ION DYNAMICS IN LINEAR RF TRAPS

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Introduction

The quadrupole linear trap is a widespread tool for many fundamental physics experiments. Compared to quadrupole trap, higher order traps present the interesting feature to generate an almost flat potential well, which induces a small RF-driven motion and low RF-heating. These traps have been widely used in the ultra cold collisions community with buffer gas cooled samples and recently, to produce Coulomb crystals of a new kind thanks to laser cooling [1]. Moreover, multipole traps have been at the heart of a promising microwave ion clock based on a double trap, a quadrupole plus a 16-pole [2].

In this context, we are setting up a novel experiment composed of three trapping regions. The purpose of this device is to study ion dynamics under laser cooling, in quadrupole and octupole RF traps, aiming at the confinement and transport of a wide panel of ion clouds from small chains to large samples ($N > 10^6$).

Multipole vs Quadrupole: General considerations

Equations of motions:

$$\ddot{x}/r_0-(q/m)k\Phi_0(t)(r/r_0)^{k-1}\cos([k-1]\theta)=0$$
$$\ddot{y}/r_0+(q/m)k\Phi_0(t)(r/r_0)^{k-1}\sin([k-1]\theta)=0$$

2k=4

$$\ddot{x}-(q/m)2\Phi_0(t)x=0$$
$$\ddot{y}+(q/m)2\Phi_0(t)y=0$$

- Linear pair of equations
- Solution: Mathieu equation
- Definition of stability criteria

2k=8

$$\ddot{x}-(q/m)4\Phi_0(t)(x^3-3y^{2x})/r_0^2=0$$
$$\ddot{y}+(q/m)4\Phi_0(t)(y^3-3x^{2y})/r_0^2=0$$

- Non-linear
- No stability region
- Local condition for adiabaticity

Quadrupole Optimisation

Search for the least perturbed harmonic potential

"Magic" ratio between trap radius (r_0) and electrode radius (r_e) [3]

$$r_0=1.156\,r_e$$

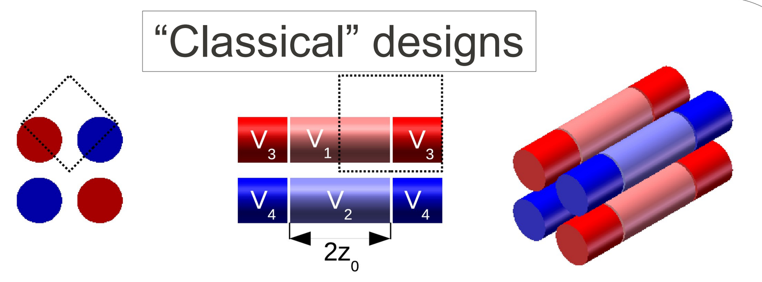
minimization of the quadrupole next's higher term

However... the role of the DC electrodes not studied in linear traps

Numerical analysis of anharmonic contribution for several geometries

"Classical" designs

#1



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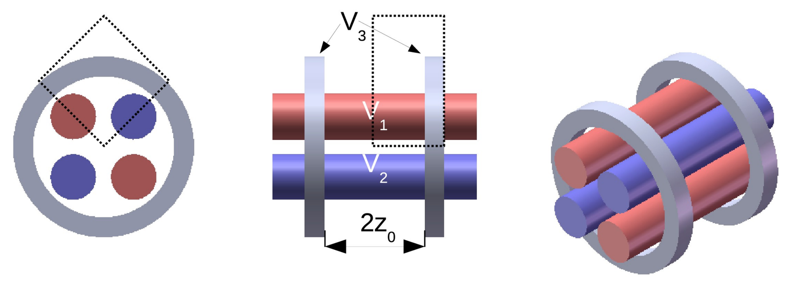
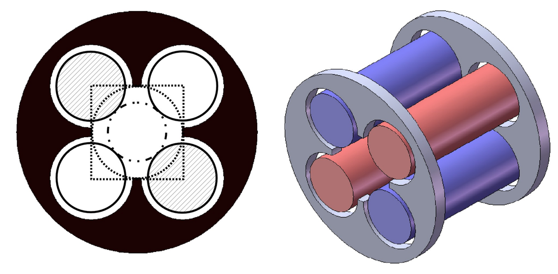


Novel design

#4



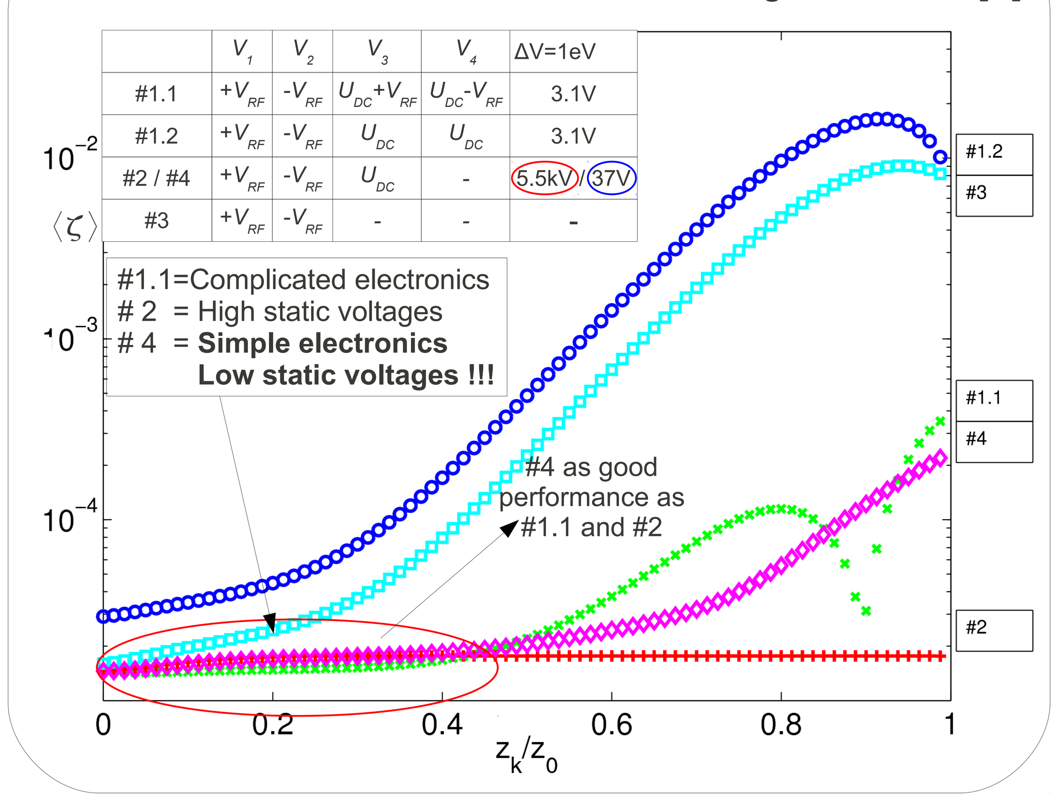
2D solution of Laplace eq. for ∞ electrodes [3]

$$\phi(x,y)=C_0+Real\left[\sum_{m=0}^{\infty}C_{4m+2}\xi^{4m+2}\right]$$
$$\xi=x+iy$$

Definition of the anharmonicity ζ

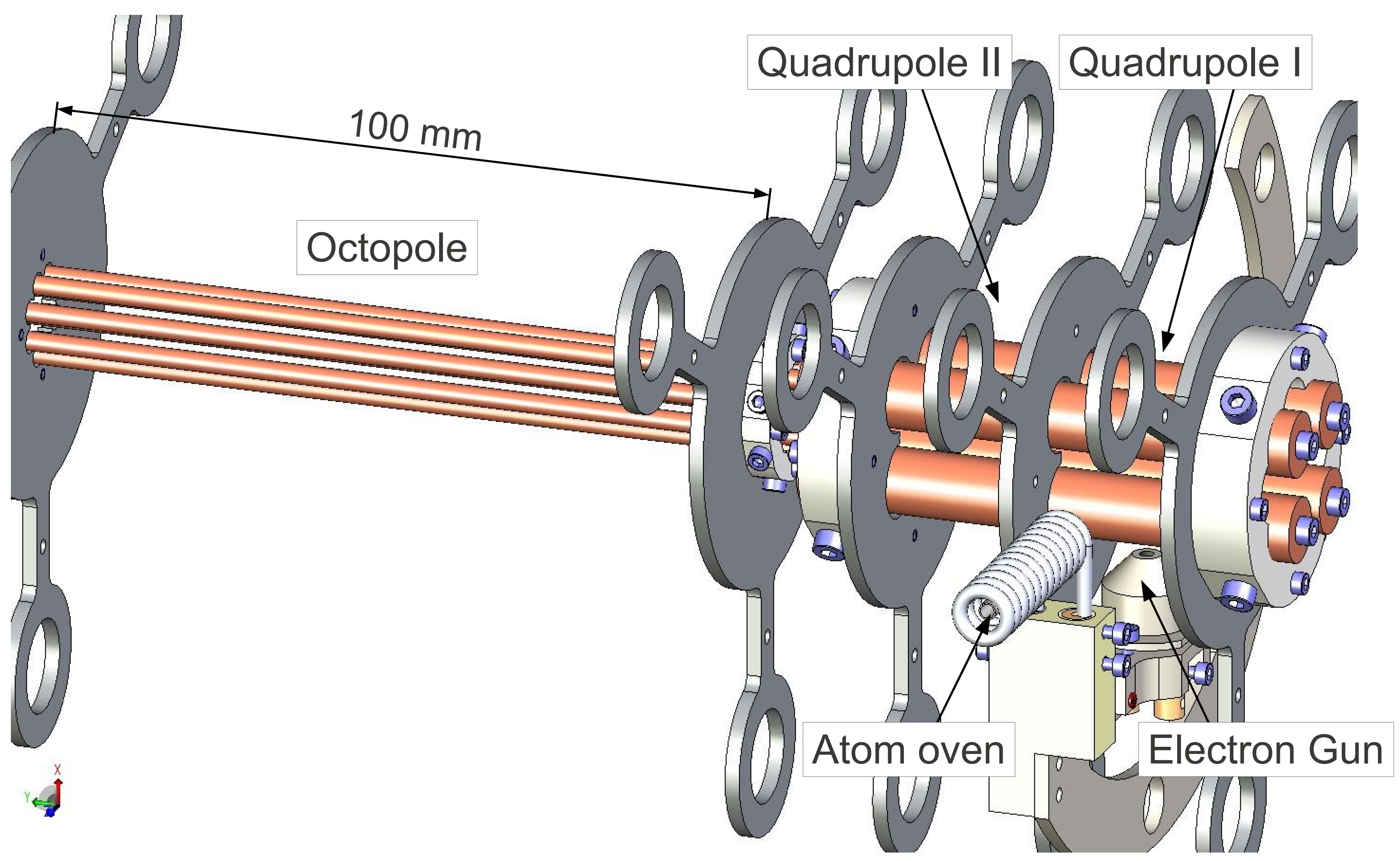
$$\phi_r=\phi_q\ \ (1+\zeta+\Delta\zeta)$$
$$\phi_q=C_0+Real\left[C_2\xi^2\right]$$
$$\zeta=\frac{Real\left[C_6\xi^6+C_{10}\xi^{10}\right]+\Delta\phi_f}{C_0+Real\left[C_2\xi^2\right]}$$
$$\Delta\zeta=\frac{\Delta\phi_s}{C_0+Real\left[C_2\xi^2\right]}$$

Anharmonic contribution for the studied geometries [4]



New experimental set-up

The set-up consists of three trapping regions of different storage potential: quadrupole-quadrupole-octupole. This configuration allows to separate the ion creation zone from the laser-cooling zone, in order to avoid perturbations of the potential created by atom deposition on the RF electrodes.



Adiabatic approximation / Pseudopotential

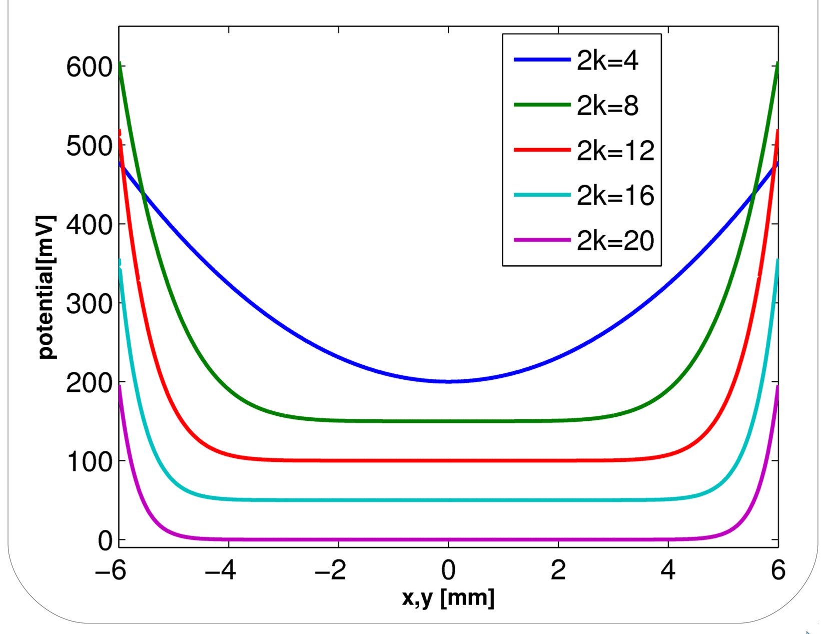
Applied potential
$$\Phi_0(t)=U+V\cos(\Omega t)$$
+
Adiabatic approx.

Resulting pseudopotential

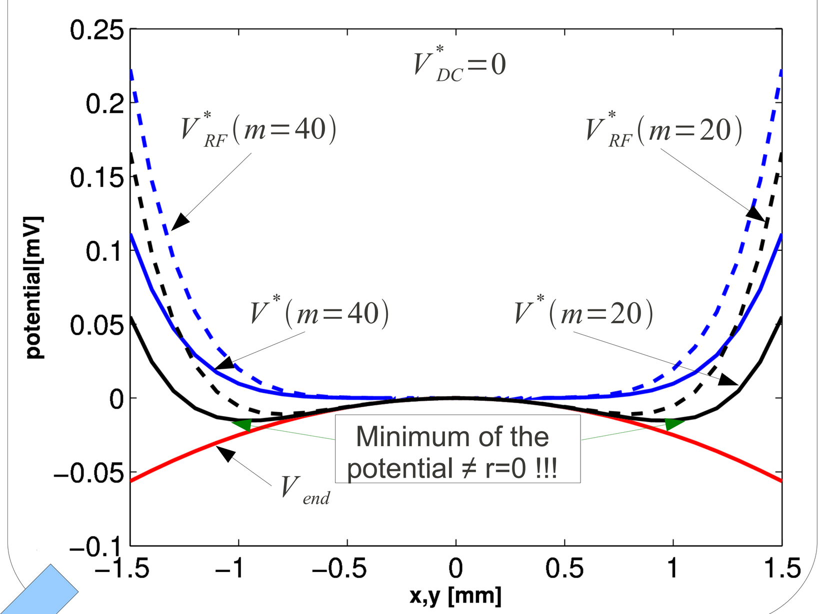
$$V^*(r)=V^*_{RF}+V^*_{DC}+V_{end};\quad V^*_{RF}=\frac{-q^2V^2k^2}{16m\Omega^2r_0^2}\left(\frac{r}{r_0}\right)^{2k-2}$$
$$V^*_{DC}=\frac{qU}{2}\left(\frac{r}{r_0}\right)^k\cos(k\theta);\quad V_{end}=\frac{q\kappa V_{end}}{z_0^2}\left(z^2-\frac{r^2}{2}\right)$$

$\Omega=2\pi\,10$MHz
$V_{RF}=400$V
$r_0=7.5$mm
$V_{DC}=0.1$V
$z_0=5$mm

V^*_{RF} for several k values
(a difference of 50mV has been added between each curve for clarity



Different contributions to the total pseudopotential V^*

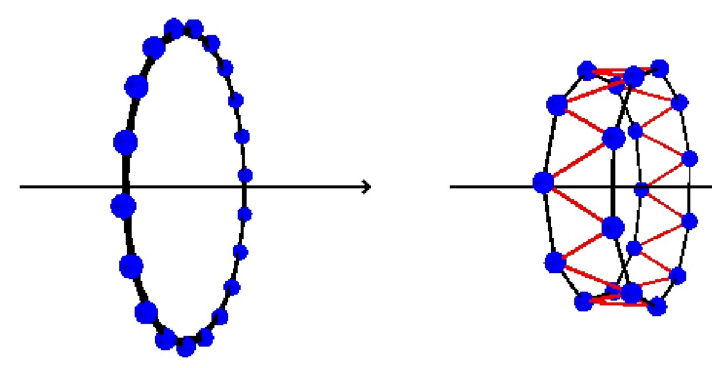


The stable Coulomb crystal is a ring!

But... One or more rings?

Exist a minimum ring radius below which, the stable structure is no longer a single ring:

$$R_l=\left(\frac{q^2/4\pi\epsilon_0}{4m\omega_z^2}\right)^{1/3}\left(\sum_{n=1}^{N/2}\frac{1}{\sin^3\frac{(2n-1)\pi}{N}}\right)^{1/3}$$
$$\simeq\left(\frac{q^2/4\pi\epsilon_0}{2m\omega_z^2}\right)^{1/3}\frac{N}{\pi}$$



Not just a curiosity: a ring of ions for optical metrology [5]

Single ion:

Requires a clock laser frequency stable at 1Hz/s level
Limited by signal to noise ratio

Stability quantify by the Allan variance $\sigma_y(\tau)=\frac{1}{\pi Q R_{S/N}}\sqrt{\frac{T_c}{\tau}};\quad R_{S/N}\propto\sqrt{N}$

Ion's Ring (advantages vs string of ions):

Each ion sees the same laser intensity
z-axis: characterized by one single oscillation frequency ω_z
Radial size independent of number of ions *N* (at first order)

Systematic Shifts induced by the RF electric field

Shift induced on the clock transition frequency f_0 due to **second-order Doppler effect** (micromotion) can be expressed as:

$$\delta f_{D2}=-f_0\frac{\omega_z^2R^2}{4(k-1)c^2}$$

The scalar dominant **Stark shift** contribution induced by the electric trapping fields can be written as:

$$\delta f_S^0(RF)=\frac{-1}{2}\Delta\alpha^0\frac{m^2}{2(k-1)q^2}\omega_z^2\Omega^2R^2$$

Uncertainty budget for the frequency transition of $|S_{1/2}, M_J=\pm1/2\rangle\rightarrow|D_{5/2}, M_J=\mp1/2\rangle$ in $^{40}Ca^+$, based on a ring in an octupole linear trap with $\omega_z/2\pi=1\,MHz$, $\Omega/2\pi=20$MHz and a RF electric field such that R = 20μm (10 ions) or 40μm (20 ions), as given in the table [5]

Effect	Conditions	Shift (Hz)	Broadening	Long-term instability
Doppler (2^e)	$R=20\,\mu$m	+6.0	±0.14	8×10^{-17}
	$R=40\,\mu$m	+24.1	±0.28	5×10^{-17}
Stark	$R=20\,\mu$m	+4.1	±0.09	6×10^{-17}
	$R=40\,\mu$m	+16.5	±0.19	3×10^{-17}
Zeeman	$\delta B\leqslant6\times10^{-7}$ G		<1	2.5×10^{-15}
BBR	$T=300\pm10$ K	+0.38(1) ± 0.05		$<10^{-16}$
Quadrupole	Trapping field	+8.0	<0.1	$\leqslant10^{-17}$
Quadrupole	Extra dc		≃0.04	$\leqslant10^{-16}$
Total	$R=20\,\mu$m	+18.5	±0.2	2.5×10^{-15}
Total	$R=40\,\mu$m	+49.0	±0.4	2.5×10^{-15}

References:
[1] K. Okada, T. Takayanagi, M. Wada, S. Ohtani, and H. A. Schuessler, Phys. Rev. A **80**, 043405 (2009)
[2] J. Prestage and G. Weaver, Proceeding of the IEEE **95**, 2235 (2007)
[3] A.J. Reuben, G.B. Smith, P. Moses, A.V. Vagov, M.D. Woods, D.B. Gordon, R. W. Munn, IJMS. **154** (1996)
[4] J. Pedregosa, C. Champenois, M. Houssin, M. Knoop; IJMS **290**, 100-105 (2010), arXiv : 1001.1403v1 [physics.ins-det]
[5] C. Champenois, M. Marciante, J. Pedregosa-Gutierrez, M. Houssin and M. Knoop and M. Kajita, Phys. Rev. A, **81**, 043410 (2010), arXiv :1003.0763v1 [quant-ph]