Laser trapping of charged particles

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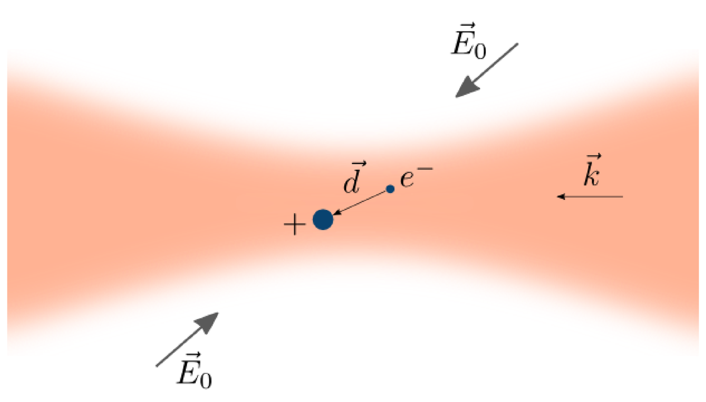
1. The general idea



We consider a charged particle that is trapped by a tightly focused Gaussian beam and an additional electrostatic field, as in the experiment reported in [1].

Dipolar traps are normally used to confine neutral atoms. Our aim is to analyze the possible effects of extra couplings between the trapping fields and the total charge of the particle.

2. Basic Hamiltonian

We model the ion as a closhed shell plus a single external valence electron. The Hamiltonian for the system is:

$$H = \sum_{j=n,e} \frac{1}{2m_j}\left[\vec{p}_j - q_j\vec{A}(\vec{r}_j)\right]^2 + \sum_{j=n,e} q_j\Phi(\vec{r}_j) + V_{\rm Coul} + H_{\rm rad} + H_{\rm rel}$$

where n, e stand for nucleus and electron respectively, $\vec{A}$ is the electromagnetic potential in Coulomb (transverse) gauge, $V_{\rm Coul}$ contains the interaction between core and electron, Φ is the electrostatic potential, $H_{\rm rad}$ is the Hamiltonian for the radiation (given by a sum of harmonic oscillators, one for each field mode), and $H_{\rm rel}$ contains relativistic corrections.

3. Power-Zienau-Wolley transformation

To study the dipolar coupling we apply the unitary transformation [2]:

$$T = e^{-iS/\hbar}, \quad S = \int dV \vec{P}_0(\vec{x})\cdot\vec{A}_<(\vec{x})$$

where the subindex $<$ indicates that a cutoff frequency must be introduced, and where:

$$\vec{P}_0(\vec{x}) = \sum_j q_j(\vec{r}_j - \vec{R})\int_0^1 d\lambda\ \delta[\vec{x} - \vec{R} - \lambda(\vec{r}_j - \vec{R})]$$

S commutes with the position operators, but the momenta are transformed as:

$$\vec{p}_j \to T\vec{p}_jT^\dagger = \vec{p}_j + \left[e^{-iS/\hbar}, \vec{p}_j\right]e^{iS/\hbar} = \vec{p}_j + \vec{\nabla}_j S$$

This is used to partially suppress the coupling between momenta and vector potential.

S commutes also with $\vec{A}(\vec{x}\,')$ and $\vec{B}(\vec{x}\,')$, but $\vec{E}$ is transformed in the way:

$$\vec{E} \to T\vec{E}T^\dagger = \vec{E} + \left[e^{-iS/\hbar}, \vec{E}\right]e^{iS/\hbar} = \vec{E} - \frac{1}{\epsilon_0}\vec{P}_{0<}^{\perp}$$

The last term corresponds to the polarization of the system of charges; this polarization field has to be projected onto its transverse part, and to the wavelengths below the cutoff.

4. Transformed Hamiltonian

After the transformation the Hamiltonian is given by:

$$H \to THT^\dagger = H_{\rm ion} + H_{\rm rad} + H_{\rm coupling} + TH_{\rm rel}T^\dagger.$$

$H_{\rm rad}$ is the same operator as before. The new Hamiltonian for the ion is:

$$H_{\rm ion} = \frac{\vec{P}^2}{2M} + \frac{\vec{p}\,^2}{2\mu} + V_{\rm Coul} + \varepsilon_{\rm dip}$$

Here we use relative motion and center of mass coordinates $\vec{r}$, $\vec{p}$, $\vec{R}$, $\vec{P}$. M is the total mass, μ is the reduced mass, and $\varepsilon_{\rm dip}$ is a dipolar self-energy. The coupling terms are:

$$H_{\rm coupling} = \sum_j q_j\Phi(\vec{r}_j) + H_{\rm trap} + \vec{p}\cdot\left(\frac{\vec{K}_e}{m_e} - \frac{\vec{K}_n}{m_n}\right) + \frac{\vec{P}}{M}\cdot\left(\vec{K}_e + \vec{K}_n\right) + \left(\frac{\vec{K}_e^2}{2m_e} + \frac{\vec{K}_n^2}{2m_n}\right)$$

Here $\vec{K}_j = \vec{\nabla}_j S - q_j\vec{A}(\vec{r}_j)$, and $H_{\rm trap}$ is responsible for the dipolar trapping:

$$H_{\rm trap} = -\int dV \vec{E}_< \cdot \vec{P}_{0<}^{\perp}$$

To lowest order in kr (where k is the wavelength, and $kr \sim 10^{-3}$ for optical frequencies), $H_{\rm trap}$ is the standard dipolar Hamiltonian $-\vec{d}\cdot\vec{E}$, with $\vec{d} = -q\vec{r}$. Here $q = |e| + Qm_e/M$ with Q the net charge of the ion. Apart from this correction, the essential description of the dipolar trapping is the same as for a neutral particle.

5. Dipolar trapping [3]

$H_{\rm trap} \simeq -\vec{d}\cdot\vec{E}$ produces an effective trapping potential for the center of mass. It causes also heating due to photon scattering (but the ratio between heating and trapping can be made smaller and smaller by increasing both the detuning and the laser intensity).

The internal dynamics are governed by the atomic Hamiltonian plus the coupling to the laser and the spontaneous emission. In a two-level picture, the laser drives transitions between a ground state $|g\rangle$ and an excited state $|e\rangle$. The internal steady-state solution can be obtained from the optical Bloch equations.

When the internal state is traced out, the dipolar Hamiltonian $-\vec{d}\cdot\vec{E}$ gives a mean force:

$$\langle\vec{F}\rangle = \vec{F}_{\rm react} + \vec{F}_{\rm dissip} = -\frac{\hbar\delta}{2}\vec{\nabla}\ln(1+s) + \frac{\hbar\Gamma}{2}\frac{s}{s+1}\vec{k}_L$$

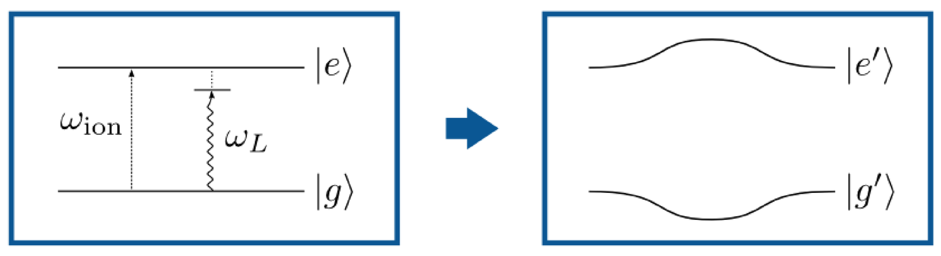
with Γ the linewidth, $\delta = \omega_L - \omega_{\rm ion}$ the detuning, and s the saturation parameter:

$$s = \frac{\rho_{ee}}{\frac{1}{2} - \rho_{ee}} = \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4},$$

where $\Omega = -\langle g|\vec{d}|e\rangle\cdot\vec{E}_0$ (for a travelling wave with wave vector $\vec{k}_L$ and amplitude $\vec{E}_0$). The first term in $\langle\vec{F}\rangle$ is associated to the intensity gradient, the second is the radiation pressure. Typically the first term is much larger, so that the effect of the dipolar coupling can be approximated by an effective potential:

$$V_{\rm eff} = \frac{\hbar\delta}{2}\ln(1+s) \simeq \frac{\hbar\delta}{2}s$$

where the last expression is for low saturation; then the potential is proportional to the laser intensity. For $\delta < 0$, the atom can be trapped in high-intensity regions.



In a dressed atom picture, the potential can be identified with the position-dependent energy shift of the atomic levels.

6. Corrections to the dipolar trapping

Higher order terms in the coupling to the internal motional degrees of freedom

The dominant terms of this form can be grouped with the dipolar coupling $-\vec{d}\cdot\vec{E}$ and give corrections to the effective potential of order $(kr)^2 \sim 10^{-6}$, associated with the second derivatives of the electric field. These corrections are independent of the net charge.

Relativistic effects

Corrections to the fine structure terms can be obtained starting from the Dirac equation with a fixed nucleus, applying a Foldy-Wouthuysen transformation, and taking the non-relativistic limit [4]. The largest term turns out to be the one given by the addition, to the spin-orbit coupling, of the external electric field given by the laser. It can also be added to the dipolar coupling $-\vec{d}\cdot\vec{E}$, modifying the coupling constant at most up to 10^{-5}. And it is also a charge-independent effect.

Coupling to the total momentum

The largest term of this kind is the standard coupling between the electromagnetic field and a charged point-like particle. To estimate its effect we make a harmonic approximation for the potential, with frequency $\omega_0 \sim 10^{-10}\omega_L$. The system is then described by:

$$H_{\rm eff} = \frac{(\vec{P} - Q\vec{A})^2}{2M} + \frac{M\omega_0^2\vec{R}^2}{2}$$

The main result of this coupling is an oscillation of very small amplitude at the field frequency. Typical energies associated to it will be dominated by the kinetic term and will be of the order of $(QA)^2/(2M)$, which is $\sim 10^{-8}$ compared to the optical depth (and $\sim 10^{-4}$ compared to the lowest trap motional energies). This is the largest charge-dependent effect in the coupling with the laser.

Time-dependence of the effective potential

The trapping potential is a result of the optical field and oscillates with time as:

$$V_{\rm dip} = V_0(\vec{R})[1 + \cos(2\omega_L t)]$$

Again we consider a harmonic approximation with frequency ω_0 for V_0. This leads to an equation analogous to the Mathieu equation for an ion in a Paul trap. The amplitude of the resulting micromotion is of the order of $(\omega_0/\omega_L)^2 \sim 10^{-20}$. However, the micromotion frequencies are very high and the associated kinetic energy may not be negligible; its order of magnitude is given by $\hbar\omega_0$ [5]. This effect is present for both neutral and charged particles.

Bibliography

[1] Ch. Schneider, M. Enderlein, T. Huber and T. Schaetz, e-print arXiv:1001.2953 (2010).
[2] C. Cohen-Tannoudji, J. Dupont-Roc and G. Grynberg. Photons and Atoms. WILEY-VCH Verlag (2004).
[3] C. Cohen-Tannoudji, J. Dupont-Roc and G. Grynberg. Atom-Photon Interactions: Basic Processes and Applications. John Wiley and Sons (1998).
[4] J. D. Bjorken and S. D. Drell. Relativistic Quantum Mechanics, McGraw-Hill (1964).
[5] J. I. Cirac, L. J. Garay, R. Blatt, A. S. Parkins and P. Zoller, *Phys. Rev. A* **49**, 421 (1994).