Putting a Soliton into Quantum Superposition

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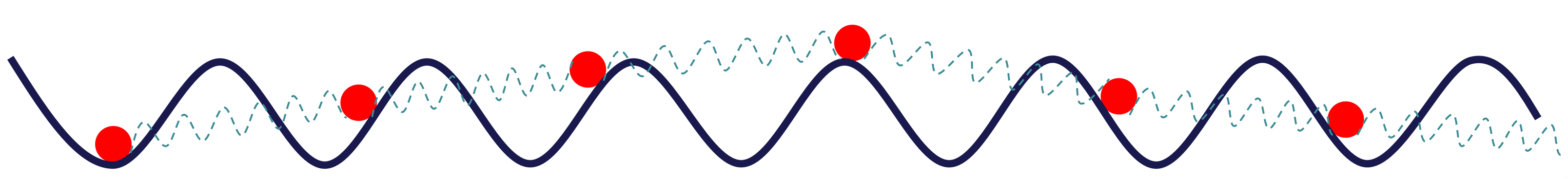
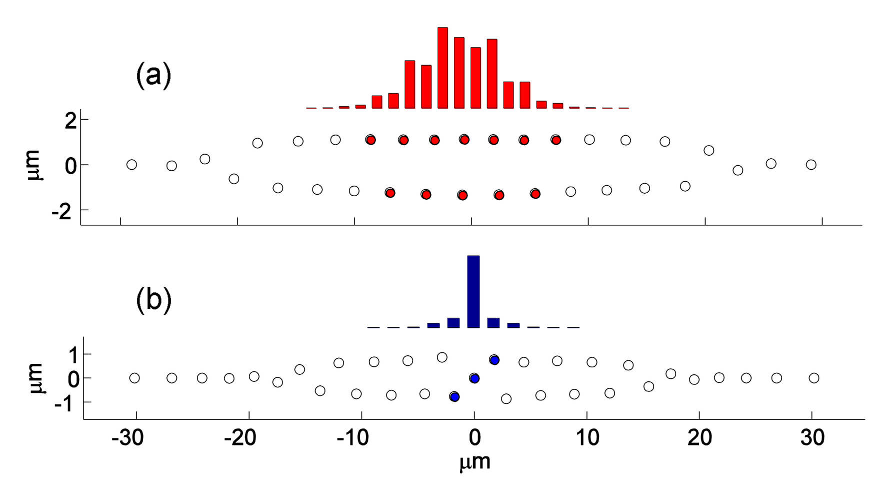
Solitons are ubiquitous in many areas of physics and the natural sciences. Quantum dynamics of solitons, however, has proven experimentally challenging to measure. We show that discrete solitons ("kinks") can be realized in existing ion traps. These solitons carry a localized, high-frequency and gap-separated internal mode. Accounting for all nonlinear interactions at the Doppler temperature, we show that coherence persists for many periods of oscillation of this mode. We suggest an experiment which could allow a first direct measurement of coherent dynamics of discrete solitons, and explore ideas for the utilization of solitons as carriers of quantum information.

(1a) Background

Solitons are localized configurations of nonlinear systems which are nonperturbative and topologically protected. In chains of coupled particles, solitons (also known as "kinks"), are discrete spatial configurations, as in the Frenkel-Kontorova model. Solitons have localized "internal modes". One mode is the soliton's translational "zero-mode". Other localized modes describe "shape-change" excitations of the soliton and are typically separated by an energy gap from the other phononic modes.

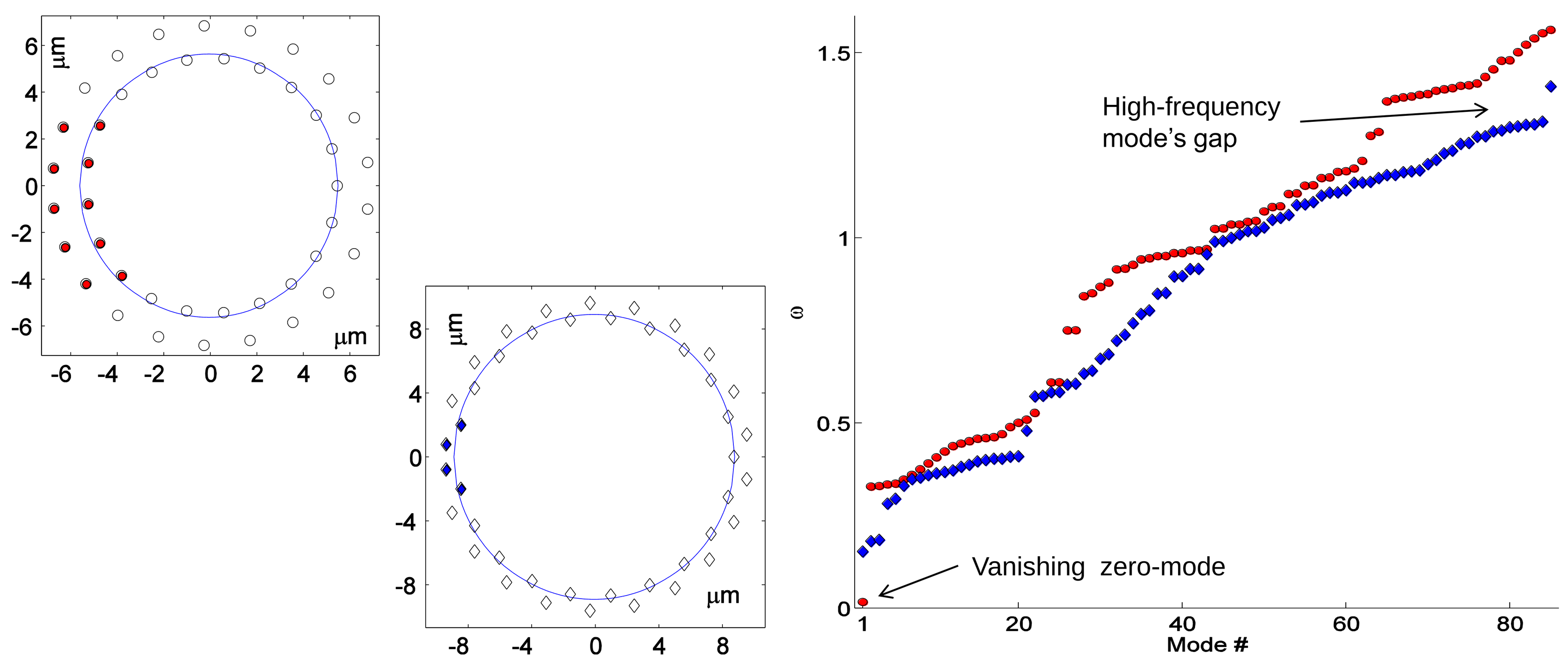


(1b) Solitons In the Linear Ion Trap



"Kink" solitons are metastable excitations of the zigzag phase of an ion chain, stable at up to ~15 times the Doppler temperature according to numerical simulations. (a) An extended kink, obtained at weak transverse trapping. A localized internal oscillation (red bars) involves ~10 ions. (b) A highly discrete kink, at strong transverse trapping. The localized internal mode (blue bars) involves mostly the 3 core ions.

(2a) Dispersion Relations



A kink is the ground state configuration in a circular trap with an odd number of ions. The kink types and dispersion relations are similar to those found in the linear trap. For an extended kink (red circles), there is no gap at the top of the spectrum and the translation mode's frequency approaches zero. For a localized kink (blue diamonds), the localized modes at the top and bottom of the spectrum are gap-separated.

(2b) QIP with the Internal Mode

The high frequency internal mode of the kink can be readily manipulated using Quantum Information Processing techniques. It lies at the top of the phonon band, separated by a **gap** from the other modes. It is **localized** to a few ions and its properties are **independent** of the total number of ions.

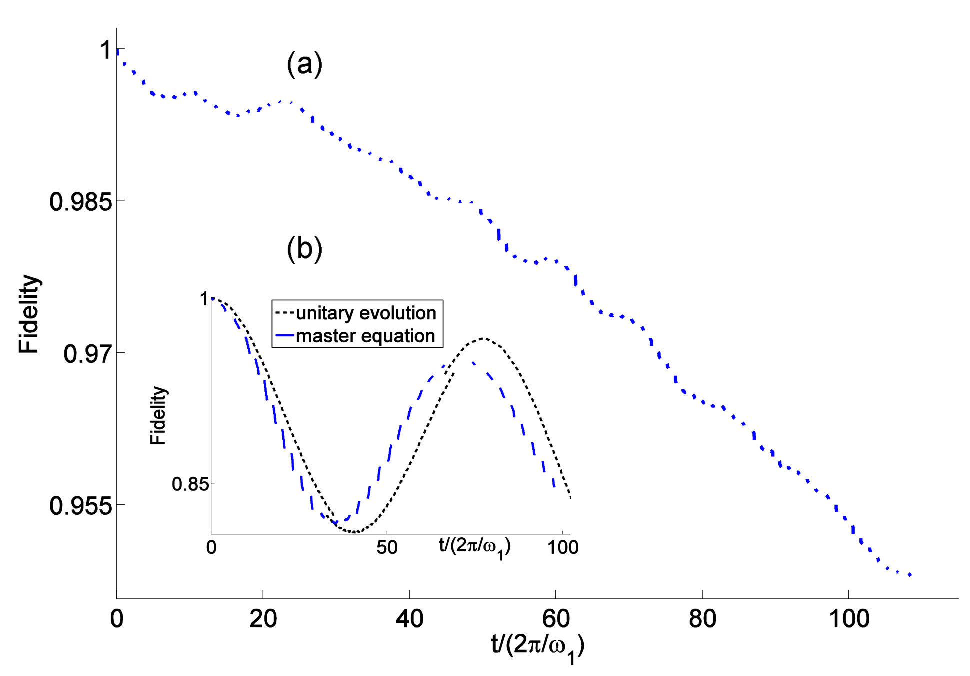
We next show that an initial superposition state $\left(|0\rangle+|1\rangle\right)$ remains coherent for time long enough to allow **cooling, preparation and measurement**.

(3) Coherent Oscillations

Expanding in a perturbative series about the classical kink configuration, we get the nondimensional Hamiltonian for the normal modes $\Theta_i = a_i^\dagger + a_i$

$$H = \sum_i \hbar\omega_i \left(a_i^\dagger a_i + \frac{1}{2} \right) + \frac{1}{3!} \sum_{ijk} L_{ijk} \hbar^{3/2} \left(8\omega_i\omega_j\omega_k \right)^{-1/2} \Theta_i\Theta_j\Theta_k + \dots$$

We derive and numerically integrate a non-Markovian master equation modeling the coherent quantum-mechanical time evolution of the high frequency mode, under the nonlinear coupling to all other modes, at the Doppler temperature.

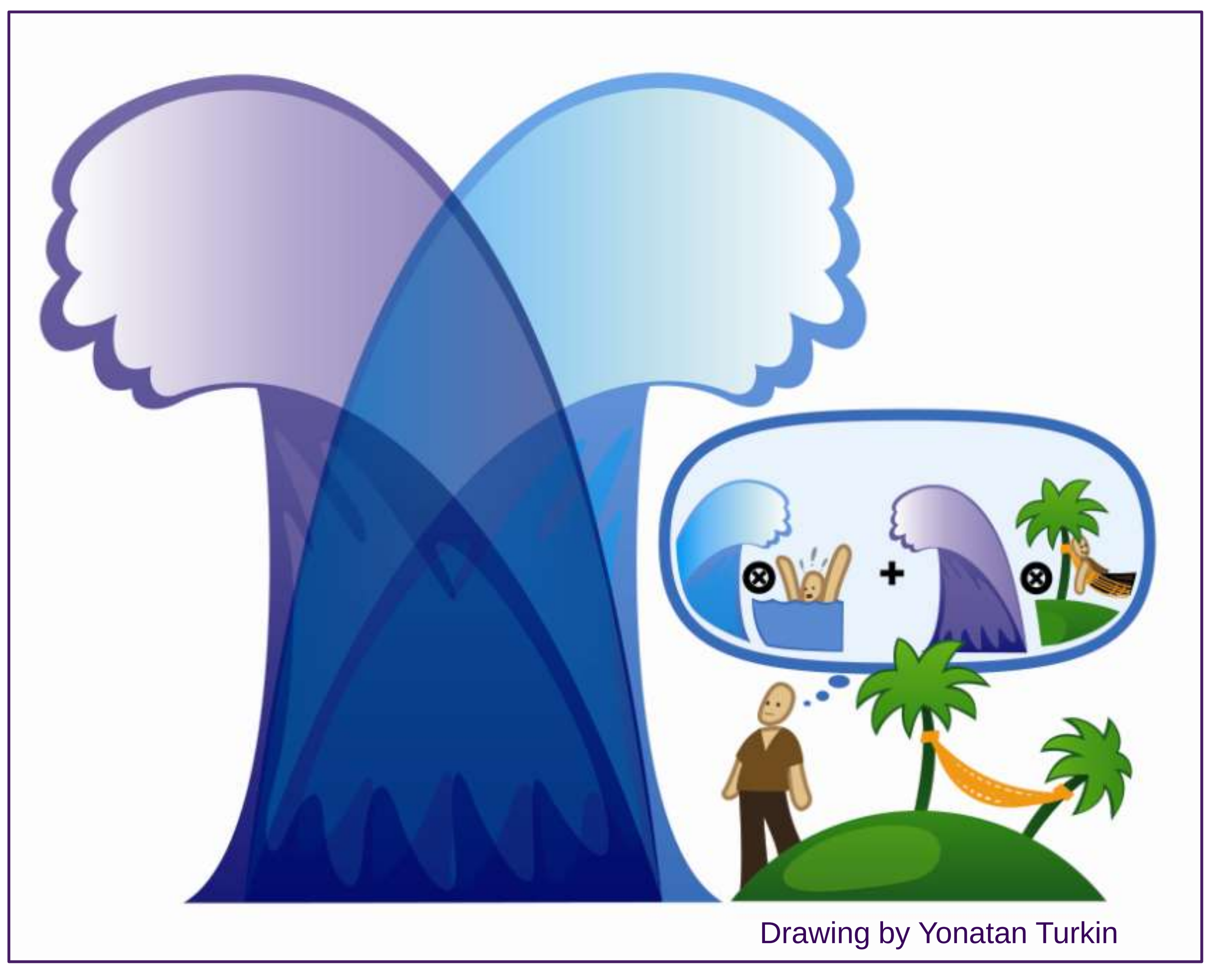


(a) Fidelity of coherent oscillations of the high-frequency mode in the linear trap. With $\nu_x/2\pi = 0.88\mathrm{MHz}$ and $\nu_y/2\pi = 8.1\mathrm{MHz}$, the mode frequencies are $\omega_1/2\pi = 11.5\mathrm{MHz}$ for the high-frequency mode, $\omega_2/2\pi = 10.6\mathrm{MHz}$ for the next mode, and $\omega_{\mathrm{low}}/2\pi = 2.1\mathrm{MHz}$. The inter-ion separation at the kink center is $1.7\mu\mathrm{m}$. (b) Fidelity near a three-phonon resonance, using the master equation (blue), and exact unitary evolution (dashed).

(4) Soliton in a Superposition

To Summarize :

- Discrete **kink solitons** can be found in the ion trap and are **stable**.
- The kink has a localized **internal mode** which is independent of the number of ions in the trap and separated by an energy **gap** from the other modes.
- The **coherence** of this mode persists for long times in a **Doppler**-cooled trap.
- Ion trap **QIP** techniques can be used to manipulate this mode, which could allow the first direct measurement of **quantum dynamics** of a soliton.



Drawing by Yonatan Turkin

References

H. Landa et al., Phys. Rev. Lett **104**, 043004 (2010), and see references therein.

Acknowledgments

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