Friedrich-Alexander-Universität Erlangen-Nürnberg

SAOT erlangen graduate school in advanced optical technologies

Universidad del País Vasco Euskal Herriko Unibertsitatea

ULg UNIVERSITÉ de Liège

Generation of Total Angular Momentum Eigenstates in Remote Qubits

Andreas Maser[1,*], Uwe Schilling[1], Thierry Bastin[2], Enrique Solano[3], Christoph Thiel[1], Joachim von Zanthier[1,†]

[1]Institut für Optik, Information und Photonik, Friedrich-Alexander Universität Erlangen-Nürnberg, Erlangen, Germany
[2]Institut de Physique Nucléaire, Atomique et de Spectroscopie, Université de Liège au Sart Tilman, Liège, Belgium
[3]Departamento de Química-Física, Universidad del País Vasco - Euskal Herriko Unibertsitatea, Bilbao, Spain

*email: andreas.maser@physik.uni-erlangen.de †email: Joachim.vonZanthier@physik.uni-erlangen.de

Abstract:

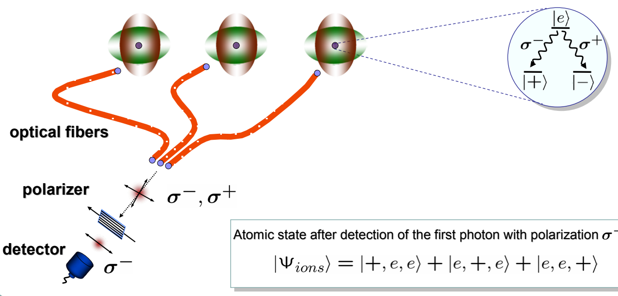
We propose a method which allows to couple N remote single photon emitters with a Λ-configuration to form any symmetric N-qubit state [1,2] or any (symmetric or non-symmetric) total angular momentum eigenstates of the N-qubit compound [3], using linear optics only.

References:

[1] C.Thiel *et al.*, Phys. Rev. Lett. **99**, 193602 (2007)
[2] T. Bastin *et al.*, Phys. Rev. Lett. **102**, 053601 (2009)
[3] A. Maser *et al.*, Phys. Rev. A **79**, 033833 (2009)
[4] A. Maser *et al.*, Phys. Rev. A **81**, 053842 (2010)

I. How to Entangle Remote Emitters by Projection

Example for three trapped ions

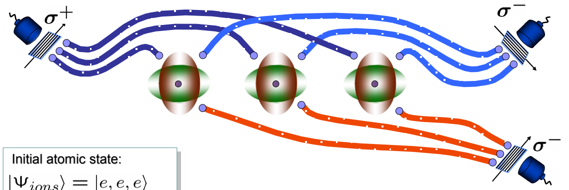


Atomic state after detection of the first photon with polarization σ^-

$$|\Psi_{ions}\rangle = |+,e,e\rangle + |e,+,e\rangle + |e,e,+\rangle$$

II. Generation of long-living W-state

Detection of three photons



Initial atomic state:

$$|\Psi_{ions}\rangle = |e,e,e\rangle$$

Detection of a σ⁻-polarized photon

$$|\Psi_{ions}\rangle = |+,e,e\rangle + |e,+,e\rangle + |e,e,+\rangle$$

Detection of a σ⁻-polarized photon

$$|\Psi_{ions}\rangle = |+,+,e\rangle + |+,e,+\rangle + |e,+,+\rangle$$

Detection of a σ⁺-polarized photon

$$|\Psi_{ions}\rangle = |+,+,-\rangle + |+,-,+\rangle + |-,+,+\rangle$$

III. Generation of any symmetric state

Usage of elliptical polarizers allows the generation of any symmetric N-qubit state

Any symmetric N-qubit state $|\psi\rangle$ can be expressed as a sum of symmetric Dicke states $|D_N(k)\rangle$, with $|D_N(k)\rangle$ being the symmetric Dicke state with k $|+\rangle$ excitations.

$$|\psi\rangle = \sum_{k=0}^{N} c_k |D_N(k)\rangle$$

Example: 3-qubit Dicke states

$$|D_3(0)\rangle = |---\rangle$$
$$|D_3(1)\rangle = \frac{1}{\sqrt{3}}(|+--\rangle + |-+-\rangle + |--+\rangle)$$
$$|D_3(2)\rangle = \frac{1}{\sqrt{3}}(|++-\rangle + |+-+\rangle + |-++\rangle)$$
$$|D_3(3)\rangle = |+++\rangle$$

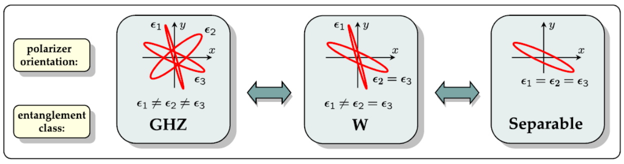
The coefficients c_k are used to construct the polynomial $P(z)$.

$$P(z) = \sum_{k=0}^{N} (-1)^{N-k} \binom{N}{k}^{\frac{1}{2}} c_k z^k$$

The roots z_i of this polynomial define the polarizer orientations $\epsilon_i = \alpha_i\sigma^+ + \beta_i\sigma^-$.

$$z_i = \frac{\alpha_i}{\beta_i}$$

In case of a 3-qubit state there is furthermore a simple correspondence between polarizer orientation and the entanglement class of the generated state [2].



IV. Generation of all total angular momentum eigenstates

Explanation of the algorithm: mimicking the coupling of angular momentum

Total angular momentum (TAM) eigenstates $|S,m\rangle$ are defined as the eigenstates of the total angular momentum operator $\hat{S}$ and its z-component $\hat{S}_z$.

$$\hat{\mathbf{S}}^2 |S,m\rangle = S(S+1)\hbar^2 |S,m\rangle$$
$$\hat{S}_z |S,m\rangle = m\hbar |S,m\rangle$$

Generally, N qubits can be coupled to 2^N different quantum states.

1. Example: 2-qubit system

spin-1 triplet	$\|S,m\rangle$
$\|++\rangle$	$\|1,1\rangle$
$\frac{1}{\sqrt{2}}(\|+-\rangle + \|-+\rangle)$	$\|1,0\rangle$
$\|--\rangle$	$\|1,-1\rangle$

spin-0 singlet	$\|S,m\rangle$
$\frac{1}{\sqrt{2}}(\|+-\rangle - \|-+\rangle)$	$\|0,0\rangle$

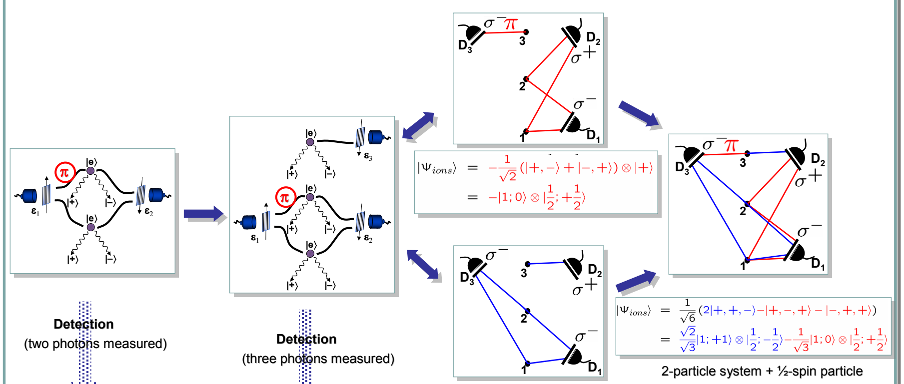
2. Example: 3-qubit system

spin-$\frac{3}{2}$ quartet	$\|S,m\rangle$
$\|+++\rangle$	$\|\frac{3}{2},+\frac{3}{2}\rangle$
$\frac{1}{\sqrt{3}}(\|++-\rangle + \|+-+\rangle + \|-++\rangle)$	$\|\frac{3}{2},+\frac{1}{2}\rangle$
$\frac{1}{\sqrt{3}}(\|+--\rangle + \|-+-\rangle + \|--+\rangle)$	$\|\frac{3}{2},-\frac{1}{2}\rangle$
$\|---\rangle$	$\|\frac{3}{2},-\frac{3}{2}\rangle$

We exemplify our method for this state !

first spin-$\frac{1}{2}$ doublet	$\|S,m\rangle$
$\frac{1}{\sqrt{6}}(2\|++-\rangle - \|+-+\rangle - \|-++\rangle)$	$\|\frac{1}{2},+\frac{1}{2}\rangle$
$\frac{1}{\sqrt{6}}(\|+--\rangle + \|-+-\rangle - 2\|--+\rangle)$	$\|\frac{1}{2},-\frac{1}{2}\rangle$

second spin-$\frac{1}{2}$ doublet	$\|S,m\rangle$
$\frac{1}{\sqrt{2}}(\|+-+\rangle - \|-++\rangle)$	$\|\frac{1}{2},+\frac{1}{2}\rangle$
$\frac{1}{\sqrt{2}}(\|+--\rangle - \|-+-\rangle)$	$\|\frac{1}{2},-\frac{1}{2}\rangle$



$$|\Psi_{ions}\rangle = -\frac{1}{\sqrt{2}}(|+,-\rangle + |-,+\rangle) \otimes |+\rangle = -|1;0\rangle \otimes |\tfrac{1}{2};+\tfrac{1}{2}\rangle$$

$$|\Psi_{ions}\rangle = 2|+,+\rangle \otimes |-\rangle = 2|1;+1\rangle \otimes |\tfrac{1}{2};-\tfrac{1}{2}\rangle$$

Pick out two particular setups

$$|\Psi_{ions}\rangle = \frac{1}{\sqrt{6}}(2|+,+,-\rangle - |+,-,+\rangle - |-,+,+\rangle) = \frac{\sqrt{2}}{\sqrt{3}}|1;+1\rangle \otimes |\tfrac{1}{2};-\tfrac{1}{2}\rangle - \frac{1}{\sqrt{3}}|1;0\rangle \otimes |\tfrac{1}{2};+\tfrac{1}{2}\rangle$$

2-particle system + ½-spin particle
=
3-particle system

Detection
(two photons measured)

ϵ_1	ϵ_2	$\|state\rangle$
σ^-	σ^-	$\|++\rangle$
σ^-	σ^+	$\|+-\rangle + \|-+\rangle$
σ^-	σ^+	$\|+-\rangle - \|-+\rangle$
σ^+	σ^+	$\|--\rangle$

Including π shift

Detection
(three photons measured)

ϵ_1	ϵ_2	ϵ_3	$\|state\rangle$
σ^-	σ^-	$\sigma^\mp$	$\|++\rangle \otimes \|\pm\rangle$
σ^-	σ^+	$\sigma^\mp$	$(\|+-\rangle + \|-+\rangle) \otimes \|\pm\rangle$
σ^-	σ^+	$\sigma^\mp$	$(\|+-\rangle - \|-+\rangle) \otimes \|\pm\rangle$
σ^+	σ^+	$\sigma^\mp$	$\|--\rangle \otimes \|\pm\rangle$

Including π shift

➡ This approach corresponds to a successive coupling of angular momenta.

➡ We have found an algorithm that transforms the coupling of angular momenta into explicit experimental setups.

V. From symmetric Dicke states to any N-qubit state

Results and future research

Symmetric Dicke states [1]:
Generation of Symmetric Dicke States of Remote Qubits with Linear Optics

All symmetric states [2]:
Operational Determination of Multiqubit Entanglement Classes via Tuning of Local Operations

Total Angular Momentum Eigenstates [3]:
Generation of total angular momentum eigenstates in remote qubits

Minor changes allow the generation of entangled photonic qubits using single photon sources (e.g., trapped atoms/ions):

Method for the generation of polarization entangled photons [4]:
Versatile source of polarization-entangled photons

The total angular momentum eigenstates form a basis of the Hilbertspace of an N-qubit system. So far only superpositions of the symmetric basis states are possible.

➡ **Generalization of this method may lead to the generation of *any* N-qubit state**