Smooth composite pulses in coherent atomic excitation

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Abstract

We present a systematic theoretical approach for construction of an arbitrarily flat excitation profile in a two-level system using composite pulses. These consist of a sequence or pulses having specific relative phases. If the phases are chosen appropriately, they nullify the first few terms in an expansion versus a desired parameter. In such way a flat profile is accomplished.

1. Introduction

A two-state coherently-driven quantum system is described by the Schrödinger equation

$$\mathrm{i}\hbar\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{c}(t) = \mathbf{H}(t)\mathbf{c}(t),$$

where $\mathbf{c}(t) = [c_1(t), c_2(t)]^T$ is a vector-column containing the two probability amplitudes of the two states. The Hamiltonian is

$$\mathbf{H}(t) = \frac{\hbar}{2}\begin{bmatrix} 0 & \Omega(t)\,\mathrm{e}^{-\mathrm{i}D(t)} \\ \Omega(t)^*\,\mathrm{e}^{\mathrm{i}D(t)} & 0 \end{bmatrix},$$

with $D(t) = \int_{t_i}^{t}\Delta(t')\,\mathrm{d}t'$, where $\Delta = \omega_0 - \omega$ is the detuning between the laser carrier frequency ω and the Bohr transition frequency ω_0. The Rabi frequency $\Omega(t) = -\mathbf{d}.\mathbf{E}(t)/\hbar$ parameterizes the coupling between the electric field with an envelope $\mathbf{E}(t)$ and the transition dipole moment $\mathbf{d}$ of the system.
The evolution of the amplitudes is usually described by the propagator $\mathbf{U}$, which is defined as the operator, connecting the initial amplitudes $\mathbf{c}(t_i)$ with the final $\mathbf{c}(t_f)$,

$$\mathbf{c}(t_f) = \mathbf{U}(t_f, t_i)\mathbf{c}(t_i).$$

Resonance model

On resonance the Schrödinger equation has a solution for arbitrary shape of the pulse, and the propagator is

$$\mathbf{U}(t,-\infty) = \begin{bmatrix} \cos\left(A(t)/2\right) & -\mathrm{i}\sin\left(A(t)/2\right) \\ -\mathrm{i}\sin\left(A(t)/2\right) & \cos\left(A(t)/2\right) \end{bmatrix},$$

where

$$A(t) = \int_{-\infty}^{t}\Omega(t')\,\mathrm{d}t'$$

and the transition probability is $P = |U_{21}|^2 = \sin^2(A/2)$. For $A = \pi$ we have complete population inversion (CPI), $P = 1$. However, if the pulse area deviates from π with α, we have

$$P = 1 - \alpha^2/4 + \mathcal{O}(\alpha^4).$$

2. Composite pulses

We introduce a constant phase shift ϕ in the Rabi frequency $\boxed{\Omega(t) \to \Omega(t)\,\mathrm{e}^{\mathrm{i}\phi}}$ and the amplitudes in the Schrödinger equation change like $\boxed{\mathbf{c}(t) \to \Phi\mathbf{c}(t)}$, with

$$\Phi = \begin{bmatrix} \mathrm{e}^{\mathrm{i}\phi/2} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\phi/2} \end{bmatrix}$$

Then the phase shifted propagator will be

$$\mathbf{U}_\phi = \mathbf{\Phi}^\dagger\mathbf{U}\mathbf{\Phi} = \begin{bmatrix} a & b\,\mathrm{e}^{-\mathrm{i}\phi} \\ -b^*\,\mathrm{e}^{\mathrm{i}\phi} & a^* \end{bmatrix}$$

We consider a composite pulse consisted of a sequence of three resonant pulses, each with a pulse area equal to $\pi + \alpha$, and a phase $\phi_i\,(i = 1, 2, 3)$. We demand a symmetric excitation profile which leads to $\phi_1 = \phi_3$, and since the global phase has no physical meaning, we will take these for zero: $\phi_1 = \phi_3 = 0$. The phase in the second pulse is $\phi_2 = \phi$.

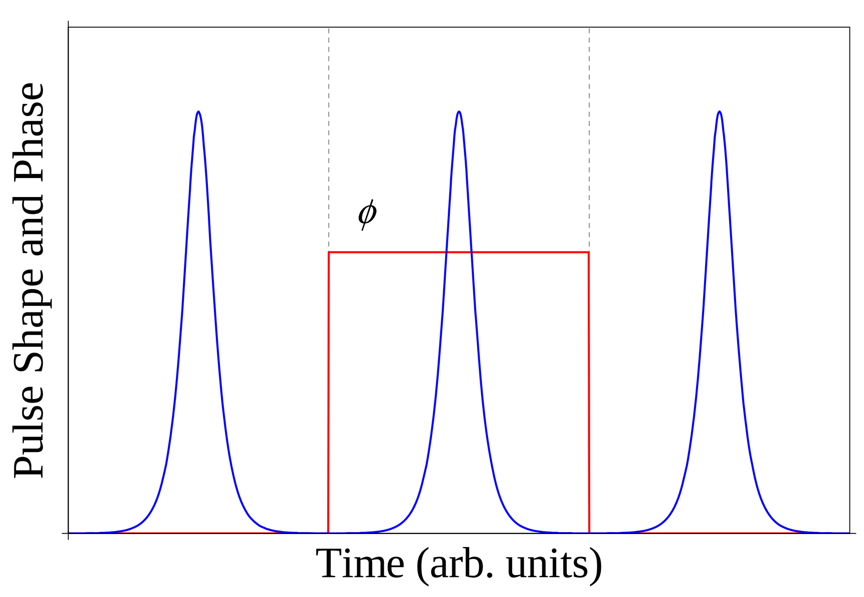

Figure 1: *A sequence of three pulses, the second one having a relative phase* ϕ.

We calculate the total propagator by multiplying the phased propagators for each pulse,

$$\mathbf{U}_{\text{tot}} = \mathbf{U}_0\mathbf{U}_\phi\mathbf{U}_0 = \begin{bmatrix} a_{\text{tot}} & b_{\text{tot}} \\ -b^*_{\text{tot}} & a^*_{\text{tot}} \end{bmatrix},$$

where

$$a_{\text{tot}} = \sin(\alpha/2)\left[\cos\phi + \cos\alpha\,(1+\cos\phi)\right]$$
$$b_{\text{tot}} = \mathrm{i}\cos(\alpha/2)\left[\mathrm{i}\sin\phi - 1 + \cos\alpha\,(1+\cos\phi)\right]$$

We expand a_{tot} around $\alpha = 0$ and nullify the first term in the expansion:

$$a_{\text{tot}} = \left(\tfrac{1}{2} + \cos\phi\right)\alpha + \mathcal{O}(\alpha^3) \quad\Longrightarrow\quad \boxed{\phi = \pm 2\pi/3}$$

The transition probability for $\phi = 2\pi/3$ is

$\boxed{P_{\text{tot}} = 1 - \alpha^6/64 + \mathcal{O}(\alpha^8)}$, better than $\boxed{P_{\text{single}} = 1 - \alpha^2/4 + \mathcal{O}(\alpha^4)}$

We can apply the same procedure for a sequence of N pulses and we obtain the following phases:

Table 1: *Phases, measured in* π/N*, for different number of pulses* N.

$N=1$	0
$N=3$	0 2 0
$N=5$	0 4 2 4 0
$N=7$	0 6 4 8 4 6 0
$N=9$	0 8 6 12 8 12 6 8 0
$N=11$	0 10 8 16 12 18 12 16 8 10 0
$N=13$	0 12 10 20 16 24 18 24 16 20 10 12 0
$\vdots$	$\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$ $\vdots$

The relative phases fulfill the following formula (in units π/N):

$$\phi_k^N = \phi_k^{N-1} + 2\left\lfloor\frac{k}{2}\right\rfloor$$

The symmetric-profile condition imposes

$$\phi_k^N = \phi_{N-k+1}^N, \quad \phi_1^N = \phi_N^N = 0$$

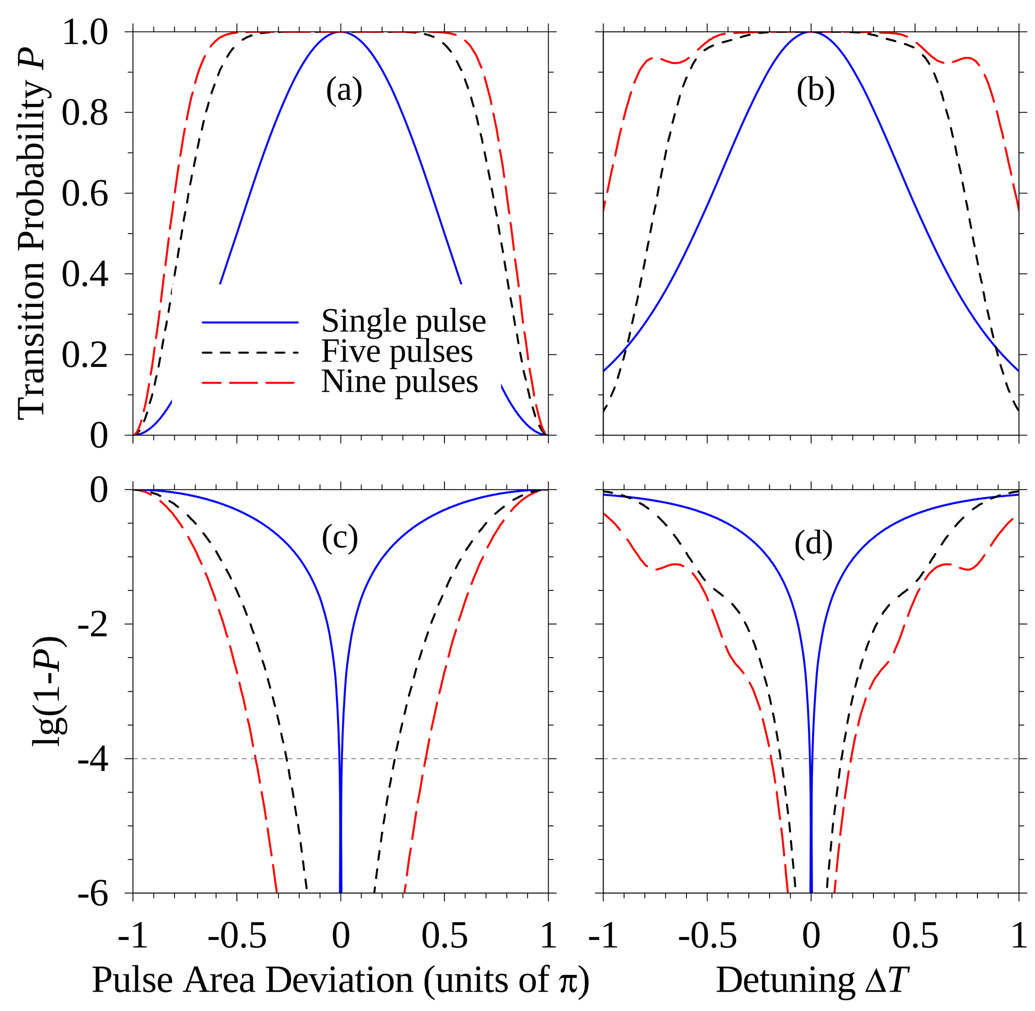

Figure 2: *Transition probability for a single hyperbolic-secant pulse and for a sequence of five and nine pulses versus (a): pulse area deviation, with relative phases as given in Table 1 and (b): detuning, where the relative phases for five pulses are* $(0, 1, -2, 1, 0)\pi/6$*, and for nine pulses have the approximate numerical values* $(0, -2.72, 4.49, 3.73, 1.30, 3.73, 4.49, -2.72, 0)$*. Frames (c) and (d) show the common logarithm of* $(1-P)$*, which is an estimate of the error of the corresponding sequence of pulses.*

3. Non-resonant excitation

We can apply the same method for non-resonant models to obtain excitation profile, which is flat versus the detuning. For instance, we consider the Rosen-Zener model, where

$$\Omega(t) = \Omega_0\,\text{sech}\,(t/T), \quad \Delta(t) = \Delta_0 = \text{const}$$

The parameters in the propagator for this model are

$$a = \frac{\Gamma^2\left(\frac{1}{2} + \mathrm{i}\delta\right)}{\Gamma\left(\frac{1}{2} + \mathrm{i}\delta - \alpha\right)\Gamma\left(\frac{1}{2} + \mathrm{i}\delta + \alpha\right)}$$
$$b = -\mathrm{i}\frac{\sin\pi\alpha}{\cosh\pi\delta}$$

The transition probability is

$$P = \frac{\sin^2\pi\alpha}{\cosh^2\pi\delta} = 1 - \pi^2\delta^2 + \mathcal{O}(\delta^4)$$

For a sequence of three pulses, with $\phi = \pm\pi/3$, we obtain

$$P = 1 - (4\pi^2\ln^2 2)\delta^4 + \mathcal{O}(\delta^6)$$

This method allows also obtaining a profile, which is insensitive both to pulse area and detuning. In this case we need to nullify terms in the Taylor expansion in respect to both parameters.

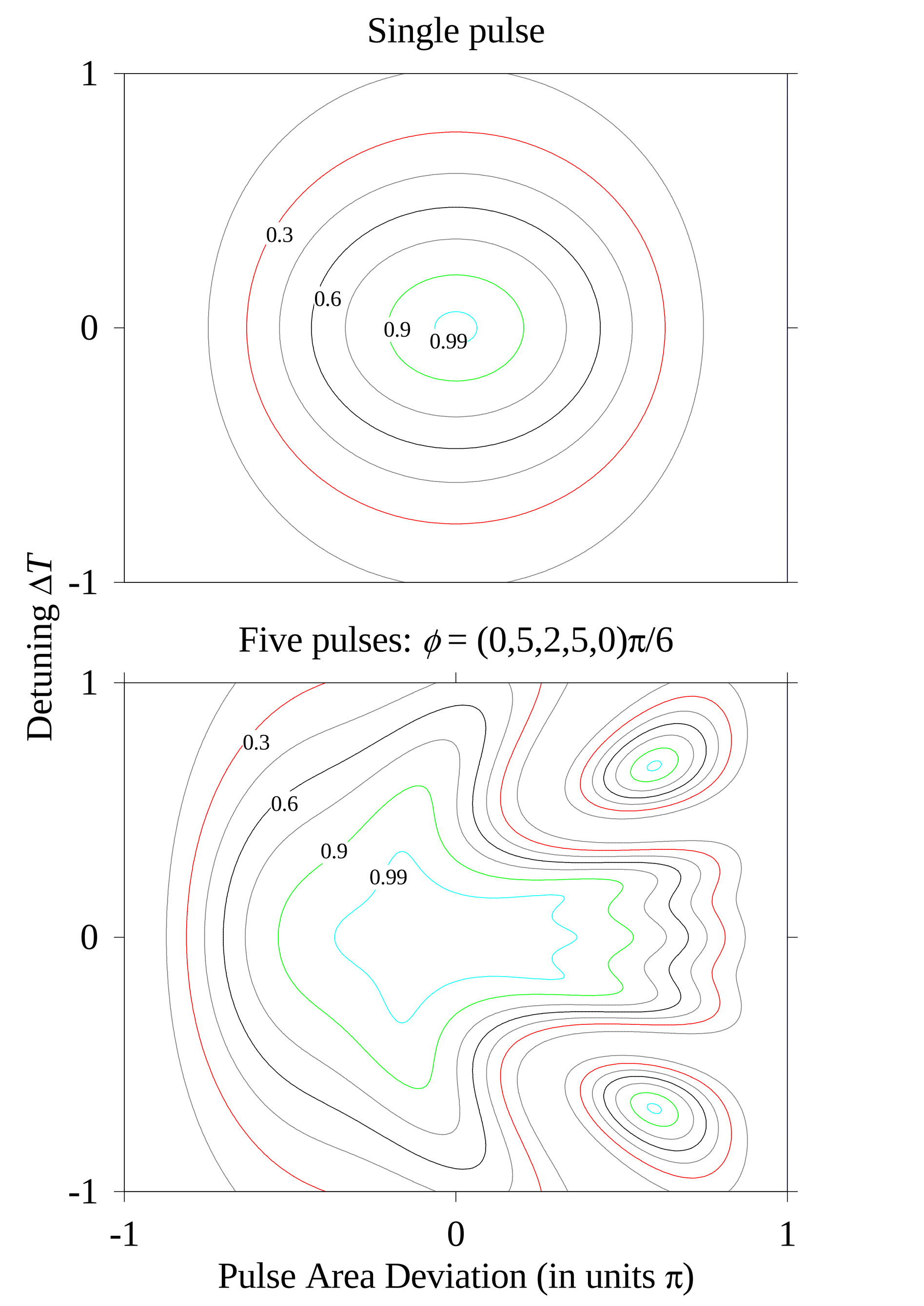

Figure 3: *Transition probability versus detuning and pulse area deviation for a single pulse (upper frame) and for a five-pulses sequence (lower frame), where the relative phases are* $(0, 5, 2, 5, 0)\pi/6$.

The described technique can be applied for any pulse shape. If we do not have an exact solution for the propagator, than a numerical interpolation is used.

4. Excitation with chirped composite pulses

Chirped pulses are usually used in order to obtain complete population inversion (CPI) by adiabatic following (RAP). Examples of chirped models are:

$$\boxed{\begin{aligned}\Omega(t) &= \Omega_0\,\text{sech}\,(t/T)\\ \Delta(t) &= B\tanh(t/T)\end{aligned}} \qquad \boxed{\begin{aligned}\Omega(t) &= \Omega_0\,\mathrm{e}^{-t^2/T^2}\\ \Delta(t) &= Ct\end{aligned}}$$

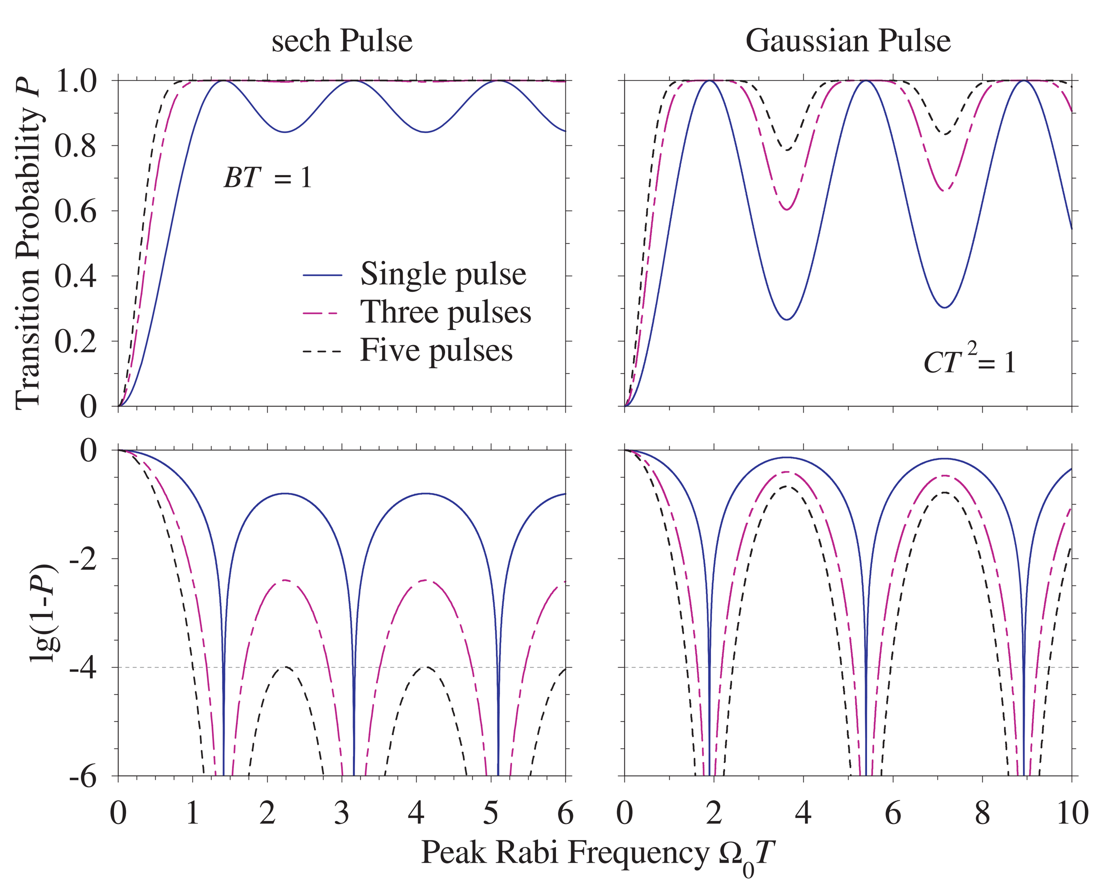

Figure 4: *Transition probabilities for chirped pulses.*

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