Single-shot realisation of multiply-conditional quantum gates

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Abstract

We propose a new approach for implementation of highly conditional quantum gates in ion traps. The advantage of the method is that it requires a small number of pulses of small area and particular phase, addressing only one (target) ion. The implementation proposed is very insensitive to the applied Rabi frequency, thereby avoiding significant experimental obstacles: imprecise calibration, fluctuation and unfavourable spatial distribution of the laser intensity, etc.

Ion trap

We exploit a linear chain of N ions, each with two relevant internal states $|0\rangle$ and $|1\rangle$.

Figure 1: *A chain of N ionic qubits*

Depending on the detuning δ, the interaction Hamiltonian

$$\hat{H}_{int}(t) = \frac{\hbar}{2}\Omega_0\sigma_+ \exp\left\{i\eta\left(\hat{a}e^{-i\omega_{tr}t} + \hat{a}^\dagger e^{i\omega_{tr}t}\right)\right\} e^{i(\phi-\delta t)} + \text{H.c.},$$

couples certain internal and motional states. If $\delta = s\ \omega_{tr} = (l-m)\omega_{tr}$, where ω_{tr} is the trap frequency, the laser couples the manifold of states $|g\rangle|n\rangle$ to $|e\rangle|n+s\rangle$. The coupling for $s<0$ is

$$\Omega_{n,n+s} = \Omega_{n+s,n} = \Omega_0 e^{-\eta^2/2}\eta^{|s|}\sqrt{\frac{(n+s)!}{n!}}L_n^{|s|}(\eta^2),$$

where η is the Lamb-Dicke parameter and $L_b^a(x)$ is the generalized Laguerre polynomial.

Composite pulse technique

One can perform a complete population transfer ($|1\rangle \to |2\rangle$) in a system with a resonant pulse of area π. The propagator is

$$U(A) = \begin{bmatrix} \cos\frac{1}{2}A & -i\sin\frac{1}{2}A \\ -i\sin\frac{1}{2}A & \cos\frac{1}{2}A \end{bmatrix},$$

where $A = \int \Omega t \mathrm{d}t$ is the pulse area. If $A=\pi$ we perform a complete transfer. The fidelity of the (single pulse) gate is 1, if we manage to achieve exact area of π. Since this is experimentally impossible, very often adiabatic transfer techniques robust to intensity fluctuation are used. However the latter require long interaction times and very large pulse areas.
However one can combine the advantages of both resonant pulses (small area) and adiabatic transfer (robustness) by using *composite pulses*.
Assume the pulse has some initial phase φ. The corresponding propagator is

$$U(A,\varphi) = \begin{bmatrix} \cos\frac{1}{2}A & -i\mathrm{e}^{-i\varphi}\sin\frac{1}{2}A \\ -i\mathrm{e}^{i\varphi}\sin\frac{1}{2}A & \cos\frac{1}{2}A \end{bmatrix}.$$

By *composite pulse* we mean a sequence of time-separated pulses producing the propagator

$$U(A,\varphi_1,\varphi_2,\ldots,\varphi_N) = U(A)\,U(A,\varphi_1)\ldots U(A,\varphi_N)\ldots U(A,\varphi_1)\,U(A),$$

where the phases $\varphi_1,\ldots\varphi_N$ are appropriately selected.

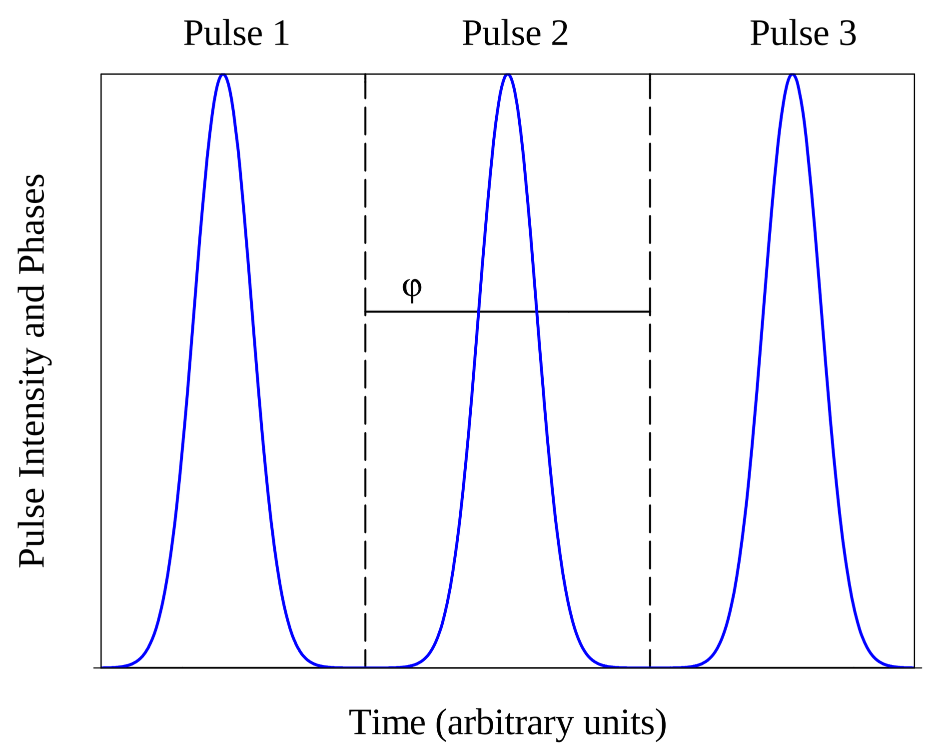

Figure 2: *A sequence of 3 pulses of equal area A and a fixed relative phase:* $U = U(A)\,U(A,\varphi)\,U(A)$

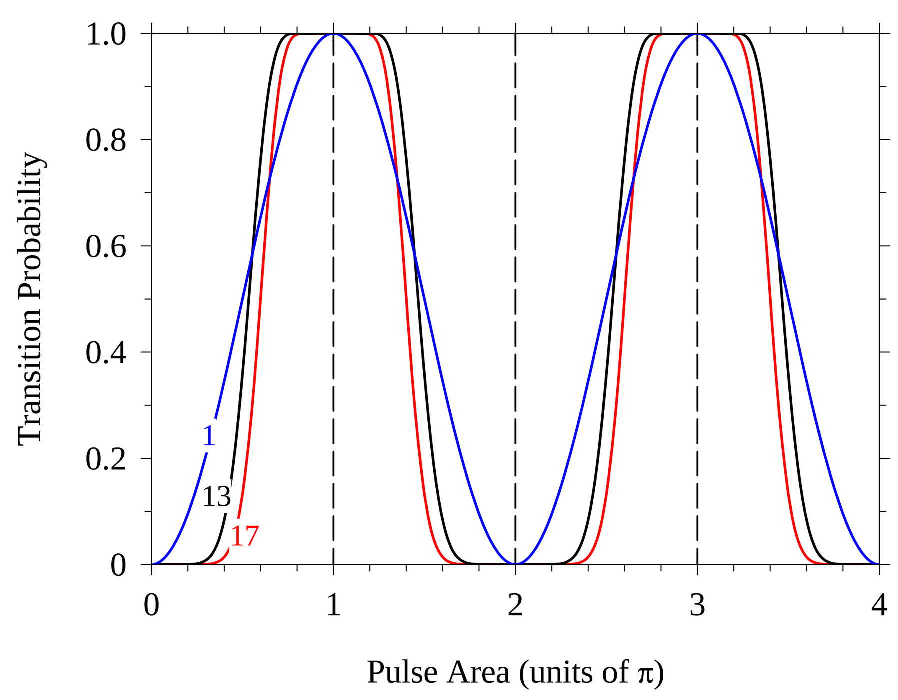

Figure 3: *Excitation profile (transition probability) of a qubit for a sequence of 1 (blue), 13 (black) and 17 (red) pulses of defined relative laser phases.*

Implementation of the Toffoli gate

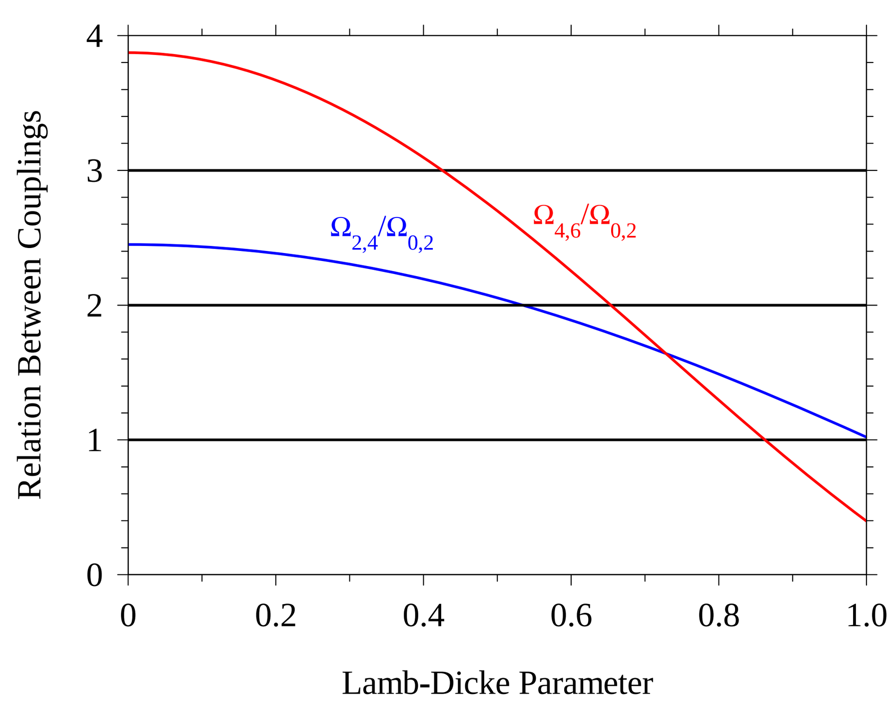

Figure 4: *Coupling strengths vs the Lamb-Dicke parameter for the related transitions.*

1.Initialization

First we prepare the ions in a state of definite phonon number of the COM mode:

$$|\Psi\rangle = |\psi\rangle|n\rangle,$$

where $|\psi\rangle$ is some N-particle quantum state.

2.Flip the state of each ion by local consecutive red-sideband pulses

As a result we dress the quantum states with a different number of phonons, depending on the excitation number. If initially $n=3$:

- $|000\rangle|3\rangle \longrightarrow |111\rangle|0\rangle$
- $(|100\rangle,|010\rangle,|001\rangle)\,|3\rangle \longrightarrow (|011\rangle,|101\rangle,|110\rangle)\,|2\rangle$
- $(|011\rangle,|101\rangle,|110\rangle)\,|3\rangle \longrightarrow (|100\rangle,|010\rangle,|001\rangle)\,|4\rangle$
- $|111\rangle|3\rangle \longrightarrow |000\rangle|6\rangle$

3.Address the target ion with a composite pulse on the second red-sideband

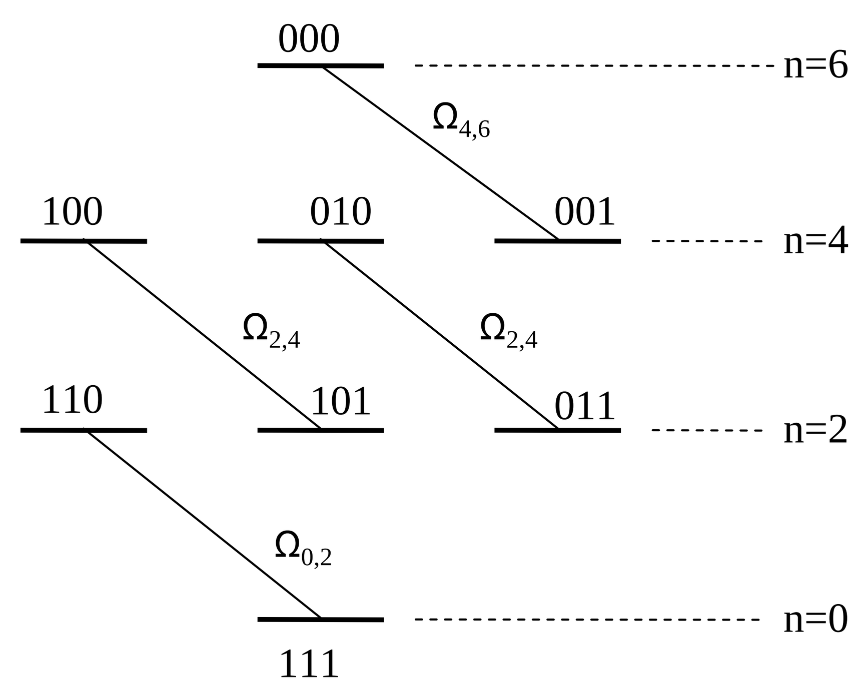

Figure 5: *Linkage diagram when only the target ion is addressed.*

For appropriate values of the Lamb-Dicke parameter and the pulse area (or Rabi frequency), $\Omega_{0,2}$ drives complete transfer (area of around π), whereas the rest drive complete return (area of around 2π).

4.Flip the state of each ion by local consecutive red-sideband pulses

This is in order to restore the original states.

Fidelity

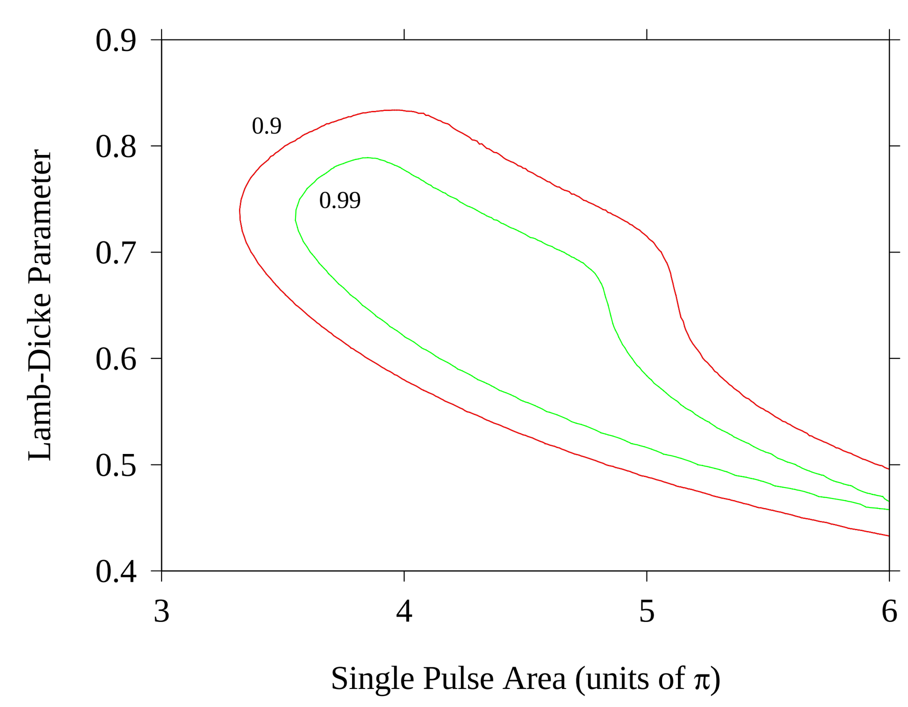

Figure 6: *Numerically obtained fidelity of the Toffoli gate, achieved by a composite pulse containing 13 second red-sideband pulses.*

Local addressing

One can improve significantly the local addressing by means of a composite pulse.

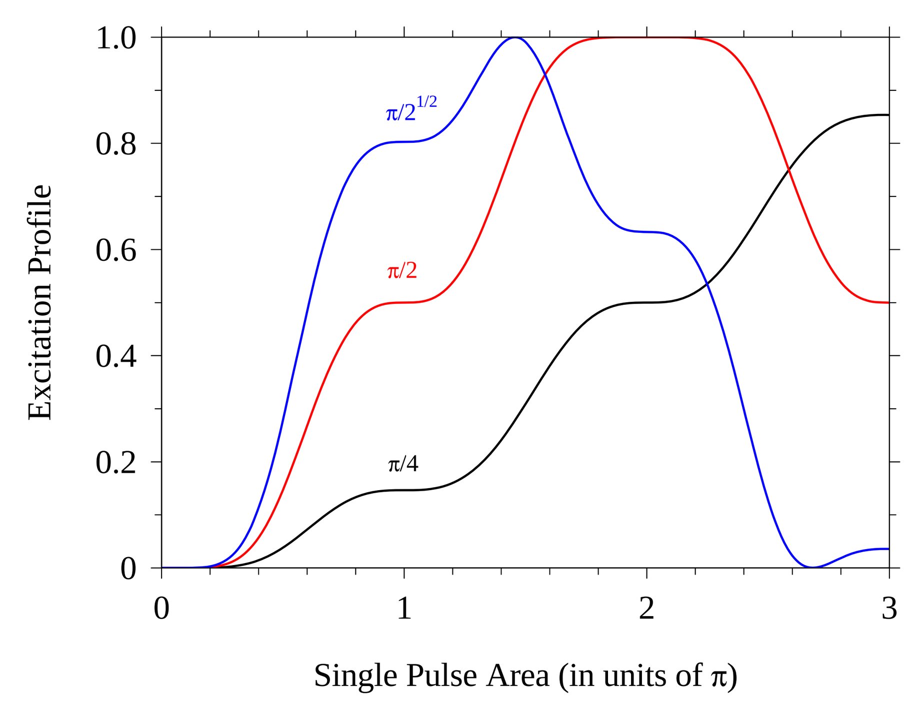

Figure 7: *Stabilisation of excitation by a composite pulse of the type* $U(A,0)U(A,2\varphi)U(\alpha A,\varphi)U(A,2\varphi)U(A,0)$, *where* $\varphi = \arccos(\alpha/4)$.