The CPW-cavity planar Penning trap

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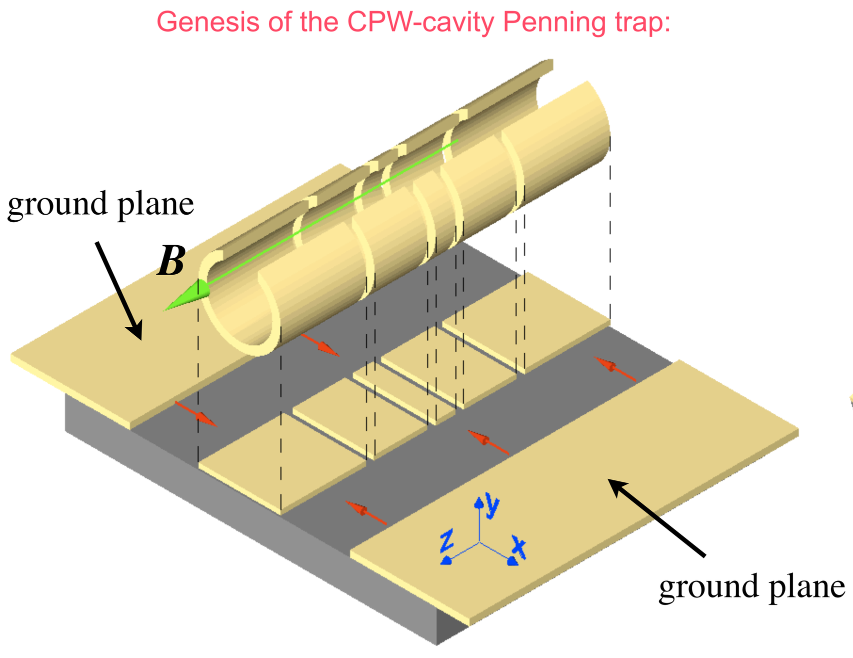
Introduction

The Penning trap is an essential tool in modern atomic physics. Some of the most precise measurements of nature constants are performed in Penning traps, such as the *g*-factor of the free electron, the masses of ions, etc. The dynamics of a trapped electron can be controlled with great accuracy, at the level of inducing quantum-jumps between the Fock-states of the trap[1]. Additionally, the spin can be coherently manipulated and monitored non-destructively, by means of the continuous Stern-Gerlach effect. A single electron can be trapped for months, highly isolated from the environment and protected against decoherence. Thus, electrons in cryogenic Penning traps are outstanding quantum laboratories and have been proposed for implementing a quantum processor[2]. We present a novel planar Penning trap design: *the CPW-cavity trap*. The trap results from the projection of the cylindrical trap onto a surface. The trap is also a planar microwave cavity, similar to those used in Circuit-QED experiments[3]. Moreover, the CPW-cavity behaves as an elliptical Penning trap[4,5], where the magnetron motion could be almost completely suppressed.

[1] L Brown *et al*, Rev Mod Phys **58**, 233 (1986)
[2] G Ciaramicoli *et al*, Phys Rev Lett **91**, 17901 (2003)
[3] A Wallraff *et al*, Nature (London) **431**, 162 (2004)
[4] M Breitenfeldt *et al*, Int J Mass Spectrom **275**, 34 (2008)
[5] M Kretzschmar, Int J Mass Spectrom **275**, 21 (2008)

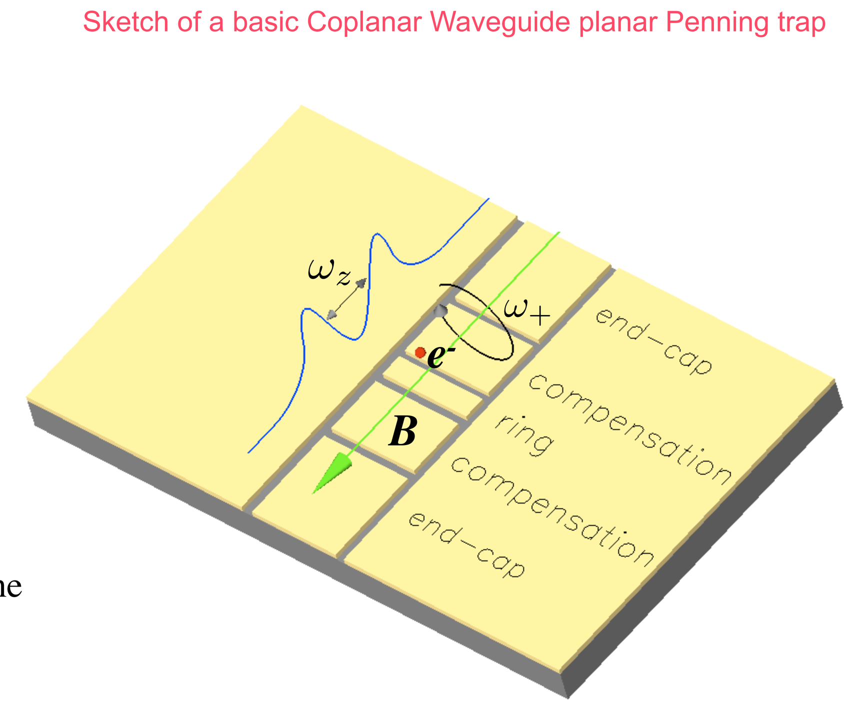
The coplanar waveguide-cavity Penning trap

Genesis of the CPW-cavity Penning trap:



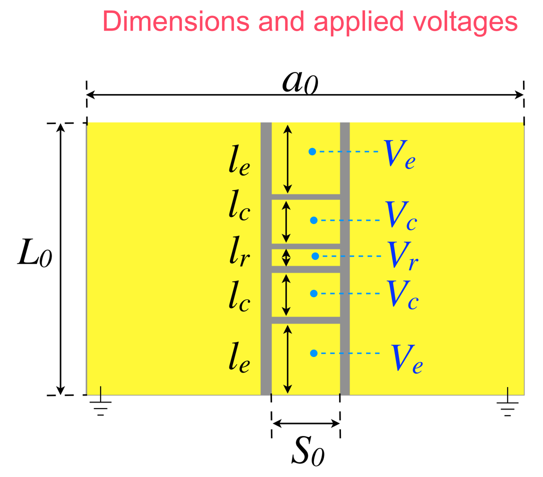
The CPW-cavity Penning trap results from the projection of the well-known cylindrical 3D trap onto a plane. The original five rings of the cylindrical trap become planar rectangular segments. Additionally, two external ground planes are added, for shielding and for setting a 0-voltage reference. The resulting structure is the *CPW-cavity Penning trap*.

Sketch of a basic Coplanar Waveguide planar Penning trap



The CPW-cavity Penning trap consists of two *end-caps*, two *compensation electrodes* and one central *"ring"*. Their task is similar to the equivalent ones of the 3D cylindrical trap. The magnetic field is parallel to the chip's surface, thus the axial potential is symmetric along $\boldsymbol{u_z}$. The trap forms a microwave transmission-line: a *Coplanar Waveguide* (CPW).

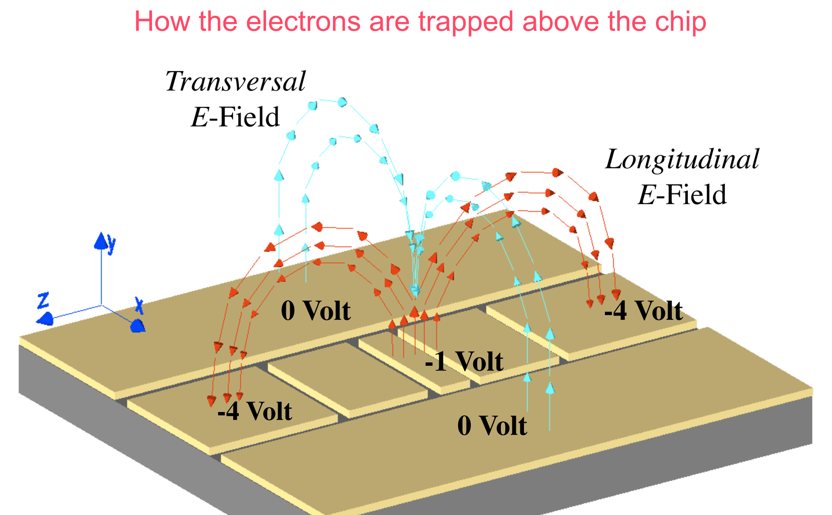
Dimensions and applied voltages



For trapping electrons the applied voltages must fulfill the condition:

$$V_e < V_c \sim V_r < 0$$

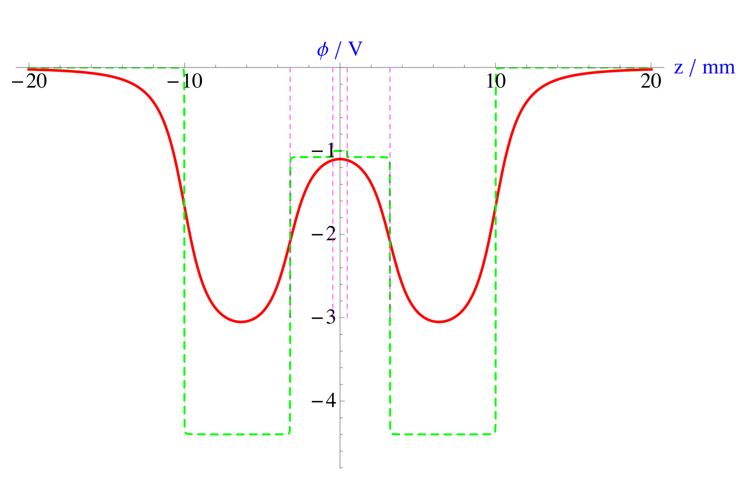
How the electrons are trapped above the chip

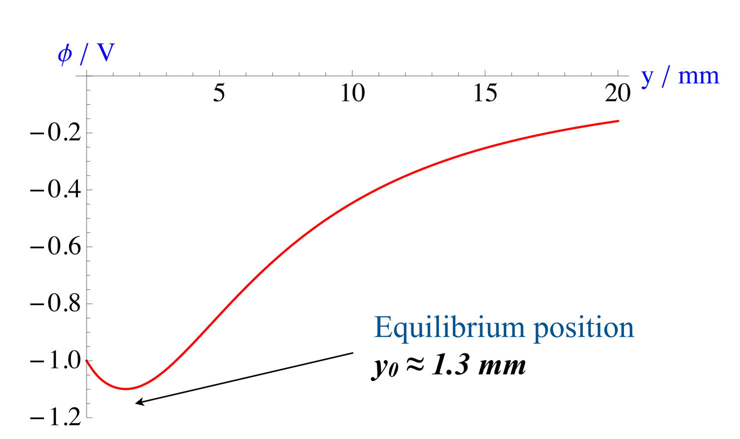


The "longitudinal" electric field points from the ring towards the end-caps while the "transversal" field points from the ground planes towards the ring. At some height $\boldsymbol{y_0}$ along the vertical axes $\boldsymbol{u_y}$ ($x = 0, z = 0$) both field components cancel. The electrons are trapped around the equilibrium position $\boldsymbol{y_0}$.

Example of the electrostatic trapping potential

Example (all mm and Volt): $l_e = 6.8\ ,\ l_c = 2.75\ ,\ l_r = 0.92\ ,\ S_0 = 7.0\ ,\ V_e = -4.4\ ,\ V_c = -1.0735\ ,\ V_r = -1.0$





The ideal elliptical Penning trap

The CPW-cavity behaves in general as an *elliptical Penning trap.* This is due to the asymmetry between the $\boldsymbol{x}$ and $\boldsymbol{y}$ coordinates, which results in a potential around the equilibrium position $\boldsymbol{y_0}$ of the form:

$$\phi_{quad}(x,y,z) = \underbrace{C_{002} \cdot \left(z^2 - \frac{x^2 + (y-y_0)^2}{2} \right)}_{\text{standard circular trap}} + \underbrace{\frac{1}{2} C_{002}\, \epsilon \cdot (x^2 - y^2)}_{\text{elliptical part}}$$

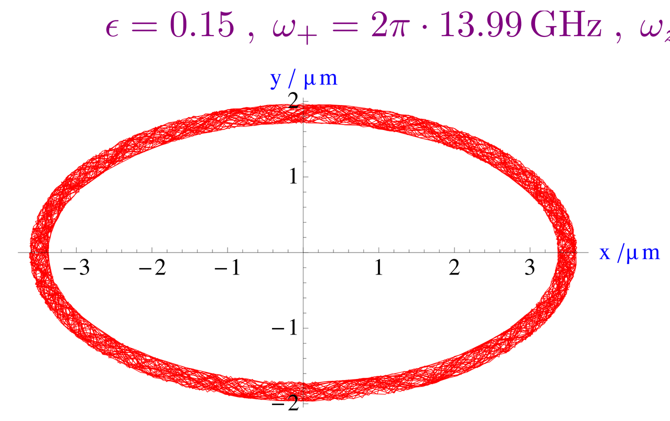
where the the coefficients C_{ijk} are given by: $C_{ijk} = \frac{1}{i!\,j!\,k!} \cdot \frac{\partial^{i+j+k} \phi(x,y,z)}{\partial x^i\, \partial y^j\, \partial z^k}\Big|_{(0,y_0,0)}$ and the ellipticity: $\epsilon = \frac{1-\xi}{1+\xi} \quad ; \quad \xi = \frac{C_{200}}{C_{020}}$

The ideal elliptical trap can be calculated analytically[5]. The motion of the particle is a superposition of three periodic oscillations: the cyclotron, axial and magnetron, with frequencies ω_+, ω_z and ω_-, respectively. The expressions for these eigenfrequencies are[4,5]:

$$\omega_+ = \sqrt{\frac{1}{2}(\omega_c^2 - \omega_z^2) + \frac{1}{2}\sqrt{\omega_c^2\,\omega_1^2 + \epsilon^2\,\omega_z^4}} \qquad \omega_- = \sqrt{\frac{1}{2}(\omega_c^2 - \omega_z^2) - \frac{1}{2}\sqrt{\omega_c^2\,\omega_1^2 + \epsilon^2\,\omega_z^4}} \qquad \omega_z = \sqrt{2\,C_{002}\,\frac{q}{m}} \qquad \text{with:} \quad \omega_c = \frac{q}{m}\cdot B \quad \text{and} \quad \omega_1 = \sqrt{\omega_c^2 - 2\,\omega_z^2}$$

The magnetron orbit of the electron is an ellipse, where the orientation of the major and minor axes (along $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively, or vice versa) depends on the sign of the ellipticity parameter ε. On the other hand, for any value of ε the cyclotron orbit is very closely circular. For the example calculated above (assuming $\boldsymbol{B}$ = 0.5 T and a temperature of T = 4.2 K) :

$$\epsilon = 0.15\ ,\ \omega_+ = 2\pi \cdot 13.99\,\text{GHz}\ ,\ \omega_z = 2\pi \cdot 21.6\,\text{MHz}\ ,\ \omega_- = 2\pi \cdot 16.5\,\text{kHz}$$



Plot of the radial motion of a single electron in a CPW-cavity trap. Magnetron sideband cooling has been assumed.The aspect ratio of the ellipse is 1.9. The amplitude of the axial oscillation (not visible in the plot) is 116 µm. The cyclotron amplitude is 0.13 µm.

Electric anharmonicities and compensation

In general a real elliptical trap differs considerably from the ideal case presented before. The electrostatic potential of the CPW-cavity contains several anharmonic terms which cause shifts of the motional frequencies ($\Delta\omega_+$, $\Delta\omega_z$, $\Delta\omega_-$). The shifts depend on the trapped particle's energies (E_+, E_z, E_-) and they can be big enough to make the detection of a single trapped electron impossible. Therefore, each anharmonicity C_{ijk} and its effect upon ω_+, ω_z and ω_- must be carefully considered. In particular, the Taylor series – up to the 4th order – of the potential around the equilibrium position (x = 0, $y = y_0$, z = 0) has the following expression:

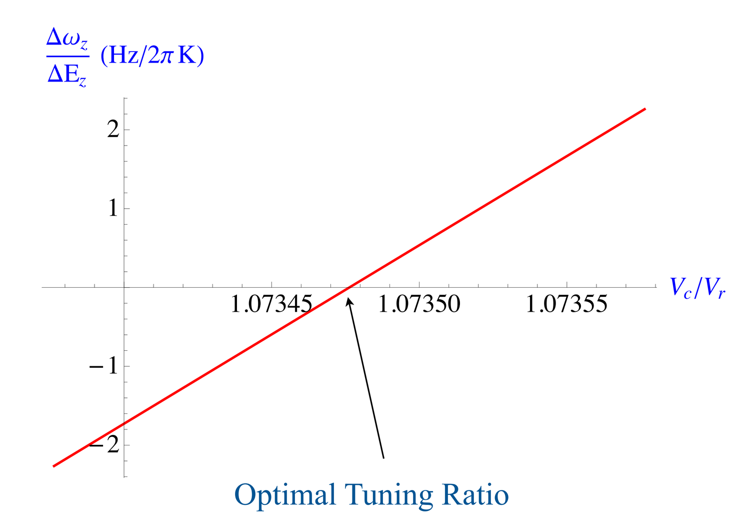
$$\begin{aligned} \phi(x,y,z) &= \phi(0,y_0,0) + \\ &+ C_{002}\,z^2 + C_{012}\,z^2(y-y_0) + C_{202}\,z^2\,x^2 + C_{022}\,z^2\,(y-y_0)^2 + C_{004}\,z^4 + \\ &+ C_{200}\,x^2 + C_{210}\,x^2\,(y-y_0) + C_{220}\,x^2\,(y-y_0)^2 + C_{400}\,x^4 + \\ &+ C_{020}\,(y-y_0)^2 + C_{030}\,(y-y_0)^3 + C_{040}\,(y-y_0)^4 + O((x,y,z)^5). \end{aligned}$$

The influence of each term can be calculated using classical canonical perturbation theory. Besides the C_{004} term, the most relevant one is the vertical asymmetry C_{012} (C_{003} has no effect upon ω_z). The respective shifts of the axial frequency are:

$$\frac{\Delta\omega_z}{\Delta E_z} = \frac{3}{4}\,\frac{\omega_z^0}{q}\,\frac{C_{004}}{C_{002}^2} \qquad\qquad \frac{\Delta\omega_z}{\Delta E_z} = -\frac{2\,q^2\,\eta_-^2}{m^3}\,\frac{C_{012}^2}{\omega_z^5}$$

C_{012} produces always a negative shift of ω_z. On the contrary, for C_{004} the sign of the shift depends on this term being positive or negative. This can be used to cancel the total dependence of ω_z upon E_z by tuning appropriately the compensation voltage V_c. This kind of compensation is in general different from making C_{004} = 0.

Compensation of the axial frequency shift



Global frequency-shift matrix

The coefficients up to 4th order produce linear frequency shifts. These can be summarized in a global frequency-shift matrix

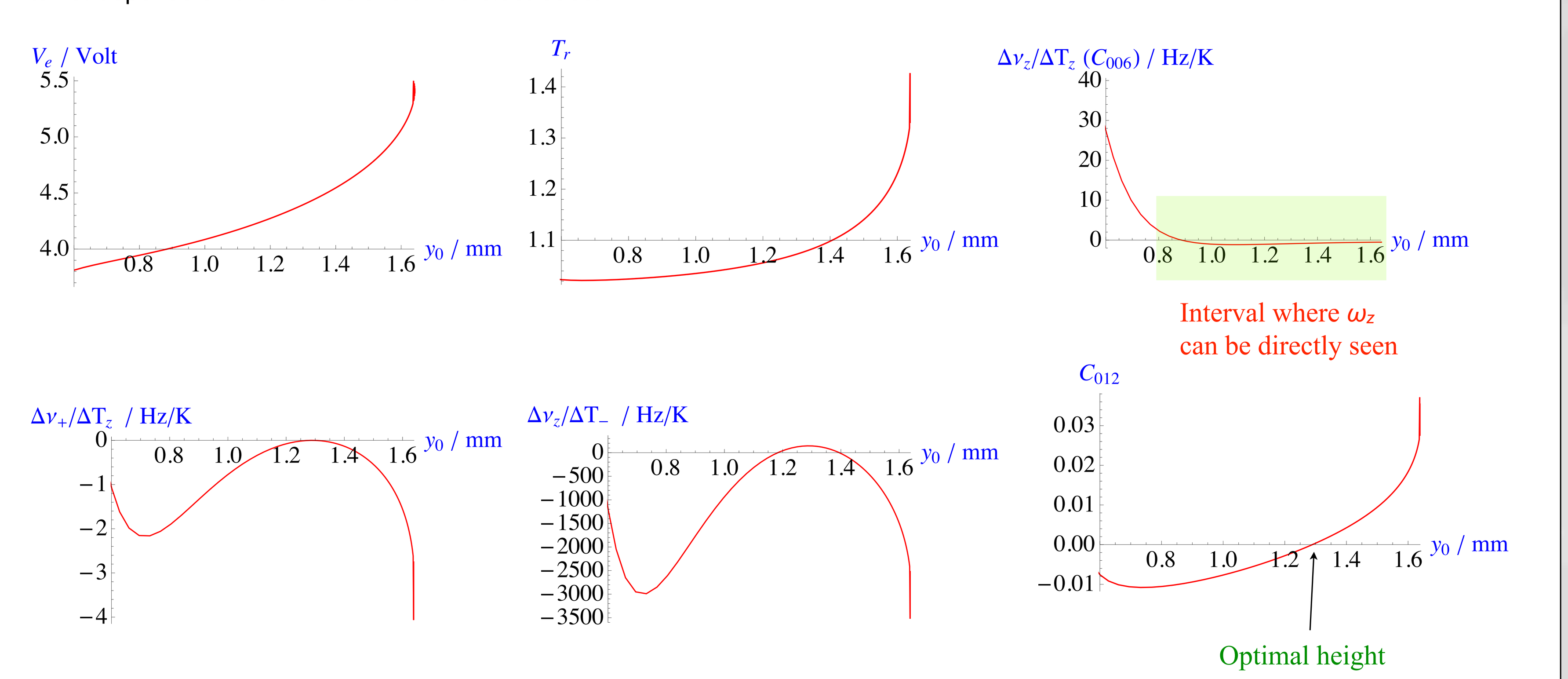
$$\begin{pmatrix} \Delta\omega_+ \\ \Delta\omega_z \\ \Delta\omega_- \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \cdot \begin{pmatrix} \Delta E_+ \\ \Delta E_z \\ \Delta E_- \end{pmatrix}$$

For the example (in *Hz/K*): $\begin{pmatrix} -2.2\cdot 10^{-6} & -0.002 & -2.07 \\ -3.3\cdot 10^{-6} & 0.0001 & 143 \\ -2.5\cdot 10^{-6} & 0.11 & -2.4 \end{pmatrix}$

The biggest shifts of the eigenfrequencies are due to the magnetron energy, E_-, which is of the order of a few mK (after applying magnetron-axial sideband coupling). The shifts due to E_+ are always negligible.

Variation of the trapping height $\boldsymbol{y_0}$

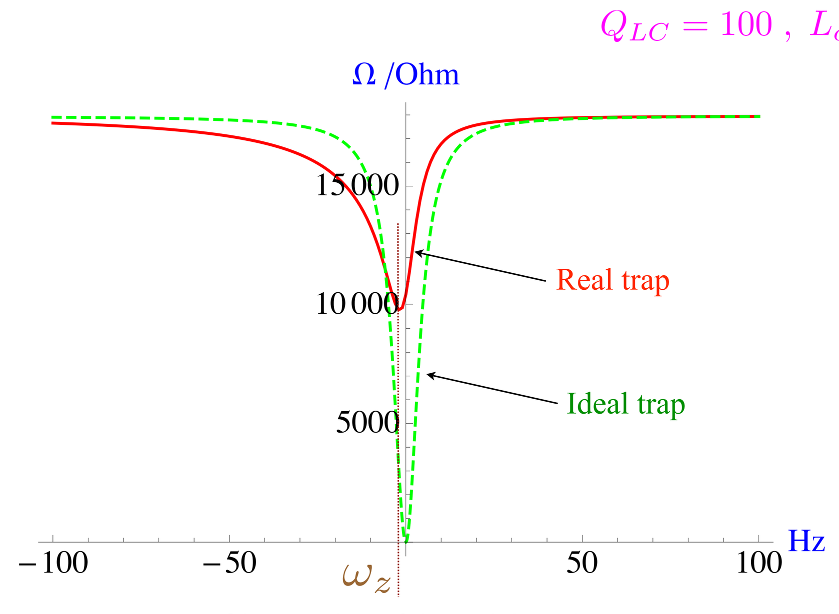
Changing the voltage applied to the end-caps, V_e, the trapping height y_0 can be tuned. For each height, the corresponding tuning ratio has to be adjusted. The trap can be compensated over a large interval of heights (~ 1 mm in the example). The span and range of that interval depends on the dimensions of the electrodes.



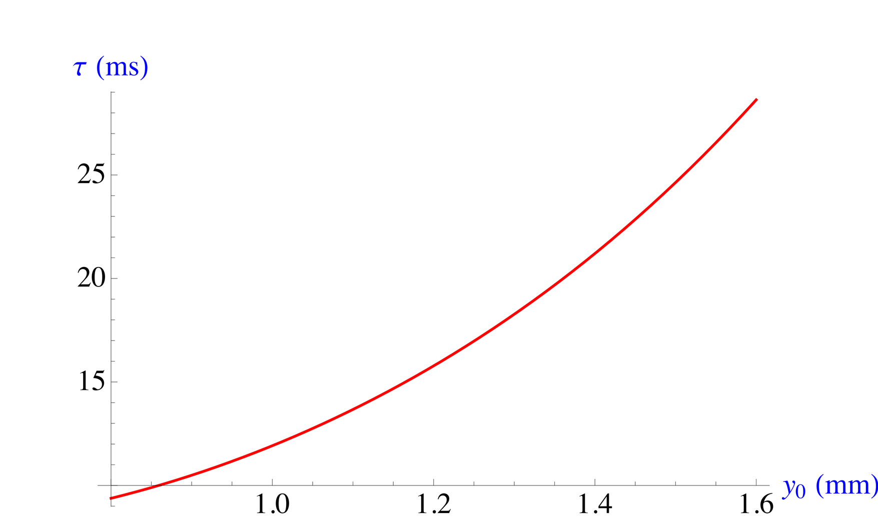
Axial detection of a single electron

The axial motion can be observed through the detection of the charges induced upon the compensation electrodes. The induced current flows along a LC-circuit, resonantly tuned with the axial frequency, and the electron motion "shortcuts" the resonance spectrum of the LC. The interaction of the electron with the LC-circuit also results in *resistive cooling* of the axial motion.

$$Q_{LC} = 100\ ,\ L_{coil} = 1.3\,\mu\text{H}$$



Observation of the axial motion



The cooling time constant as a function of the trapping height

General properties of the CPW-cavity Penning trap

- The trap can be operated with the electron positioned at different heights.
- For each height the optimal Tuning Ratio is different and must be adjusted.
- Above a maximum height and below a minimum one no optimal Tuning Ratio exists.
- At one particular height the vertical asymmetry C_{012} cancels. That height depends on the dimensions of the electrodes and might not exist for a particular trap geometry.
- For each height, the axial frequency can be tuned by rescaling the applied voltages while keeping the Tuning Ratio and the ratio V_e/V_r constant.
- The range of heights at which the trap can be operated is limited also by non-linear frequency shifts (C_{006}).
- The trap is a planar microwave cavity, such as those used in novel Circuit-QED experiments.
- The cyclotron frequency can be directly measured, observing the resonance of the microwave cavity.

Outlook

At the University of Sussex we are developing a new experimental set-up for the novel CPW-cavity planar Penning trap presented here. We plan to use a closed-cycle Pulse Tube Cooler to develop a cryogenic CPW-cavity trap, where electrons at 4.2 K will be captured. The basic theory of the CPW-cavity trap is about to be submitted by *New Journal of Physics.*

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