (CRRA) meaning that either the risk aversion is constant between correlated periods (see Chou, 1988) or that it is a constant parameter when using generalized utility functions (see Quiggin and Chambers, 2003; Segal and Spivak, 1988). However, time varying RRA, considers the change in the risk attitude of the representative investor. Following the Arrow-Pratt measure, the model-dependent estimation of the time varying RRA has been widely used. The estimation of the RRA has attracted a vast strand of literature and a common feature in most of them are the data used, where option data are chosen (see Ait-Sahlia and Lo, 2000; Jackwerth, 2000; Rosenberg and Engle, 2002; Bliss and Panigirtzoglou, 2004; Jackwerth, 2004; Bakshi and Madan, 2006; Kang and Kim, 2006; Kang et al., 2010; Kostakis et al., 2011; Barone-Adesi et al., 2014; Duan and Zhang, 2014). Furthermore, informed traders would prefer trading in the option markets, rather than the spot market, in order to benefit from their informational dominance (see Skiadopoulos et al., 2019).

Campbell (1996) models the pricing kernel directly and finds that RRA is decreasing relative to the surplus consumption ratio. Ait-Sahlia and Lo (2000) implement a non-parametric approach and estimate the RRA based on the pricing kernel while their theoretical framework lies upon the triangular relationship between the risk neutral density, the subjective density and the pricing kernel. They find that the representative investor becomes more risk averse, as the index decreases, as well as when its value is extremely high. Jackwerth (2000) uses a similar approach and mentions that although in order for the RRA to be positive, both the first and the second derivative of the representative investor's utility function must be positive, this feature is guaranteed only for the first derivative. They argue that pre-crisis the implied marginal utility function is monotonically decreasing through wealth levels, due to peoples' greedy behavior, while Rosenberg and Engle (2002) support this argument with their findings. Similarly Jackwerth (2004) estimate the absolute risk aversion (ARA) and find similar findings to Campbell (1996).

Bliss and Panigirtzoglou (2004) are based in the same theoretical framework, but adopt a new methodology to estimate the subjective density, through utilizing Berkowitz (2001) test and assuming power and exponential utility functions. They find that the RRA is inversely related to the market risk, which could be explained from the measure used as a proxy for consumption, or by the hypothesis that the risk aversion of the representative investor is volatility-dependent. Kang and Kim (2006) extend their methodology for another five utility functions (power, exponential, Hyperbolic Absolute Risk Aversion (HARA) with two parameters, log plus power and linear plus exponential functions), and find that the forecasting ability of the subjective density increases when using a more flexible utility function and that independently of the utility function, the RRA is decreasing on the gross return level, where many other have used and extended it (see Kang and Kim, 2006; Kostakis et al., 2011; Kang et al., 2014; Fabozzi et al., 2014; Li, 2018).

Jackwerth (2000), reports that before 1987 (pre-crisis), the risk aversion is positive and monotonically downwards sloping. However, after 1988 (post-crisis) he finds that the RRA is negative in some cases and is opposed to the theory assumptions. Rosenberg and Engle (2002) compare their results to those obtained by Jackwerth (2000) and they find similar results, where they also observe negative values for the ARA. Kang et al. (2014) estimate risk aversion under uncertainty, and find that the behavior of the pricing kernel is different before 2008 (pre-crisis) and after 2008 (post-crisis). They also support that the difference between the model-dependent aversion and the aversion under uncertainty, becomes wider post-crisis.

Bakshi et al. (2003) express the volatility spread, *i.e.* the difference between the realized and the implied volatility of the market, by utilizing the model-free risk neutral moments of the price's distribution, assuming a power utility function for the representative investor. The volatility, skewness and kurtosis of the distribution are calculated with respect to the volatility, cubic and quadratic contracts. The model-free risk neutral moments constructed by Bakshi et al. (2003) are not only used in literature but also tested for their forecasting ability on the market returns (see Xing et al., 2010; Conrad et al., 2013; Chang et al., 2013; Kelly and Jiang, 2014; Amaya et al., 2015). Bakshi and Madan (2006) estimate the RRA by utilizing Bakshi et al. (2003) methodology, and report that RRA estimates are positive through out their sample period and that their findings are consisted with Ait-Sahlia and Lo (2000); Bliss and Panigirtzoglou (2004). Kang et al. (2010) use the same approach but this time they formalize Bakshi and Madan (2006) formula in the physical measure. Their analysis is based on the realised and the adjusted implied volatility, where they are both decreasing, throughout the sample period used (2001-2007).

Duan and Zhang (2014), use the Bakshi et al. (2003) formulas, in order to estimate the market premium, so as to generate the forward-looking premium. They estimate the RRA, through using a five year rolling window of data, and they report estimates consisted with those produced by Ait-Sahlia and Lo (2000); Bliss and Panigirtzoglou (2004). However, when they apply Bakshi and Madan (2006) methodology, and they estimate RRA based on ex-post sample moments, they find that the estimates increase in a very high level, while when using the full sample and the forward-looking physical moments, the estimates are consisted with the ones reported in literature. Kang et al. (2014) utilize a model under uncertainty to estimate the pricing kernel and the RRA. They find that for a wealth level higher than 1, absolute risk aversion obtains negative values, such as the uncertainty premium, which is consistent with Jackwerth (2000).

Skiadopoulos et al. (2019) further develop a methodology for exploiting the RRA and test its forward-looking behavior in combination with its ability to forecast the real economic activity. The theoretical framework of their paper is similar with Kang et al. (2010); Duan and Zhang (2014), using the model-free methodology of Bakshi et al. (2003). Their estimates are consisted with the ones presented in the previous literature (see Ait-Sahlia and Lo, 2000; Rosenberg and Engle, 2002; Bakshi et al., 2003; Bliss and Panigirtzoglou, 2004; Kang et al., 2010; Kang and Kim, 2006; Barone-Adesi et al., 2014; Duan and Zhang, 2014). Although they find that RRA is not affected by the two crises on 1987 (Russian crisis), 2001 (U.S. recession), in contrast with Jackwerth (2000) and Rosenberg and Engle (2002), they mention that it is significantly high during the 2008 crisis and it gradually decreases after that year. They also apply Bliss and Panigirtzoglou (2004) methodology, but argue that the RRA estimates extracted from their methodology has a very weak explanatory power, possibly coming from the transformation steps of the subjective density function.

The empirical analysis adopted in their paper is based on the U.S., South Korea, UK, Japan and Germany markets, while their predictability tests are based on Kostakis et al. (2015). They find that the RRA can successfully predict the real economic activity (REA) in the U.S. and in South Korea, and is significant both in in-sample and out-of-sample testing, while the two estimates (RRA and REA) are inversely related. On the other hand, they support that the reason that led the RRA estimates fail to predict the REA in U.K., Japan and Germany, is the low liquidity in the option markets of these countries and the relatively weaker option trading compared to U.S and South Korea.

3 Estimating the Relative Risk Aversion

This paper mainly adopts the Kang et al. (2010)'s model-free approach to estimate the relative risk aversion (RRA). The approach is now one of the most widely accepted approach in the literature (also see Duan and Zhang, 2014; Skiadopoulos et al., 2019, and many others). It is a further development of Bakshi and Madan (2006)'s methodology which uses Bakshi et al. (2003)'s formulation to derive risk-neutral moments of the option's underlying asset prices and obtain RRA from the difference between the risk-neutral and physical moments of the underlying prices.

I assume a fixed RRA between each pair of neighboring expiry dates of the option, *i.e.* within a month before the third Friday every month. At each date t within the month $\tau := 30$ days, I compute the RRA within each month following Kang et al. (2010):

$$\arg\min_{\gamma} \left(\frac{\gamma^2}{2}\sigma_{q,t}^2(\tau)(\kappa_{q,t}(\tau)-3) + \gamma\sigma_{q,t}(\tau)\theta_{q,t}(\tau) - \frac{\sigma_{p,t}^2(\tau)-\sigma_{q,t}^2(\tau)}{\sigma_{q,t}^2(\tau)} \right)^2, \tag{1}$$

where γ denotes the RRA, and $\sigma_{q,t}(\tau)^2, \theta_{q,t}(\tau)$ and $\kappa_{q,t}(\tau)$ denote risk-neutral variance, skewness and kurtosis, respectively, and are calculated following Bakshi et al. (2003); Jiang and Tian (2005):

$$\mu_t(\tau) = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V_t(\tau) - \frac{e^{r\tau}}{6}W_t(\tau) - \frac{e^{r\tau}}{24}X_t(\tau) \tag{2a}$$

$$\sigma_{q,t}(\tau) = e^{r\tau}V_t(\tau)\,, \tag{2b}$$

$$\theta_{q,t}(\tau) = \frac{e^{r\tau}(W_t(\tau) - 3\mu_t(\tau)V_t(\tau)) + 2\mu_t(\tau)^3}{[e^{r\tau}V_t(\tau) - \mu_t(\tau)^2]^{3/2}}\,, \tag{2c}$$

$$\kappa_{q,t}(\tau) = \frac{e^{r\tau}(X_t(\tau) - 4\mu_t(\tau)W_t(\tau) + 6\mu_t(\tau)^2V_t(\tau)) - 3\mu_t(\tau)^4}{[e^{r\tau}V_t(\tau) - \mu_t(\tau)^2]^2} \tag{2d}$$

where r is the risk free rate, $V_t(\tau), W_t(\tau), X_t(\tau)$ are the volatility, cubic and quadratic contract prices calculated at each date t within the month τ. Note that, I take Jiang and Tian (2005)'s definition to compute the volatility (2b) because the way the model-free variance measure is constructed should coincide with what is measured in the realised setting. Following Skiadopoulos et al. (2019) the realised measurement is represented by a quadratic variation over period t to T, and therefore the variance formula needs to be adjusted, through removing the μ term. I can revise the skewness and kurtosis definitions (2c $\sim$ 2d) from Bakshi et al. (2003); Skiadopoulos et al. (2019) in the same manner. But this change would only imply a negligible difference to the results obtained.

Bakshi et al. (2003) estimates the risk-neutral moments as follows:

$$V(t,\tau) = \int_{S_t}^{\infty} \frac{2\left(1 - \ln\left[\frac{K}{S_t}\right]\right)}{K^2} C(t,\tau;K) + \int_0^{S_t} \frac{2\left(1 + \ln\left[\frac{S_t}{K}\right]\right)}{K^2} P(t,\tau;K) \tag{3a}$$

$$W(t,\tau) = \int_{S_t}^{\infty} \frac{6\ln\left[\frac{K}{S_t}\right] - 3\ln\left[\frac{K}{S_t}\right]^2}{K^2} C(t,\tau;K) - \int_0^{S_t} \frac{6\ln\left[\frac{S_t}{K}\right] + 3\ln\left[\frac{S_t}{K}\right]^2}{K^2} P(t,\tau;K) \tag{3b}$$

$$X(t,\tau) = \int_{S_t}^{\infty} \frac{12\ln\left[\frac{K}{S_t}\right]^2 - 4\ln\left[\frac{K}{S_t}\right]^3}{K^2} C(t,\tau;K) + \int_0^{S_t} \frac{12\ln\left[\frac{S_t}{K}\right]^2 + 4\ln\left[\frac{S_t}{K}\right]^3}{K^2} P(t,\tau;K) \tag{3c}$$

where $C(t,\tau;K)$ and $P(t,\tau;K)$ are call and put prices at time t with the month τ. K is the strike price. S_t is the price of the underlying forward price at time t. Note that, to compute the integrals in (3), I need to obtain a continuous call and put price function at each date t. The two, most prominent methods in literature for extracting this function are either to fit a smoothing spline to the implied volatility - strike smile (see Shimko, 1993), or to the Black-Scholes delta - strike price (see Malz, 1997). This paper takes the former approach because Shimko (1993) allows for the extrapolation in the range of the existing strike prices, where Malz (1997) extrapolates the spline without allowing tail limitations. I then extract the pseudo-implied volatility values from a set of equally spaced strikes with the available strike range (see Kang et al., 2014; Skiadopoulos et al., 2019).

The RRA estimation (1) also requires the physical realised variance and so, at each day t, I compute it $\sigma^2_{p,t}(s)$, between time $t-s$ and t following Andersen et al. (2003), Skiadopoulos et al. (2019) and others:

$$\sigma^2_{p,t}(s) = \sum_{i=t-s}^{t} \sigma_i^2 + \sum_{i=t-s}^{t} R_i^2, \quad R = \ln(S_t^{Op}) - \ln(S_{t-1}^{Cl}), \tag{4}$$

with σ_i^2 being the daily realised variance and R_i^2 being the overnight returns calculated as the log difference of the opening price of each day, S_t^{Op} and the closing price of the previous day, S_{t-1}^{Cl}. This paper considers two different horizons for the realised variance calculation:

(I) $s = \tau$, *i.e.* I take the realised variance of the month prior to date t; and

(II) $s = \tau - t$, *i.e.* I take the realised variance of the number of days to maturity prior to date t, given the data used on the options expiring at the end of each month τ.

Case I follows Bakshi et al. (2003); Bakshi and Madan (2006); Kang et al. (2010); Duan and Zhang (2014);

Skiadopoulos et al. (2019) and many others and Case II is the variation I propose: the risk-neutral moments are taken with the horizon of number of days to expiry and so, I believe the realised variance should be considered in the same manner.

4 Data and Preliminary Observations on Monthly RRA

This paper computes the IRA from 2000 to 2017, mainly using three sets of the data. First, I download from Wharton Research Data Services (WRDS) database, the contractual data and the daily implied volatility data of the out-of-the-money European options written on the S&P 500 index. The S&P 500 index options expire on the third of Friday of each month. I limit my data download for each option within a month prior to its expiry date, *i.e.* between its expiry date and the previous expiry date. I also discard data on options (1) with implied volatility larger than one; or (2) with the number of strikes less than five observed. The reason being that I need at least five available strike prices data, in order for the spline to extrapolate smoothly through the implied volatility-strike space. See Table 1 for descriptive statistics of my cleaned option data.

Table 1: Statistical description of my cleaned S&P 500 index option data. No. of options and option prices are for those with different strike prices within each month. Standard deviation, skewness and kurtosis are the moments of the call or put prices within each month.

	No. of options		Option prices			Standard Deviation			Skewness			Kurtosis		
	Min	Max	Min	Max	Ave	Min	Max	Ave	Min	Max	Ave	Min	Max	Ave
Call	6	95	0.44	160.5	15.957	0.021	50.074	9.142	-1.357	5.617	1.302	-1.885	32.231	1.042
Put	6	123	0.47	151.5	16.406	0.021	39.447	6.381	-1.647	10.309	1.053	-1.811	118.164	0.944

Second, I download the daily realised volatility data of the index from the Realised Library of the Oxford Man Institute of Quantitative Finance. The realised volatility is computed using 5-minute returns on the index by the Institute. I add to the daily realised volatility the overnight realised volatility computed using the overnight returns which is computed using the opening price of each day and the closing price of the previous day (see Skiadopoulos et al., 2019). The daily opening and closing prices of the index is downloaded from Thomson Reuters Eikon.

The risk-free rate used in this paper is the ICE 3-month LIBOR rates reported by Thomson Reuters Eikon given that they are the most liquid short-term rates and considered to best reflect the realized market borrowing and lending rates. I do not use the 1-month risk-free rate matching the options' expiry dates because the treasury bills with a very short maturity, are often illiquid and exposed to price manipulation from the central banks (see Bliss and Panigirtzoglou, 2004).

Figure 1 depicts my monthly RRA estimates between 2000 and 2017, obtained following the methodology presented in Section 3. Also see Appendix C for the yearly minimum, maximum and average RRA values. Overall, my estimates are within the range of -0.58 and 5.36. This is broadly consistent with those computed in the literature: Figure 2 depicts the RRA estimates computed by a variety of methodologies in the literature. Kang et al. (2010); Duan and Zhang (2014); Skiadopoulos et al. (2019) use the model-free approach applied in this paper, but employ the Generalized Method of Moments to estimate RRA. Their

RRA estimates are reportedly within the range of [2.086, 5.72], [1.8, 7.1] and [2.27, 9.55], respectively. Notably, Skiadopoulos et al. (2019) uses the largest period of time reported in literature, producing the RRA estimates between 2001 and 2015, while Duan and Zhang (2014) findings are for a smaller period between 2001 and 2010 which is included in my sample period.

Figure 1: The highest, lowest and average monthly RRA estimates within each year between 2000 and 2017.

(I) $s = \tau$ (II) $s = \tau - t$

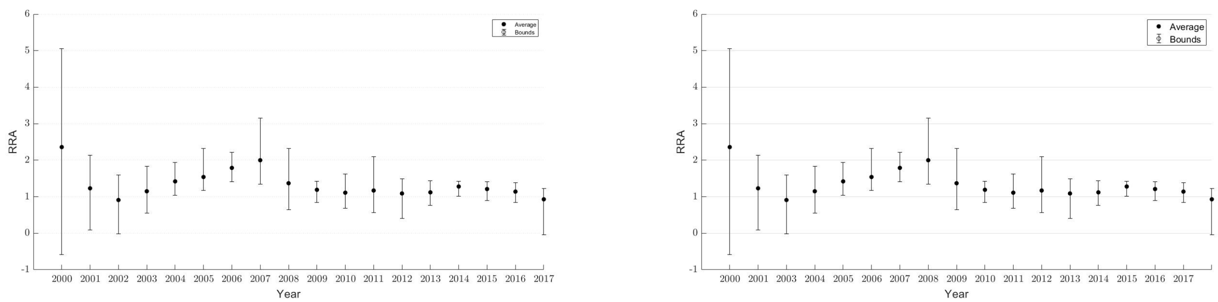
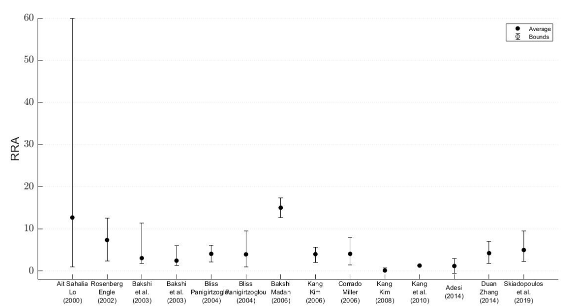


Figure 2: The RRA estimates reported in literature.



In the following I report three major observations I have made on estimating my monthly RRA estimates.

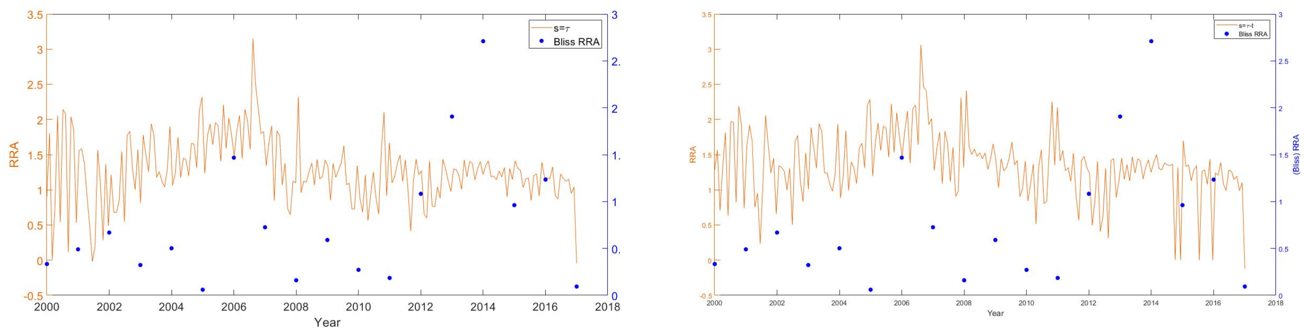
(1) Significant model risk in estimating method and data

From Figure 2, I observe a significant impact on the RRA estimates due to a change of estimating approach. Ait-Sahlia and Lo (2000) uses a utility-based model and reports the largest range of RRA values, obtaining estimates between 1 and 60. Rosenberg and Engle (2002) uses a similar approach within the same sample period, but reports a smaller range of RRA estimates between 2.36 and 12.55. Bliss and Panigirtzoglou (2004) parametrises the underlying price distribution density function under the physical measure and finds the RRA between 0.95 and 9.5. Kang and Kim (2006) adopts Bliss and Panigirtzoglou (2004)'s approach and manages to stabilise the RRA estimates, *i.e.* between 0.1 and 0.7.

I would like to note that I have attempted to adopt Bliss and Panigirtzoglou (2004)'s methodology to estimate RRA before my adoption of Bakshi et al. (2003); Bakshi and Madan (2006); Kang et al. (2010); Skiadopoulos et al. (2019)'s methodology. I discard Bliss and Panigirtzoglou (2004)'s approach because it can only generate yearly RRA estimates and I believe it is too-restrictive to assume RRA stays constant throughout any year. Anyhow, I report in Figure 3, my RRA estimates obtained by using either of these strands of approaches. Also see Appendix A for details of my adoption of the Bliss and Panigirtzoglou (2004) methodology.

Figure 3: The RRA estimates using Bliss and Panigirtzoglou (2004)'s model (in blue) or Bakshi et al. (2003)'s model (in orange) between 2000 and 2017. Bliss and Panigirtzoglou (2004)'s model generates yearly estimates of RRA, while Bakshi et al. (2003)'s model generates monthly estimates.

(I) $s = \tau$ (II) $s = \tau - t$



Figures 3 shows a drastic difference between the two sets of RRA estimates. Bliss and Panigirtzoglou (2004)'s methodology seems to generate more volatile estimates relative to Bakshi et al. (2003)'s method. They also peak at different years: under Bakshi et al. (2003)'s method, my RRA estimates peaks at 2007 which is consistent with Skiadopoulos et al. (2019)'s finding while under Bliss and Panigirtzoglou (2004)'s method, my estimates peaks in 2015. It is interesting to note that, Duan and Zhang (2014) who adopts a model-free framework similar to Bakshi et al. (2003) and others, find his estimates consistent to Bliss and Panigirtzoglou (2004), opposing my findings above. This grants a more detailed comparison with Duan and Zhang (2014)'s results as part of my future research.

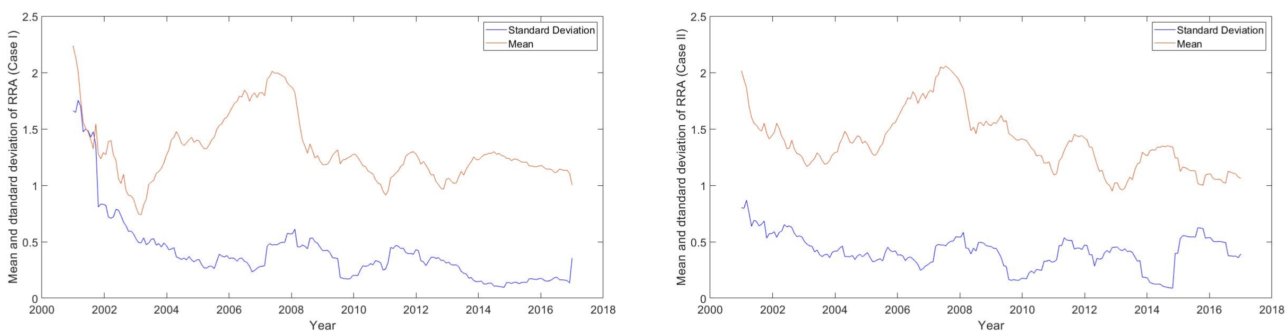
Like us, the above mentioned papers use the S&P 500 index options and the index prices to estimate RRA. There are several others who use other indices to compute RRA. Bakshi et al. (2003) use S&P 100 and report a larger range of RRA, γ being between 1.76 and 11.39, compared to my findings for a time period prior to the one used in this paper. Bliss and Panigirtzoglou (2004) estimates RRA using the S&P 500 and FTSE 100 index, separately. Although the estimates assuming a power utility function are very similar between both indices, the estimates assuming an exponential utility function have a broader range for the S&P 500 and a slightly lower mean than those for FTSE 100. Kang et al. (2010) uses FTSE 100 index, and reports a range of estimates lower than Bliss and Panigirtzoglou (2004)'s mean estimates.

(2) Monthly RRA stabilises over time and peak during 2008 crisis

Figures 1 and 3 both show that from 2000 to 2017, RRA stabilises around its mean over time. This is further confirmed by Figure 4 which depicts the mean and standard deviation of monthly RRA estimates in a 12-month rolling window within the sample period. I am the first to observe this pattern which I need to further validate in my future research. Moreover, I find that RRA estimates peak around the crisis period which is similar to Skiadopoulos et al. (2019)'s findings. This opposes Bliss and Panigirtzoglou (2004) assumption that the risk aversion is inversely related to the market risk. Prior to the 2008 crisis, RRA has been high. Although during the crisis period, RRA rises a little and reverts back to a low level afterwards, but is overall lower in comparison to the 2000-2002 period. So far I have not found any literature on the potential reasons for RRA to stabilise over time, and I believe it might be an interesting phenomenon to study for my future studies.

Figure 4: The 12-month rolling window average and standard deviation of the estimated RRA from 2000 to 2017.

(I) $s = \tau$ (II) $s = \tau - t$



(3) Negative RRA estimates around 2000-2002 and 2017

Figures 1 and 3 both show a large spread of the RRA estimates at the beginning of the sample period 2000-2017. Particularly, I observe negative RRA estimates in 2000, 2002 and 2017 under Model I ($s = \tau$) and in 2017 under Model 2 ($s = \tau - t$). While I have not found any literature focusing on the RRA estimates around similar time periods but I am not the first to obtain negative RRA estimates: Jackwerth (2000); Barone-Adesi et al. (2014) have found and discussed negative RRA estimates around the crisis period. Jackwerth (2000) propose that before the crisis the prices' risk-neutral and physical moments (mean, median and mode), lie close to each other, while post-crisis this feature changes. This change potentially causes negative RRA estimates to be produced. Barone-Adesi et al. (2014) obtains the RRA estimates by calibrating the pricing kernel and reports negative RRA estimates in 2007.

In the classic utility theory, RRA should not be negative for a risk-averse market 'representative' investor under the Arrow-Pratt measure: by definition it is a fraction of the second and the first order derivative of the utility function of the representative investor and therefore, is always positive in theory: the first order derivative of the utility function is always positive (*i.e.* the utility value increases with wealth), as is the second order derivative (*i.e.* the utility function is concave). The negative RRA estimates seem to me a piece of supportive evidence of Kahneman and Tversky (1979)'s prospect theory: when the market is placed in the domain of losses, it can portrait a risk-loving attitude towards the asset price volatility. Indeed, both Jackwerth (2000); Barone-Adesi et al. (2014)'s approaches are theoretically developed based on the prospect theory, which I may consider to extend in my future research on implied RRA.

5 RRA *v.s.* S&P 500 Risk-Return Characteristics

This section explores the explanatory power of the RRA estimates on (a) the S&P 500 index returns and (b) the return variations, *i.e.* the VIX index, separately. For each regression, I include the lagged value up to three lags prior to t for each of the independent variables, and for the dependent variable. This is mainly made in order to control the autocorrelation existence in my model. More lags could be added, but they would made the model more complicated and less intuitive.

For each of these scenarios I run a multi-variate regression with all the independent variables on the dependent variables and eliminate highly correlated variables to avoid the multicollinearity problem that can bias my regression results. I then only keep the significant independent variables and further remove amongst them those with a correlation with others higher than $\sqrt{R^2}$. I then re-run the regression and further remove insignificant independent variables. This process is repeated until I am left with significant independent variables which also pass my multicollinearity check. The autocorrelation (DW), heteroscedasticity (BP), normality (JB) test results are also included as part of my regression results alongside the R^2 and the adjusted R^2.

5.1 A Naïve Regression Model

Table 2 reports my regression results on S&P returns and VIX using RRA estimates obtained under model I and model II, separately. The returns used for this analysis are the monthly returns calculated as the log difference of the price at time t and $t - \tau$. For each regression I report the results with all independent variables and the results with only the significant independents that passes the multicollinearity check.

The results show that for both S& P 500 and VIX indices, their first and second lags are significant. This is consisted with the literature, which imposes that returns are autocorrelated and therefore the return lags, in my case the returns at time $t-1, t-2$ have explanatory power on the returns at time t (see Fama and French, 1988; Chen, 1991; Avramov et al., 2006). Theory suggests that this is a result coming from the leverage effect on the returns, *i.e.* of the negative relationship of the returns with their volatility changes (see Ait-Sahalia et al., 2013) and thus the lags are necessary to control the autocorrelation of the models.

My RRA estimates, both from model I and model II, are found significant for the S&P 500 returns, where for model I they are significant up to the first lag, i.e. RRA and RRA lag 1, where for model II only the RRA lag 1 is significant. This seems to suggest that RRA may have a causal relationship with S&P 500 returns. I plan to verify this using the standard, Granger (1969) causality test in my future research. It is important to note that the rolling window standard deviation is also significant for the S& P 500 index, and it is shown to be inversely related to the returns. This further confirms that RRA has explanatory power over returns.

On contrast, RRA seems to lose its explanatory power when used to explain the VIX. The RRA estimates I obtained through both models, in combination with their rolling window standard deviation, are all found insignificant when used as explanatory variables for the VIX index. Note that the VIX index is known to capture the S&P 500 risk level, and can be used as a proxy for market risk. Therefore my findings are consistent with my previous observation, that the RRA does not react to the market risk, opposing Bliss and Panigirtzoglou (2004)'s assumption.

Moreover, all regressions are shown to be a "good fit" to the data, by failing the F-test. On the other hand, adjusted R^2 for all regressions are low, indicating that there may be omit independent variables. Furthermore, the regression using model II RRA, on S&P 500 returns, seems to fail the heteroscedasticity check, whilst residuals normality is rejected for all the models. I therefore expand the pool of independent variables and launch a set of comprehensive regression analyses.

5.2 A Comprehensive Regression Model

This subsection explores the explanatory power of the RRA estimates on the S&P 500 index returns and return variations with the Fama-French factors and a few macro-economic factors added to the multi-variate regression. It is important to include these factors as they are all suggested by the literature as the most significant and fundamental factors that explain the stock market returns and return variations. In following I first discuss these factors in details and in relation to my RRA estimates and then show my regression results using all these control variables alongside my RRA estimates and their standard deviations on the S&P 500 index returns and return variations.

Fama-French Factors

The Fama-French three factors, originated by Fama and French (1993), have been used extensively in literature as artificial variables that can explain a proportion of the market returns variation. Liew (1999) test the forecasting ability of the SMB, HML and WML factors on the growth of a country's GDP and find that SMB and HML have a forecasting ability that remains significant even after business cycle variables are included in the regressions. Petkova (2006) further supports the significance of the Fama-French factors by showing that when they are added in a regression alongside other variables such as dividend yield and term spread shocks, the default spread, and one-month Treasury-bill yield etc., these other factors lose their forecasting ability.

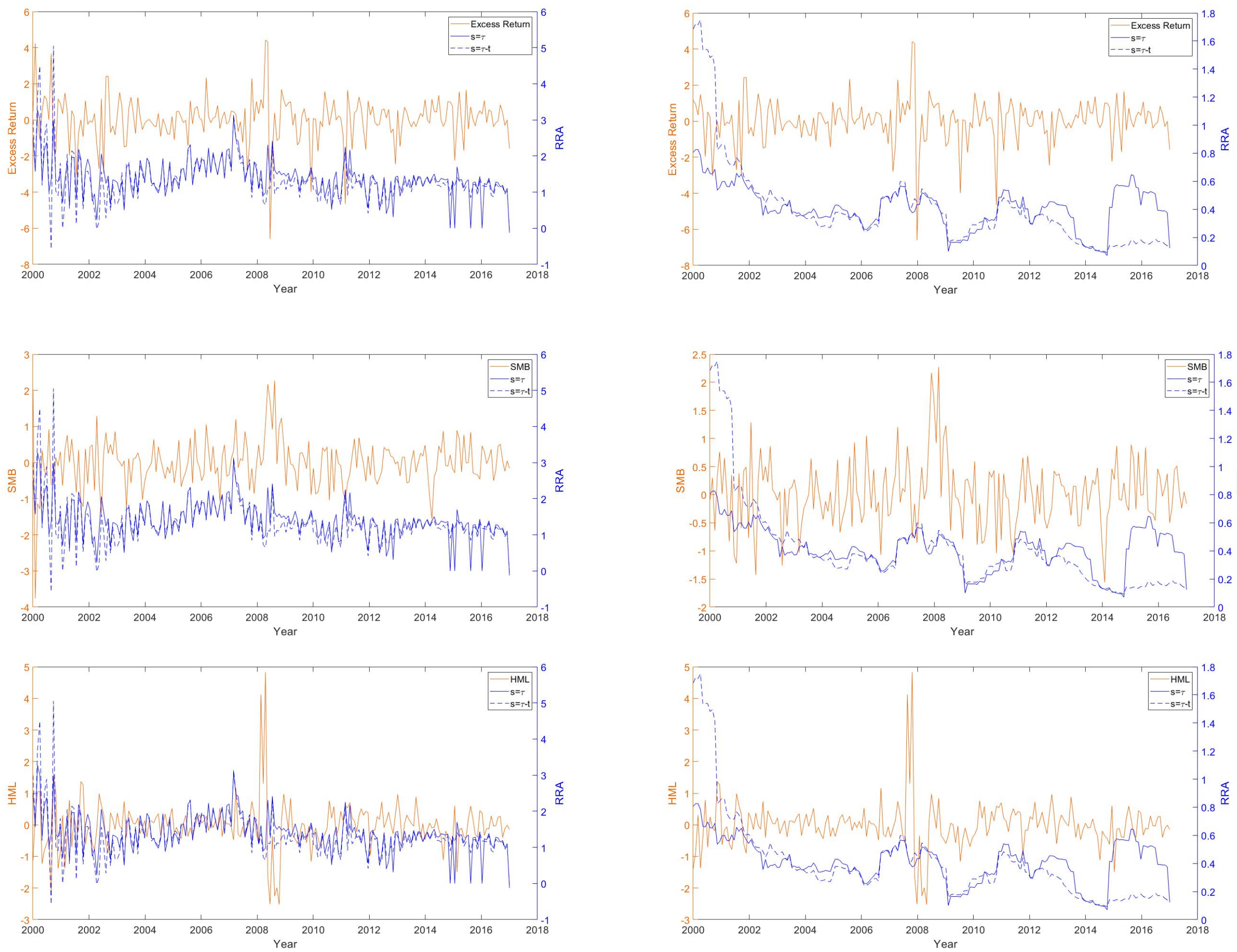
I download the monthly Fama and French factor data downloaded from Kenneth R. French's website, and plot them against my RRA estimates and their standard deviation in Figure 5. It is hard to see any correlation between my RRA estimates and any of the three F-F factors. Indeed, the correlation coefficients between any F-F factors and the estimated RRAs is no higher than 0.065 in absolute terms for both model I and II. Moreover, all F-F factors presents a shock around 2008 crisis period, but this is not the case for the RRA estimates.

Macroeconomic Factors

The Gross Domestic Product (GDP) reflects the economic growth and it is extensively used in literature. Although it is a monetary measure of goods and services value and should not reflect the living cost nor the inflation rate, Cole et al. (2008) reports that future GDP growth is positively related to bank stock

Figure 5: The Fama and French 3 factors (top: market premium; middle: SMB; bottom: HML, depicted by the orange solid lines) *v.s.* the RRA estimates (left) and standard deviation of RRA estimates within a 12-month rolling window (right) between 2000 and 2017 under model I (blue solid line) and model II (blue dotted line).

(I) Fama-French factors *v.s.* γ (II) Fama-French factors *v.s.* σ_γ



returns, Vassalou (2003) extends the research of Liew (1999) and shows that when a news-related GDP factor is included in the analysis, F-F SMB and HML factors can lose their explanatory ability.

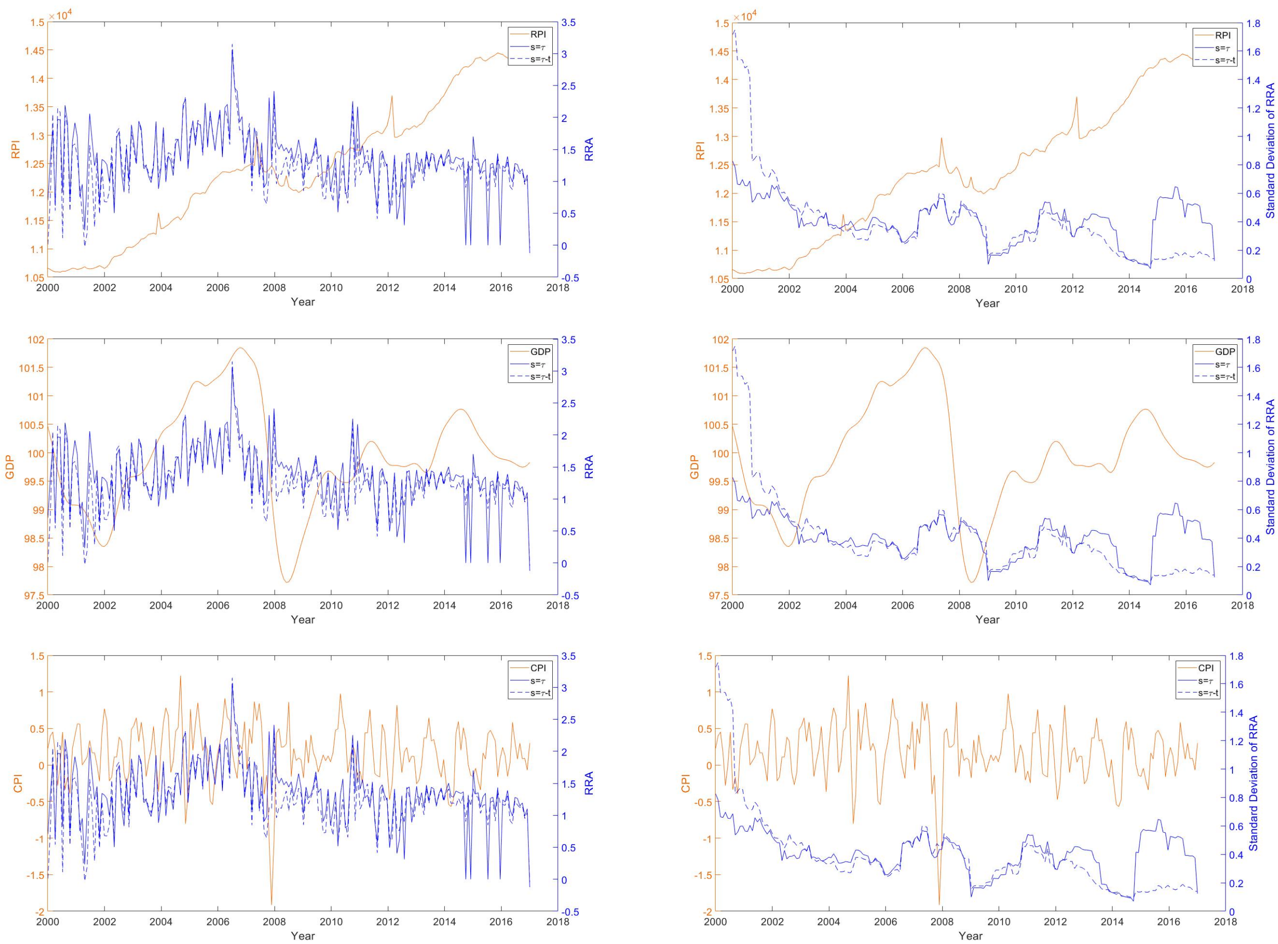
The Consumer Price Index (CPI) represents the average cost of living in a country and it is constructed as a weighted average of the prices of the goods in the representative consumer's basket. It has been commonly used in literature as a controlling variable of stock returns, mainly as a measure of inflation (see Gultekin, 1983). Flannery and Protopapadakis (2002) show that the CPI has a predictive power on aggregate U.S. stock returns, as they are documented as negatively correlated in the US market (see Humpe and Macmillan, 2007).

The Retail Price Index (RPI) is a measure of inflation, very similar with the CPI, but different in its construction: while the CPI is measured as a geometric mean of the representative consumer's basket goods, RPI is an arithmetic mean. Moreover, the CPI is linked with the payments a government does, *e.g.* pension, and RPI with the payments a government receives, *e.g.* taxes. Therefore, they could act cooperatively in my analysis, and further assist me to produce a robust model on explaining stock returns, unless they are highly-correlated and induce a multicollinearity problem to my regression.

I download the monthly data on GDP, RPI, CPI from FRED (the Federal Reserve Economic Data). I plot them against my RRA estimates and their standard deviation in Figure 6. RRA estimates under both model I and II show a negative correlation with the RPI index, as low as -0.67 (under Model II). But this may be caused by the non-stationary behavior of the index. Furthermore, the RRA standard deviation under both model I and II am positively correlated with the GDP factor ($\rho_{s=\tau} = 0.23, \rho_{s=\tau-t} = 0.39$). The CPI factor seems uncorrelated with both RRA estimates under model I and II, as well as their standard deviations (with maximum correlation $\rho = 0.08$).

Figure 6: The macroeconomic factors (top: RPI; middle: GDP; bottom: CPI, depicted by the orange solid lines) *v.s.* the RRA estimates (left) and standard deviation of RRA estimates within a 12-month rolling windoe (right) between 2000 and 2017, under model I (blue solid line) and model II (blue dotted line).

(I) Macroeconomic factors with γ (II) Macroeconomic factors with σ_γ



Now I run separate regressions on the returns (a) of the S&P 500, and (b) of the VIX index, with the independent variables including: the index returns, the VIX index values, the estimated RRA, the 12-month rolling window standard deviation of the RAA estimates, the Fama-French factors, *i.e.* SMB, HML and RP, the risk free rate, and the macroeconomic factors *i.e.* U.S. GDP, CPI, RPI. In the following I separately report my regression results using the RRA estimated under Model I and II.

Table 5 reports my final set of regression results which, from left to right, use as the dependent variable (1) the daily return of the S&P 500 on the option's expiry date *i.e.* the third Friday of each month; (2) the monthly return of the S& P 500 index, *i.e.* the return between the closing price of the previous expiry date and the price at expiry; (3) the daily VIX value on the option's expiry date; and (4) the monthly VIX value on the option's expiry date.

In line with my results from a set of naïve regressions, the return (VIX) lags are significant in explaining the return (VIX) itself. However, although in the naïve regressions, both the first and the second returns lags were significant for the S%P 500 returns, at 1% and 10% significance level respectively, after the inclusion of the Fama-French and the macroeconomic factors only the second lag remains, whose significance level is improved at 1% level. Regarding the VIX, although the first and the second lags were highly significant at 1% level in the previous analysis, after including the factors, only the first lag remains when daily data are used, at 10% significance level. Regarding the monthly data, this feature remains for model II, where the significance level is improved at 1%, whereas for model I the lags are significant up to the third lag, at 5%, 1% and 1% significance level.

The VIX and the S&P 500 returns seem to have explanatory power on each other, which is consistent with what is reported in the literature. Giot (2005) finds that future returns are negatively correlated with the present level of implied volatility indices, where Guo and Whitelaw (2006); Banerjee et al. (2007) and others, report similar results. It is therefore not a surprise that S&P 500 returns and VIX share an explanatory relationship with each other. However, the power of VIX on the S&P 500 returns seems stronger than the opposite case. For the monthly S&P 500 returns, all the VIX values up to their third lag are significant at 1% significance level, where for the VIX, only the return is found significant and not its lags.

Consistent with my previous findings, RRA estimates, their lags and their standard deviation are significant in explaining S&P 500 returns and they have no power on explaining VIX in any case. A common feature for all regressions on S&P 500 returns, is that the third lag of RRA is always significant, whereas the significance of the rest parameters depends on the case. Moreover, the Fama-French factors significance seems to be reduced. In both models either one or two parameters for each factor are significant. Note that the RP (risk premium) factor is removed during the multicollinearity tests. For the VIX index, the SMB factor and its lags are insignificant, while the two models share the same significant parameters for the HML factor depending on the data-set. The factors are significant at 1% for both models, for the S&P 500 returns, except the third lag of the SMB factor in model II, when the monthly data are used.

Amongst the macroeconomic factors used, the GDP factor is entirely removed from the regressions. This opposes Skiadopoulos et al. (2019)'s finding on a inverse relationship between GDP and RRA. But their finding is under the aim of explaining the US real economic activity while ours are for explaining stock returns. Moreover, they add the GDP factor as the log deviation of its values depending on the impact period, and this might be the reason I obtain different results. Recall that Liew (1999) has argued that the inclusion of GDP can make F-F SMB and HML factors to lose significance. For this reason, the absence of the GDP factor might have caused the SMB and HML factors to resurface as significant variables in my regression.

The results show that RPI is only significant under the second model, where by contrast, CPI is always significant when used in the S&P 500 returns regressions. This is because the RPI factor and the risk-free rate share a high correlation, and RPI has been removed at the multicollinearity tests. As discussed in my control variable section, RPI is linked with the government taxes. This may explain its high correlation with the risk free rate. Furthermore, both RPI and CPI am removed when they are used to explain VIX. Interestingly, Bekaert and Hoerova (2014) supports that the conditional market variance predicts the real economic activity. By contrast, my results show that this does not hold in the reversed case.

As part of the robustness check, both models fail the $F-$test, implying that the regressions are "good fits". Compared to my previous results the R^2 of the regressions are highly improved, where both models produce very similar values. However, heteroscedasticity test is failed for all the cases where the S&P 500 returns are the dependent variable. Also, the normality test is failed alongside the unfavourable results on heteroscedasticity checks. This potentially calls for future research in including more relevant control variables in my sets of regressions.

6 Future Research

6.1 Causality between RRA and S&P 500 Returns

my comprehensive regression results show that when using RRA to explain the S&P 500 returns, S&P 500 return lag 1 loses significance under both Models I and II. I therefore hypothesize that RRA may be closely related the S&P 500 return lag 1, although their correlation is merely -0.08 under model I and -0.19 under model II. It is intuitive for them to be negatively correlated: the higher is the return, the more driven are the investors to invest with a lower risk tolerance.

The left half of Table 8 represents my regression results on RRA using S&P 500 returns and lags. The result shows a significant explanatory power of S&P 500 return lag 1 on RRA. Moreover, under model II, return lag 1 becomes the sole variable that is significant amongst all return variables. This leads me to believe that RRA is formed following the return lag 1 and in turn, then explains the return itself. This maybe an interesting finding but to confirm this, I need to run a set of comprehensive regressions including all control variables and check whether the finding still stands.

As I have mentioned, RRA seems to be negatively affected by return lag 1; by contrast, my previous comprehensive regression results show that RRA is positively correlated with return. This leads me to hypothesize that RRA delivers a negative impact from return lag 1 to return: RRA rises following a negative return lag 1 and in turn, increases the return itself. This is not directly observable from the positive correlation between the return and its lag 1, which is 0.24 in my sample period. I will further test this hypothesis in my future research.

It is worth mentioning that RRA is not the only risk aversion variables that show a close connection with return lags. In fact, the standard deviation of RRA is highly correlated wtih return lags. It further confirms the tight relationship between RRA and S&P 500 returns. But how this standard deviation of RRA affects RRA and returns, calls for further research.

There are a few issues with my results above. First, although the regressions models all seem to be good fits to the data, they fail the heteroscedasticity and normality checks. Also the adjusted R^2 is just around 30%. These observations indicate that my regression results maybe biased in some way. In line with my previous argument, I shall perform a set of comprehensive regressions to see whether these statistical issues still exist.

Although from my comprehensive regression results VIX and its lags still carry explanatory power of S&P 500 returns alongside RRA which indicates a lack of connection between VIX and its lags, and RRA, I still run a set of regressions on RRA using VIX and its lags. I report the results in the right half of Table 8 which show a certain level of explanatory power of VIX and its lags on RRA. This set of results are inconclusive – they show that VIX and its lags are related with RRA but in an unclear form. Coudert and Gex (2008) have argued that RRA reacts ahead of the crisis period which could be some findings I validate in my future research.

6.2 RRA and S&P 500 Returns: A Lesson from the 2008 Crisis

Splitting my sample period (2000-2017) to pre-crisis (2000-2007) and post-crisis (2009-2017) periods, seems to improve the correlation estimates between the F-F factors and RRA estimates. Under model II, the RRA-HML correlation moves from -0.13 for the full sample to 0.24 pre-crisis. This correlation under model I moves from -0.06 for the full sample to 0.15 pre-crisis. Perhaps more importantly, the correlation between HML and the standard deviation of RRA moves from -0.006 and -0.059, for model I and II respectively, for the full sample to -0.24 pre-crisis under both models, but to 0.22 post-crisis under model I and 0.33 under model II. These observations indicate potential, drastic changes to my results once I move my regressions from the full sample to pre-/post-crisis periods.

I note that control variables' inter-relationships also change by this split of sample period. In the full sample, the GDP factor is strongly positively correlated with the RPI index ($\rho = 0.93$), whilst both factors are negatively correlated with the RRA estimates producing $-0.59 < \rho < -0.36$ and positively correlated with the standard deviation estimates that range between $0.46 < \rho < 0.55$. During the post-crisis period, however, this feature has almost vanished with standard deviation and RRAs producing a correlation range $-0.13 < \rho < -0.04$. The only relationship that remains, is between the model II RRA and the RPI, GDP factors ($\rho_{RPI} = -0.65, \rho_{GDP} = -0.38$). The CPI factor is uncorrelated with all the parameters in all sets of results.

7 Conclusions

This paper attempts to understand the empirical relationship between RRA and S&P 500 returns and return variations. To the best of my knowledge, I am the first to do so although the RRA impact on an asset's return-risk trade-offs has been well established long ago (since Arrow, 1965; Markowitz, 1952, and many others). I first construct and generate monthly RRA estimates using Kang et al. (2010)'s model-free approach, and compare my estimates with those presented in the literature. I find: (1) that RRA estimates vary significantly across different choices of the estimation model and the underlying index; (2) that RRA stabilises over time and peak during 2008 crisis which is broadly consistent with the literature (such as Skiadopoulos et al., 2019); and (3) RRA estimates can go negative which has been identified before by (such as Jackwerth, 2000; Barone-Adesi et al., 2014).

Second, I run regressions using RRA on the monthly S&P 500 index returns and the VIX index, respectively, with and without control variables such as the risk-free rate, F-F factors and macro-economic factors (*e.g.* GDP, RPI and CPI). I find in all cases, a significant explanatory power of RRA, some of its lags and its standard deviation over a yearly window, on S&P 500 returns, but none of these variables are significant in explaining VIX. Although this is my biggest finding, it needs further validation because my regressions seem unable to pass all robustness checks.

I have made a further attempt to understand how RRA affects the S&P 500 returns and formed several hypotheses. I run a set of naïve regression using returns on RRA and the results show that RRA may be formed negatively following the return lag 1 and in turn affects the return itself positively. I believe this is an immediate hypothesis for me to work on in my future research.

I also observe drastic changes in the coefficients of the correlations between RRA estimates and control variables. This potentially indicates changes in my already observed relationships captured by my regressions. So my future research will not only be conducted over the full sample period (2000-2017) but also to pre-/post-periods (before/after 2008). I believe this exercise will shed more lights on the relationship between RRA and S&P returns as well as their causality relationship if any.

Through my analyses, I also find three other ideas that call for a deeper look. First, my all-time highest RRA estimates using Bakshi et al. (2003); Kang et al. (2010)'s model-free approach, and using Bliss and Panigirtzoglou (2004)'s approach, occur at different years (2007 and 2015 respectively). This is inconsistent with the findings of Duan and Zhang (2014) who adopts a model-free approach and generates RRA estimates consistent with Bliss and Panigirtzoglou (2004)'s results. I plan to compare in detail my estimation against Duan and Zhang (2014)'s work in order to understand this inconsistency. Second, my observed negative RRA potentially supports Kahneman and Tversky (1979)'s prospect theory in that the market can become risk-loving when placed in the domain of loss. So I may consider to adopt or develop a theoretical approach for RRA estimation, following Jackwerth (2000); Barone-Adesi et al. (2014), in my future studies. Third, my RRA estimates stabilise over my sample period on which I have not found any similar evidence in the literature nor any study that investigate the potential causes of this pattern.

Table 2: Regression results on monthly S&P returns and VIX using RRA estimates under Model I and II. Lag x denotes the return (VIX) at time $t - x$ when regressing on the return (VIX) value at time t.

	S&P returns							
	Model I				Model II			
	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat
α	0.007 0.011	0.54 0.614	0.003 0.01	0.797 0.258	0.028 0.015	0.07 1.822	0.039 0.012	0.001 3.325
lag_1	0.289 0.073	0 3.943	0.277 0.072	0 3.826	0.252 0.075	0.001 3.383	0.224 0.072	0.002 3.131
lag_2	−0.162 0.075	0.032 −2.16	−0.134 0.071	0.058 −1.906	−0.165 0.076	0.031 −2.178	−0.142 0.073	0.055 −1.932
lag_3	0.101 0.073	0.169 1.381			0.098 0.074	0.187 1.324		
γ	0.021 0.006	0.001 3.34	0.018 0.006	0.001 3.241	0.005 0.006	0.436 0.781		
γ_1	−0.01 0.006	0.094 −1.684	−0.011 0.006	0.073 −1.802	−0.011 0.006	0.07 −1.82	−0.01 0.006	0.092 −1.695
γ_2	0.001 0.006	0.923 0.097			0.003 0.006	0.631 0.481		
γ_3	−0.008 0.007	0.213 −1.25			−0.001 0.006	0.875 −0.158		
σ_γ	−0.024 0.01	0.023 −2.286	−0.026 0.01	0.011 −2.568	−0.051 0.021	0.018 −2.378	−0.059 0.021	0.005 −2.877
R^2	0.164		0.151		0.126		0.114	
R^2_{adj}	0.129		0.128		0.0884		0.096	
DW	2.008	0.478	1.976	0.389	1.994	0.446	1.951	0.328
JB	387.94	0	409.38	0	230.1	0	313.28	0
BP	14.603	0.067	7.442	0.19	21.64	0.006	13.731	0.008
F	4.614	0	6.766	0	3.376	0	6.182	0

	VIX							
	Model I				Model II			
	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat
α	−0.032 0.074	0.67 −0.427			−0.01 0.092	0.915 −0.106		
lag_1	−0.322 0.074	0 −4.371	−0.332 0.071	0 −4.664	−0.362 0.076	0 −4.795	−0.332 0.071	0 −4.664
lag_2	−0.18 0.078	0.022 −2.314	−0.191 0.071	0.008 −2.689	−0.232 0.081	0.005 −2.874	−0.191 0.071	0.008 −2.689
lag_3	−0.027 0.075	0.719 −0.361			−0.023 0.076	0.762 −0.303		
γ	−0.049 0.042	0.247 −1.161			0.048 0.04	0.231 1.201		
γ_1	0.043 0.039	0.269 1.109			0.01 0.039	0.79 0.266		
γ_2	0.011 0.038	0.771 0.292			−0.017 0.038	0.649 −0.456		
γ_3	0.003 0.042	0.942 0.073			−0.059 0.04	0.14 −1.482		
σ_γ	0.044 0.063	0.49 0.691			0.079 0.124	0.526 0.636		
R^2	0.124		0.11		0.125		0.11	
R^2_{adj}	0.086		0.101		0.088		0.101	
DW	1.988	0.044	1.973	0.432	1.945	0.334	1.973	0.432
JB	148.64	0	131.52	0	113.91	0	131.52	0
BP	8.178	0.416	1.63	0.443	9.103	0.334	1.63	0.443
F	3.318	0	11.96	0	3.64	0	11.96	0

Table 5: Regression results on the returns (a) of the S&P 500, and (b) of the VIX index, with: the index returns, the VIX index values, the estimated RRA, the 12-month rolling window standard deviation of the RAA, the F-F factors, *i.e.* SMB, HML and RP, the risk free rate, and the macroeconomic factors *i.e.* U.S. GDP, CPI, RPI.

	Model I: $s=\tau$							
	S&P rtn daily		S&P rtn monthly		VIX daily		VIX monthly	
	Coef. Std. Er.	P-Val T-stat	Coef. Std. Er.	P-Val T-stat	Coef. Std. Er.	P-Val T-stat	Coef. Std. Er.	P-Val T-stat
α	−0.005 0.002	0.005 −2.83	−0.005 0.007	0.542 −0.611	−0.005 0.003	0.114 −1.589	−0.034 0.022	0.116 −1.581
r					−4.612 0.299	0 −15.442	−6.882 1.28	0 −5.376
r_2	−0.277 0.065	0 −4.239	−0.171 0.064	0.009 −2.658	−1.171 0.417	0.006 −2.808		
γ			0.025 0.005	0 4.987				
γ_1								
γ_2								
γ_3	0.002 0.001	0.072 1.811	−0.011 0.005	0.027 −2.23				
σ_γ			−0.032 0.008	0 −4.142				
SMB	0.002 0.001	0.022 2.316						
SMB_1	0.002 0.001	0.001 3.283	−0.009 0.003	0.013 −2.512				
SMB_2	0.003 0.001	0 3.549						
SMB_3			−0.007 0.004	0.035 −2.126				
HML					0.01 0.005	0.043 2.035		
HML_1					0.011 0.005	0.024 2.283	0.059 0.022	0.008 2.67
HML_2	−0.117 0.008	0 −15.176					0.042 0.021	0.048 1.991
RF	−0.03 0.012	0.012 −2.529						
VIX	−0.014 0.008	0.076 −1.785	−0.031 0.009	0.001 −3.512				
VIX_1			−0.106 0.009	0 −11.502	−0.128 0.074	0.086 −1.727	1.795 0.902	0.048 1.99
VIX_2			−0.064 0.01	0 −6.746			−0.386 0.066	0 −5.836
VIX_3			−0.037 0.01	0 −3.618			−0.233 0.066	0.001 −3.527
RPI								
CPI	0.003 0.001	0.026 2.243						
CPI_3	0.003 0.002	0.068 1.833	0.015 0.005	0.005 2.845				
F	23.17	0	14.23	0	32.84	0	23.65	0
R^2	0.651		0.507		0.558		0.269	
R^2_{adj}	0.632		0.477		0.547		0.246	
DW	2.014	0.496	1.834	0.093	2.023	0.562	1.952	0.346
JB	188.7	0	4.484	0.075	133.24	0	74	0
BP	52.561	0	48.879	0	10.597	0.06	15.993	0.014

	Model II							
	S&P daily rtn		S&P monthly rtn		VIX daily		VIX monthly	
	Coef. Std. Er.	P-Val T-stat	Coef. Std. Er.	P-Val T-stat	Coef. Std. Er.	P-Val T-stat	Coef. Std. Er.	P-Val T-stat
α	0.001 0.003	0.807 0.245	−0.051 0.029	0.075 −1.793	−0.005 0.003	0.114 −1.589	0.034 0.047	0.475 0.716
r					−4.612 0.299	0 −15.442	−6.652 1.282	0 −5.187
r_2	−0.277 0.065	0 −4.255	−0.173 0.067	0.011 −2.578	−1.171 0.417	0.006 −2.808		
γ			0.021 0.005	0 4.202				
γ_1	−0.002 0.001	0.049 −1.979						
γ_2							−0.058 0.036	0.105 −1.628
γ_3	0.003 0.001	0.031 2.169	−0.01 0.005	0.041 −2.061				
σ_γ	−0.007 0.004	0.085 −1.733	−0.047 0.017	0.005 −2.824				
SMB	0.003 0.001	0.009 2.659						
SMB_1			−0.008 0.004	0.03 −2.192				
SMB_2								
SMB_3			−0.006 0.004	0.081 −1.756				
HML	0.003 0.001	0.001 3.515	0.006 0.003	0.042 2.048	0.01 0.005	0.043 2.035		
HML_1	0.003 0.001	0.001 3.346			0.011 0.005	0.024 2.283	0.054 0.022	0.016 2.426
HML_2							0.044 0.021	0.041 2.058
RF							2.479 0.992	0.013 2.5
VIX	−0.115 0.008	0 −14.947	−0.04 0.009	0 −4.429				
VIX_1			−0.112 0.01	0 −11.489	−0.128 0.074	0.086 −1.727	−0.392 0.066	0 −5.941
VIX_2	−0.027 0.012	0.021 −2.331	−0.068 0.01	0 −6.823			−0.24 0.066	0 −3.641
VIX_3			−0.039 0.011	0 −3.674				
RPI			0 0	0.017 2.414				
CPI	0.003 0.001	0.036 2.116						
CPI_3	0.003 0.001	0.063 1.873	0.014 0.006	0.012 2.53				
F	9.12	0	12.34	0	32.84	0	22.25	0
R^2	0.658		0.494		0.558		0.279	
R^2_{adj}	0.64		0.458		0.547		0.252	
DW	2.04	0.541	1.894	0.17	2.023	0.562	1.949	0.34
JB	137.84	0	3.235	0.146	133.24	0	83.513	0
BP	52.777	0	50.559	0	10.597	0.059	14.237	0.047

Table 8: Regression results on γ under Model I and II using monthly S&P returns and VIX. Lag x denotes the return (VIX) at time $t-x$ when regressing on the return (VIX) value at time t.

	x=*S&P returns*								x=*VIX*							
	Model I				Model II				Model I				Model II			
	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat	Coeff. Std. Err.	P-Val T-stat
α	0.377 0.123	0.002 3.073	0.442 0.103	0 4.283	0.651 0.179	0 3.641	0.652 0.121	0 5.386	0.424 0.124	0.001 3.422	0.402 0.115	0.001 3.498	0.621 0.161	0 3.869	0.831 0.095	0 8.792
x	2.62 0.784	0.001 3.34	2.638 0.764	0.001 3.453	0.679 0.869	0.436 0.781			0.135 0.067	0.044 2.024	0.121 0.065	0.064 1.861	0.05 0.07	0.475 0.715		
x_1	−2.198 0.828	0.009 −2.655	−2.309 0.804	0.005 −2.871	−2.568 0.898	0.005 −2.86	−2.07 0.815	0.012 −2.541	0.133 0.066	0.044 2.03	0.141 0.065	0.032 2.157	0.193 0.067	0.004 2.883		
x_2	1.189 0.836	0.156 1.423	1.443 0.767	0.062 1.88	0.342 0.914	0.709 0.374			0.413 0.066	0 6.283	0.42 0.065	0 6.46	0.342 0.068	0 5.038	0.388 0.065	0 5.949
x_3	−0.023 0.81	0.977 −0.029			−0.334 0.889	0.707 −0.376			−0.056 0.109	0.606 −0.517			−0.148 0.224	0.51 −0.661		
γ_1	0.178 0.067	0.008 2.669	0.2 0.064	0.002 3.12	0.115 0.071	0.107 1.619			−0.145 0.125	0.247 −1.161			0.158 0.132	0.231 1.201		
γ_2	0.094 0.067	0.165 1.395			0.153 0.071	0.032 2.156	0.181 0.068	0.008 2.673	0.125 0.133	0.349 0.94			0.532 0.14	0 3.809	0.457 0.132	0.001 3.459
γ_3	0.426 0.065	0 6.51	0.444 0.063	0 7.093	0.316 0.07	0 4.521	0.339 0.068	0 5.019	0.343 0.134	0.011 2.57	0.32 0.12	0.008 2.668	0.556 0.144	0 3.868	0.505 0.132	0 3.819
σ_γ	−0.006 0.116	0.957 −0.054			−0.223 0.258	0.389 −0.863			0.011 0.129	0.931 0.087			0.07 0.138	0.614 0.505		
R^2	0.332		0.325		0.216		0.199		0.315		0.302		0.27		0.221	
R^2_{adj}	0.304		0.307		0.182		0.186		0.288		0.287		0.24		0.209	
DW	1.79	0.058	1.832	0.109	1.895	0.205	1.705	0.177	1.817	0.092	1.798	0.07	1.925	0.284	1.832	0.12
JB	13.546	0.007	16.827	0.002	16.637	0.003	19.504	0.003	9.515	0.019	12.589	0.008	13.205	0.01	23.069	0.002
BP	44.079	0	20.656	0	18.006	0.021	4.03	0.258	49.19	0	3.682	0.45	21.592	0.006	1.171	0.76
F	7.922	0	18.38	0	4.46	0	15.95	0	10.8	0	20.77	0	0.698	0	18.25	0

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A My Implementation of Bliss and Panigirtzoglou (2004)

In theory, option implied Probability Density Functions (PDFs) reflect the forecasts of the prices' distributions of the underlying asset, given a specific time horizon. These PDFs are considered as risk neutral, implying that the representative agent is also risk-neutral. However, if the representative investor is not in fact risk neutral, the option implied PDFs may not represent the market's actual forecast of the underlying asset's distribution. In this case, I can deduce that the difference between the option implied (risk neutral) PDFs and the representative agent's accurate or objective forecast relies on the risk aversion of the representative agent.

The risk neutral and the objective density function, following Ait-Sahlia and Lo (2000), are connected with the relationship:

$$\frac{p(S_T)}{q(S_T)} = \lambda \frac{U'(S_T)}{U'(S_t)} \equiv \zeta(S_T), \tag{5}$$

where $p(S_T)$ and $q(S_T)$ are the density functions under the physical and the risk neutral measure respectively, λ is a constant, $U(x)$ is the utility function of the representative investor and $\zeta(S_T)$ is the pricing kernel. Therefore, by estimating or knowing any two of the three functions, allows me to estimate the remaining one.

For the purpose of my analysis, I use the exponential utility function, as the utility of the representative investor. Therefore, according the Arrow-Pratt measure of the relative risk aversion, I end up with the following:

$U(S_T)$	$U'(S_T)$	$RRA = \frac{-S_T U''(S_T)}{U'(S_T)}$
$-\frac{e^{-\gamma S_T}}{\gamma}$	$e^{-\gamma S_T}$	γS_T

At first the data are imported and cleaned, following the Data section (section 4). The methodology is separated in three sections, following Bliss and Panigirtzoglou (2004) paper. At first, the risk neutral PDF for each option expiry is estimated, then the forecast ability of the risk neutral PDFs is examined, and in conclusion, the subjective PDFs are obtained.

The PDF for the price of the underlying asset at each option maturity, $f(S_T)$, is connected with the call price function following Breeden and Litzenberger (1978):

$$f(S_T) = e^{r(T-t)} \frac{\partial^2 C(S_t, K, T, t)}{\partial K^2}\bigg|_{K=S_t}, \tag{6}$$

where S_t is the current value of the underlying asset, K is the strike price and $T-t$ is the price to maturity.

For the implementation of the calculation above, I need to obtain a continuous call price function. This is done by fitting a smoothing function to the obtained data, and specifically to the implied volatility smile, as presented in Bliss and Panigirtzoglou (2004). In order to do so, the option prices are converted to deltas - implied volatilities space using Black-Scoles formulas (see Malz, 1997). Although the implied volatility values are those backed out from the Black-Scholes model, the deltas used for the smoothing smile, are calculated by converting the strike prices to deltas, using the at-the-money implied volatility.

A natural smoothing spline if fitted to the implied volatility-delta smile, which minimizes the following function:

$$\min_{\theta} \sum_{i=1}^{N} \omega_i (IV_i - IV(\Delta_i, \theta))^2 + \lambda \int_{-\infty}^{\infty} g''(x;\theta)^2 dx \tag{7}$$

where IV_i is the implied volatility of the i^{th} option, $IV(\Delta_i, \theta)$ is the fitted implied volatility as a function of the i^{th} delta, θ is the set of the parameters defining the smoothing smile $g(x;\theta)$ and ω_i is the weight of the i^{th} option's squared fitted implied volatility error. In my case, following the Bliss and Panigirtzoglou (2004) paper, the vegas ($v = \frac{\partial C}{\partial \sigma}$) are employed as the weights of the smoothing spline, and λ, which is the smoothing factor of the spline, is set to 0.99.

Then, I fit the spline, and I obtain 5.000 points through the implied volatility smile, where the deltas are chosen so that they represent equally spaced strikes through the existing strike range. Moreover, the data extrapolated, are converted back to call prices, which makes it possible to compute the risk neutral density by taking the second derivative of the call price - strike relationship through each data point. For each of those PDFs, I create the individual integrals for each $K - f(K)$ pair, and I divide them with the total integral, so that they integrate to 1, so that I create their CDF. As a result, I end up with a time series of PDFs and CDFs, coming from the time series of option expiring in a one month time.

In order to test the forecasting ability of the estimated PDFs, I test the null hypothesis that the estimated PDF equals the true PDF, and thus, I test the hypothesis that if the realizations S_T, which are the prices of the underlying asset at expiry, are independent, the inverse probability transformations of the realizations:

$$y_t = \int_{-\infty}^{S_T} \hat{f}_t(u) du, \tag{8}$$

will be independent and uniformly distributed: $y_t \sim i.i.d.U(0,1)$. The Berkowitz (2001) test is employed, after applying a transformation of the acquired y_t, using the inverse of the standard normal cumulative density function:

$$z_t = \Phi^{-1}(y_t) = \Phi^{-1}\left(\int_{-\infty}^{S_T} \hat{f}_t(u) du \right). \tag{9}$$

The z_t time series is used to run an auto-regressive model, so I can test the null hypothesis, that the parameters are: $\mu = 0, \rho = 0$ and $Var(\epsilon_t) = 1$. The test is produced by estimating the log-likelihood ratio statistic:

$$LR_3 = 2[L(0,1,0) - L[\hat{\mu}, \hat{\sigma^2}, \hat{\rho})], \tag{10}$$

which has a $\chi^2(3)$ distribution, and obtaining the p-value of the test.

The subjective density function is estimated hypothesizing an exponential utility function. Provided that I have already estimated the risk neutral density, the subjective density is as follows:

$$p(S_T) = \frac{\frac{q(S_T)}{\zeta(S_T;S_t)}}{\int \frac{q(x)}{\zeta(x;S_t)} dx} = \frac{\frac{U'(S_t)}{\lambda U'(S_T)}}{\int \frac{U'(S_t}{\lambda U'(x)} q(x) dx} = \frac{\frac{q(S_T}{U'(S_T)}}{\int \frac{q(x)}{U'(x)} dx}. \tag{11}$$

B Monthly RRAs Estimated Each Year Between 2000 And 2017

Year	Model I			Model II		
	Minimum	Maximum	Average	Minimum	Maximum	Average
2000	-0.58	5.05	2.36	1.014	3.269	2.083
2001	0.09	2.14	1.23	0.633	2.189	1.41
2002	-0.021	1.59	0.91	0.234	2.058	1.275
2003	0.546	1.83	1.15	0.503	1.772	1.222
2004	1.04	1.94	1.42	0.883	1.941	1.429
2005	1.172	2.32	1.54	0.992	2.285	1.507
2006	1.404	2.21	1.79	1.465	2.224	1.774
2007	1.343	3.15	2	1.276	3.06	2.04
2008	0.649	2.32	1.37	0.906	2.413	1.553
2009	0.849	1.43	1.19	1.028	1.651	1.458
2010	0.682	1.62	1.11	0.841	1.68	1.266
2011	0.564	2.1	1.17	0.516	2.253	1.395
2012	0.413	1.49	1.09	0.495	1.521	1.123
2013	0.76	1.44	1.12	0.312	1.452	1.073
2014	1.008	1.42	1.28	1.124	1.501	1.336
2015	0.901	1.41	1.21	1.1	1.701	1.316
2016	0.847	1.39	1.14	0.95	1.434	1.212
2017	-0.045	1.23	0.93	-0.122	1.28	0.98

C Risk Aversion Estimates Presented in the Literature

Author	Risk Aversion	
	Minimum	Maximum
Ait-Sahlia and Lo (2000)	1	60
Jackwerth (2000)	-7	30
Rosenberg and Engle (2002)	2.36	12.55
Bakshi et al. (2003)	1.76	11.39
Bakshi et al. (2003)	1.36	5.98
Bliss and Panigirtzoglou (2004)	2.17	6.08
Bliss and Panigirtzoglou (2004)	0.947	9.502
Bakshi and Madan (2006)	12.71	17.33
Kang and Kim (2006)	2.086	5.719
Kang and Kim (2006)	0.1	0.7
Kang et al. (2010)	1.2	1.4
Barone-Adesi et al. (2014)	-0.5	3
Duan and Zhang (2014)	1.8	7.1
Skiadopoulos et al. (2019)	2.27	9.55