Quantum simulation of the Klein paradox with trapped ions

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Talk overview:

- Relativistic quantum mechanics basics
- Mapping to ion trap system
- Results

Relativistic quantum mechanics basics

Dirac equation

$$i\hbar\frac{\partial\psi}{\partial t} = \left(c\alpha\cdot\hat{p} + \beta mc^2\right)\psi$$

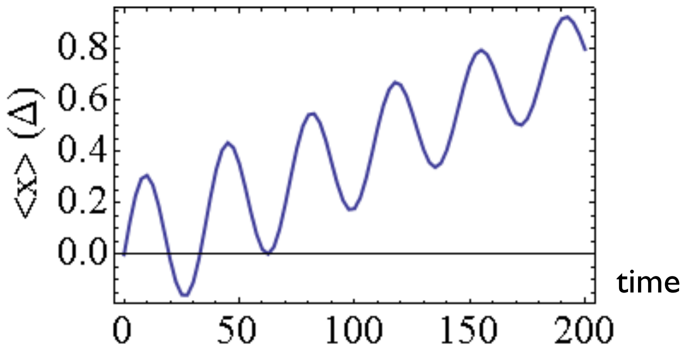
- Unites QM and relativity
- Predicts spin and anti-matter

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

$$\psi = \frac{e^{-\frac{x^2}{4\Delta^2}}}{\left(\sqrt{2\pi}2\Delta\right)^{\frac{1}{2}}}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

Predictions

Zitterbewegung

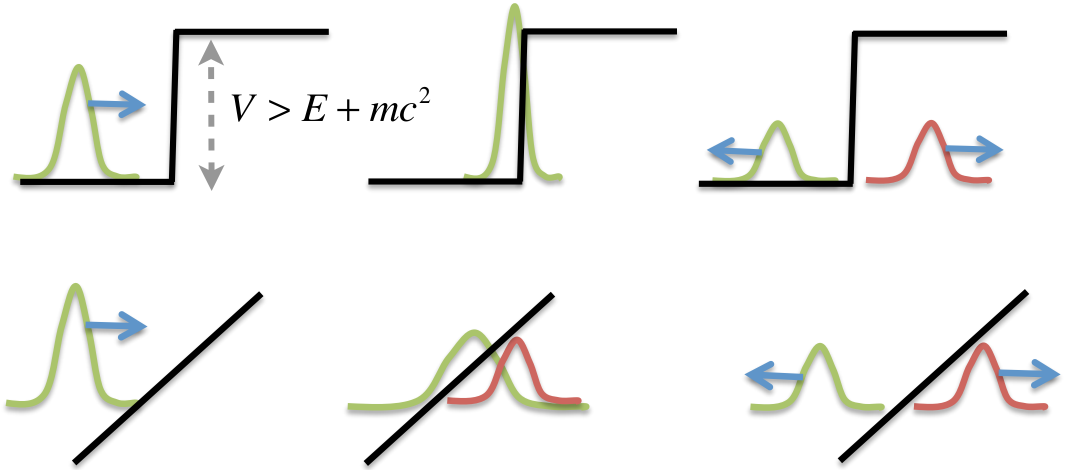


Gerritsma R., *et al.*
Nature **463**, 68, 2010.

Klein paradox

$$i\hbar\frac{\partial\psi}{\partial t} = \left(c\hat{p}\sigma_x + mc^2\sigma_z + q\phi\right)\psi$$

electric potential

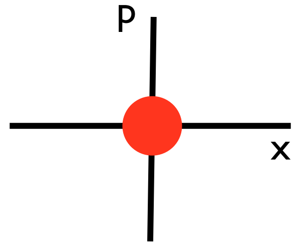


1D scattering problems already non-trivial on a classical computer (hundreds of dimensions)

How to simulate this stuff with trapped ions

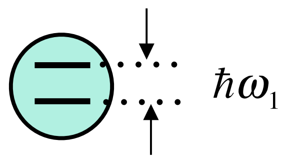
Mapping the Hilbert space

$$\psi = \frac{e^{-\frac{x^2}{4\Delta^2}}}{\left(\sqrt{2\pi}\,2\Delta\right)^{\frac{1}{2}}} \begin{pmatrix} a \\ b \end{pmatrix} \longleftrightarrow$$



COM vibrational mode of ion in harmonic trap

$\otimes$



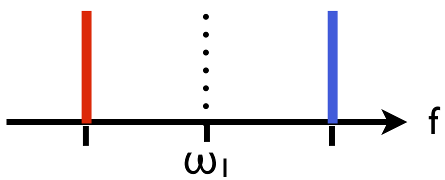
Ca^{43} ion optical qubit

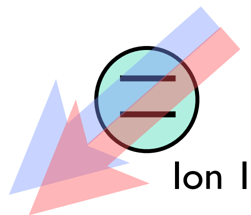
$$S_{1/2,m=1/2} \rightarrow D_{5/2,m=3/2}$$

Simulating the dynamics of a free particle

$$H = c\hat{p}\sigma_x + mc^2\sigma_z$$

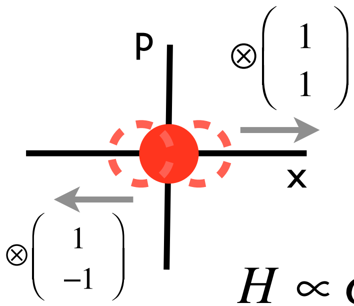
Bichromatic laser



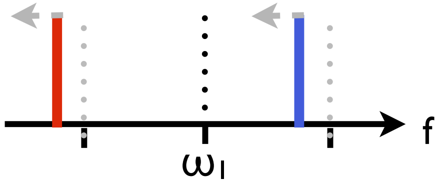


$$H = \hbar\eta\tilde{\Omega}\left(\sigma_x\cos\phi_+ - \sigma_y\sin\phi_+\right)\otimes\left(\underbrace{\left(a^\dagger + a\right)}_{\hat{x}}\cos\phi_- - i\underbrace{\left(a^\dagger - a\right)}_{\hat{p}}\sin\phi_-\right)$$

tool for state dependent ion displacement



$$H \propto \sigma_x\hat{p}$$



$$H_{bichr1} \sim \sigma_x\hat{p} + \sigma_z$$

$$H_{bichr1} = 2\eta\tilde{\Omega}\Delta\sigma_x\hat{p} + \hbar\Omega\sigma_z$$

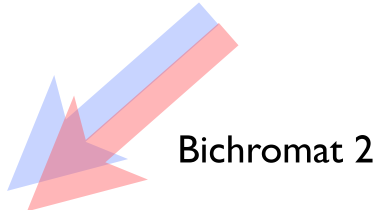
$$mc^2 := \hbar\Omega \qquad c := 2\eta\Delta\tilde{\Omega}$$

Simulation with trapped ions

Simulating a linear external electric potential

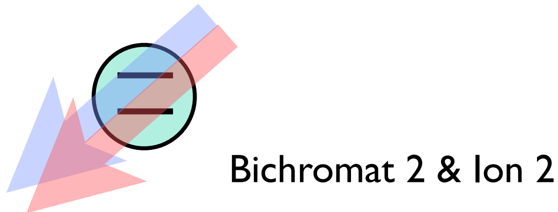
$$H = c\hat{p}\hat{\sigma}_x + mc^2\hat{\sigma}_z + q\hat{x}$$

•Add another bichromatic laser with a different phase setting



$$H_{bi2} = \hbar\eta\tilde{\Omega}\sigma_x\hat{x}$$

•Add another ion prepared in +1 eigenstate of σ_x



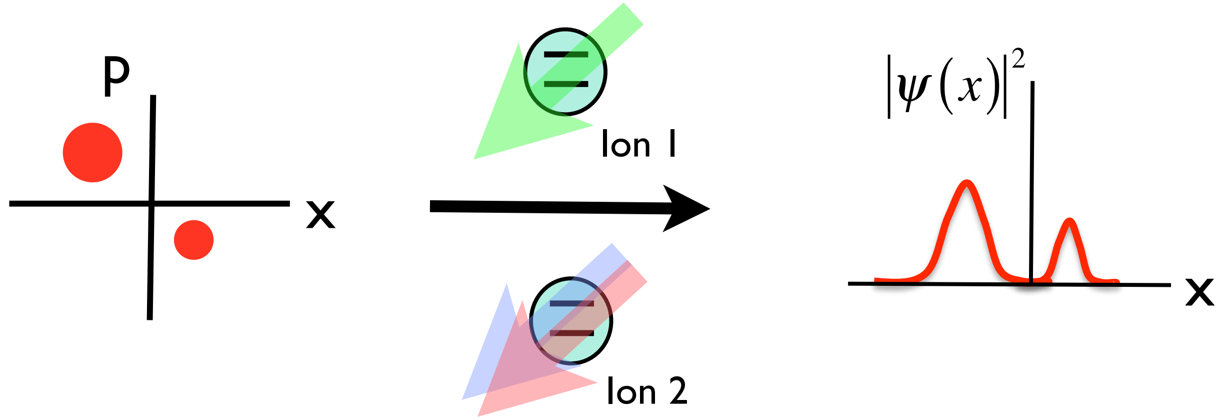
$$H_{bi2} = \hbar\eta\tilde{\Omega}\sigma_x\hat{x} \xrightarrow{|+\rangle_2} H_{bi2} = \hbar\eta\tilde{\Omega}\hat{x}$$

$$q := \hbar\eta\tilde{\Omega}$$

Simulation with trapped ions

Measurement

$|\psi(x)|^2$ $\quad |\psi(p)|^2$ $\quad \langle \hat{x} \rangle$ $\quad \langle \hat{p} \rangle$ $\quad |\psi_{\pm E}(x)|^2$



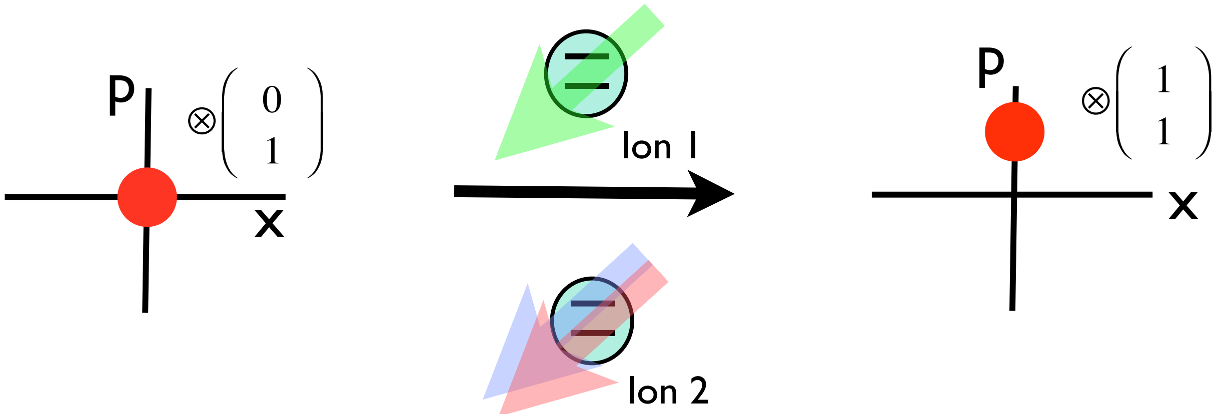
Either first project internal state into energy eigenstates OR trace it out via incoherent dissipative process

State preparation

After optical pumping and sideband cooling:

Most frequently used initial input state:

$|0,S\rangle \equiv$

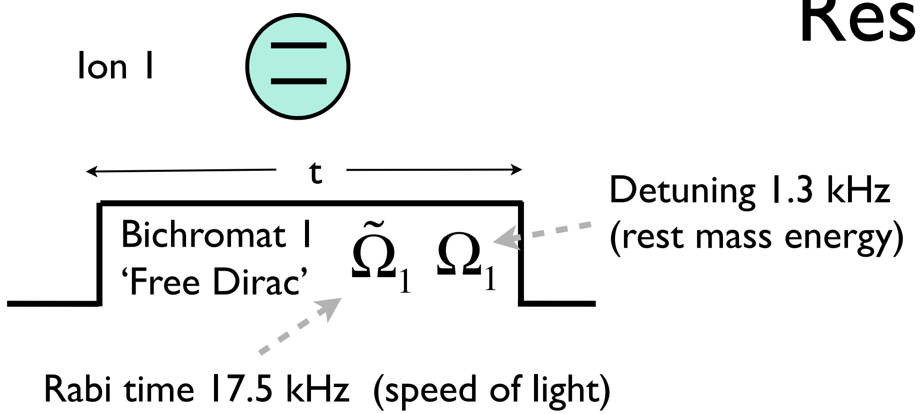


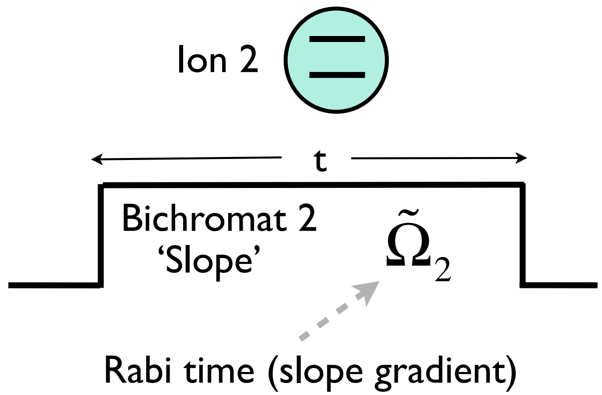
$\langle p_0 \rangle = 3.5\hbar/\Delta$

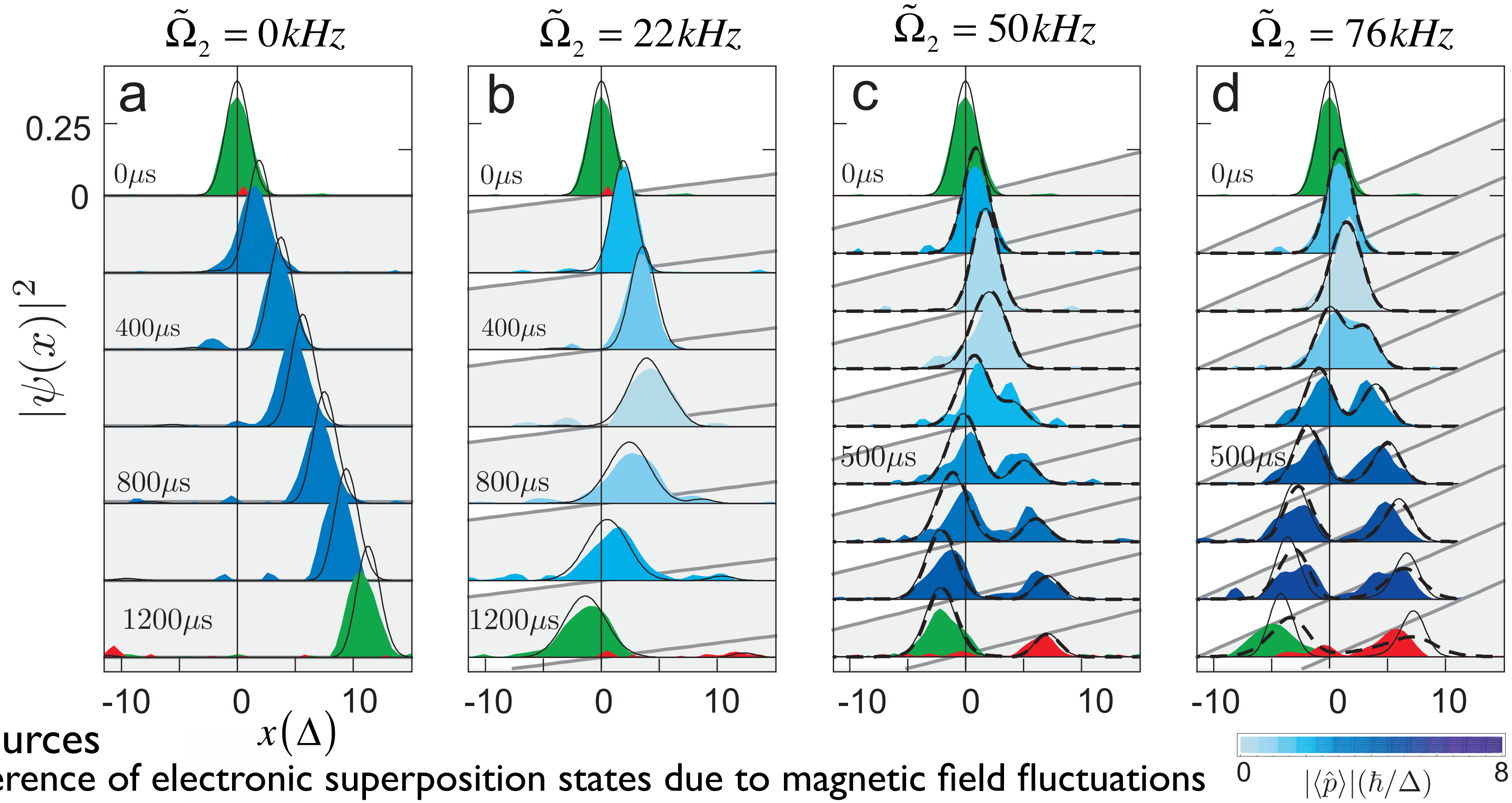
$\langle x_0 \rangle = 0$

Positive energy eigenstate

Results







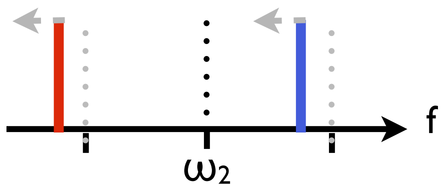
Error sources

- Decoherence of electronic superposition states due to magnetic field fluctuations
- Slow drifts in the trap frequency ~25Hz causing phase space to rotate
- Imperfections in state preparation and analysis pulses cause additional structure
- Breakdown of linearity approximation (Lamb dicke) as phonon number gets really big

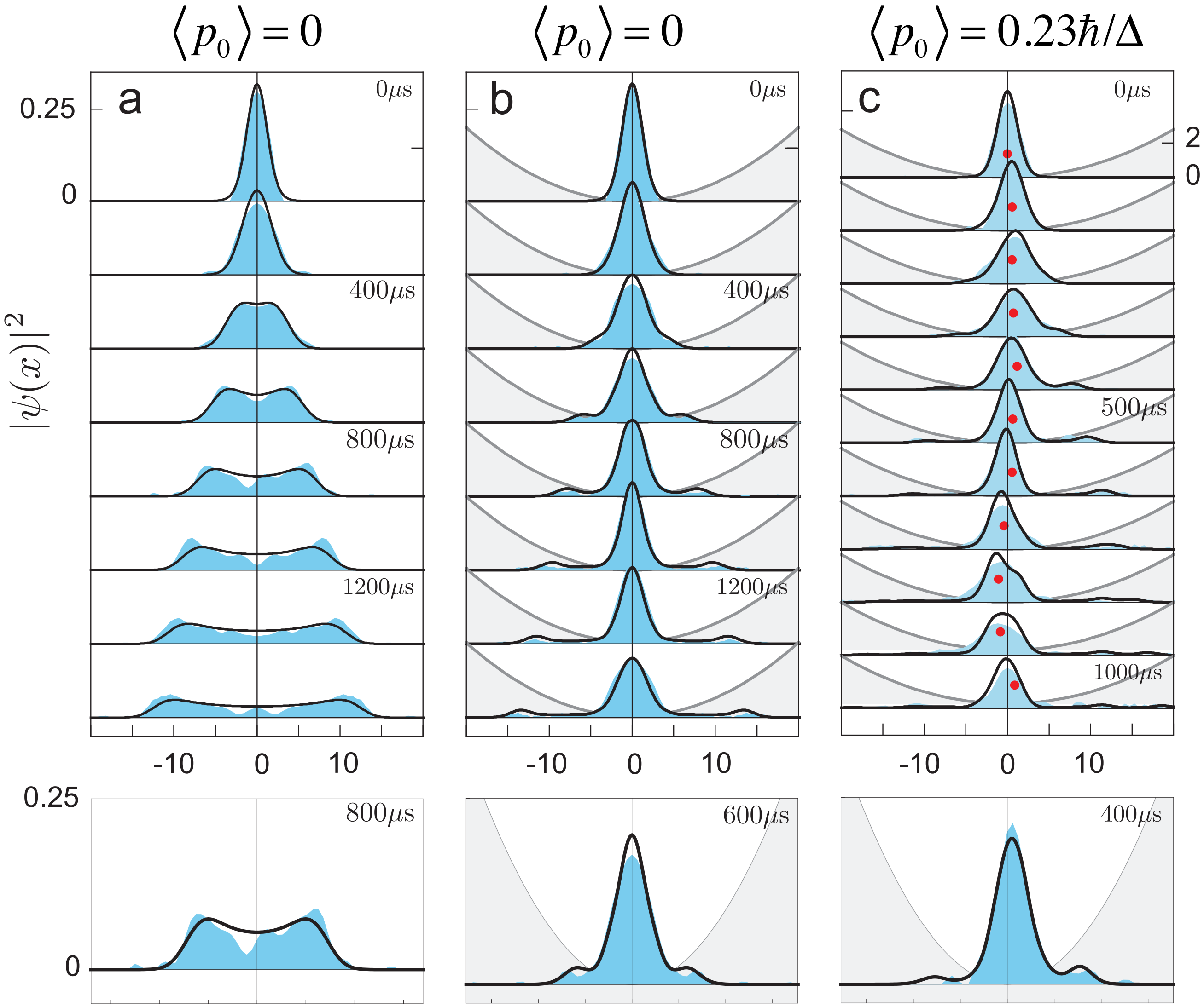
Results

x^2 potential also possible: relativistic quantum mass on a spring

Large detuning of Bichromat 2



$$H_{bichr2} = \frac{\hbar \eta^2 \tilde{\Omega}^2}{2\Omega} \hat{x}^2 \sigma_z$$



$$\frac{\eta \tilde{\Omega}}{\Omega} \ll 1$$

$$\tilde{\Omega} = 50kHz$$

$$\Omega = 33kHz$$

$$\frac{\eta \tilde{\Omega}}{\Omega} \ll 0.067$$

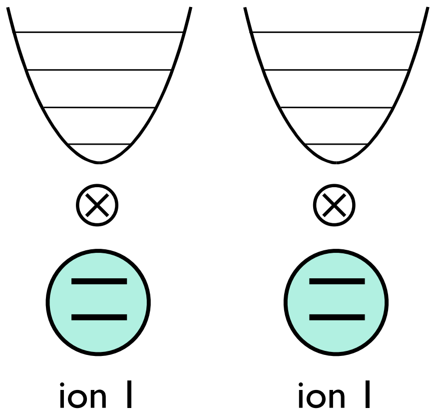
Outlook

- Other potentials

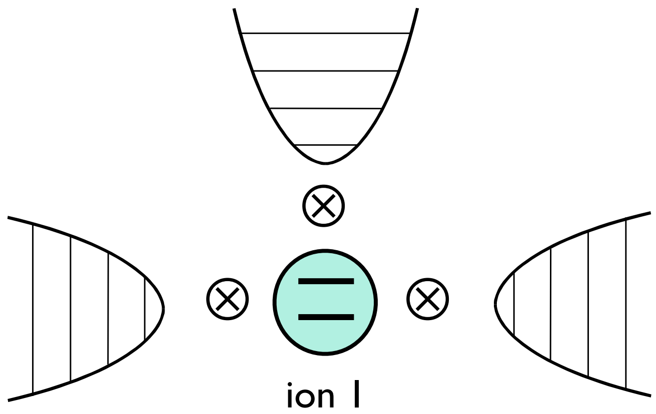
$$i\hbar\partial_t\psi = \left[c\sigma_x\left(p - \boxed{\frac{q}{c}A}\right) + \boxed{q\phi} + \left(mc^2 + \boxed{V}\right)\sigma_z - \boxed{q\tilde{V}}\sigma_y\right]\psi$$

Magnetic Electric Scalar Pseudo scalar

- Scaling up



Two Dirac particles in 1D



One Dirac particle in 3D

- Other physics

Quantized Dirac fields & Majorana physics
(theory paper coming soon....)

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