A PHOTONIC LUTTINGER LIQUID
&
SPIN-CHARGE SEPARATION IN A QUANTUM OPTICAL SYSTEM

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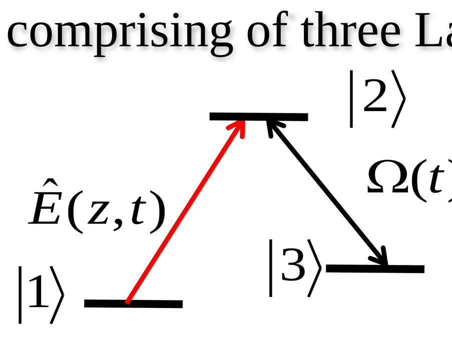
INTRODUCTION

One of the most counterintuitive characteristics of one-dimensional electron gases is spin-charge separation. In this case the electrons cease to behave as single particles comprised of spin and charge [1,2]. Instead collective excitations appear carrying only charge (no spin) or only spin (no charge) which propagate through the system with different velocities. In this work we show that stationary polaritons (light-matter excitations) [3] generated inside a hollow one-dimensional waveguide filled with atoms, can be made to generate a photonic two-component Lieb Liniger model [4].

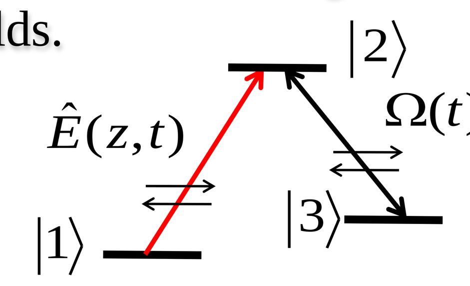
Experiments have already shown that pulses slow down and can be eventually stored in a delocalized collective state of an atomic ensemble comprising of three Lambda type of atoms [5,6].



$$\hat{\Psi}(z,t) = \cos\theta(t)\hat{E}(z,t) - \sin\theta(t)\sqrt{N}\hat{\sigma}_{13}(z,t)$$

$$v_g = c/(1+\frac{g^2 N}{\Omega^2})$$

Generalize to the case of counter propagating classical fields incident from both left and right. They will form a standing wave and trap the quantum fields.



$$\hat{\Psi} = \frac{\hat{\Psi}_+ + \hat{\Psi}_-}{2}$$

$$[\hat{\Psi}_k, \hat{\Psi}_l^\dagger] = \delta_{kl}$$

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System

For the scheme, we consider a waveguide, a hollow fiber for example, and two quantum fields $E_{1,\pm}$ and $E_{2,\pm}$ which propagate towards both the left and right directions. The fiber is filled with atoms of type **a** and **b**. The two quantum fields couple to the two species of four-level atoms **a** and **b** in the fiber, and also two classical, counter propagating control fields $\Omega_{1,\pm}$ and $\Omega_{2,\pm}$ are driving the intermediate transitions.

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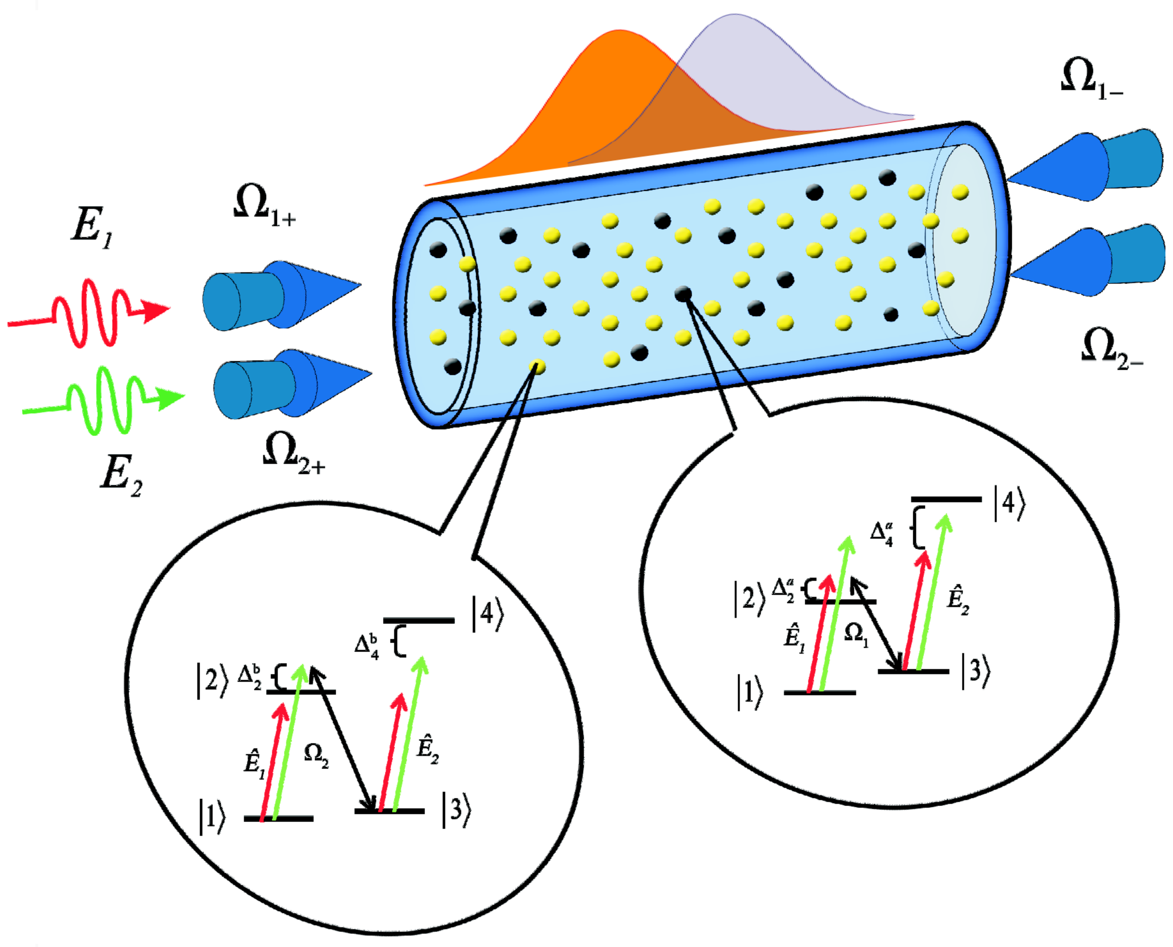

$$\begin{aligned}\mathsf{H}^{a,b} = & -\hbar n_z^{a,b}\sum_{i=1}^{2}\int dz\{-\omega_{33}^{a,b}\sigma_{33}^{a,b} + (-\omega_q^{(i)} + \Delta_2^{a,b})\sigma_{22}^{a,b} \\ & + (-\omega_{33}^{a,b} - \omega_q^{(i)} - \Delta_4^{a,b})\sigma_{44}^{a,b} \\ & + g_i^{a,b}\sqrt{2\pi}(\sigma_{21}^{a,b} + \sigma_{43}^{a,b}) \times \\ & \left(\hat{E}_{i,+}e^{i(k_{qu}^{(i)}z - \omega_{qu}^{(i)}t)} + \hat{E}_{i,-}e^{i(-k_{qu}^{(i)}z - \omega_{qu}^{(i)}t)}\right) \\ & + \sigma_{23}^{a,b} \times \\ & \left(\Omega_{i,+}(t)e^{i(k_{cl}^{(i)}z - \omega_{cl}^{(i)}t)} + \Omega_{i,-}(t)e^{-i(k_{cl}^{(i)}z + \omega_{cl}^{(i)}t)}\right) \\ & + \mathsf{H.c.}\}\end{aligned}$$

$n_z^{a,b}$ – densities of the atoms **a** and **b**

$E_{1,2\pm}$ – quantum fields $\Omega_{1,2\pm}$ – classical control fields

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Luttinger liquid of photons

Step One – Loading of the light pulses: The control fields $\Omega_{1,2+}$ are kept on until the quantum fields completely enter the fiber and then are adiabatically switched off.

Step Two – Nonlinear dynamics: Adiabatic switching on of all the control fields $\Omega_{1,2\pm}$ will create an effective Bragg grating that traps the quantum pulses. Simultaneous "shifting in" of the 4th level in each atomic species, will allow for the creation of the necessary nonlinear interactions.

$$\begin{aligned}H = & \hbar\int dz\left\{\sum_i\left[\frac{1}{2m_i}\partial_z\Psi_i^\dagger(z)\partial_z\Psi_i(z) + \frac{U_i}{2}\rho_i^2(z)\right]\right. \\ & \left. + V_{12}\rho_1(z)\rho_2(z)\right\}\end{aligned}$$

$$\rho_i(z) = \hat{\Psi}_i^\dagger(z)\hat{\Psi}_i(z) \qquad \frac{1}{m_{1,2}} = -\frac{4\Delta_2^{a,b}\nu_g^{(1,2)}}{\Gamma_{1D}^{a,b}n_z^{a,b}}$$

$$V_{12} = \frac{\pi(g_1^a)^2(g_2^a)^2\nu_g^{(1)}}{(g_2^b)^2\Delta_4^a\nu^{(1)}} + \frac{\pi(g_2^b)^2(g_1^b)^2\nu_g^{(2)}}{(g_1^a)^2\Delta_4^b\nu^{(2)}} \qquad U_{1(2)} = \frac{\Gamma_{1D}\nu_g^{1(2)}}{2\Delta_4^{a(b)}}$$

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To achieve the necessary strong repulsive interactions to reach the spin-charge separation regime, $U_{1,2}$ and $V_{1,2}$ should be **positive and larger** than the kinetic energy.

$v^{1(2)}$ are the corresponding velocities for each of the quantum fields in an empty waveguide and $v_g^{1(2)} = v^{1(2)}\Omega^2_{1(2)}/[\pi(g_{1(2)}^{a(b)})^2 n_z^{a(b)}]$ the corresponding group velocities of the untrapped pulses under the corresponding EIT conditions.

According to **Luttinger liquid** theory

$$\rho_i(z) = [\rho_{0,i} + \partial_x\phi_i(z)/\pi][1 + \cos(2\pi\rho_0 z) + 2\phi]$$

with $\Psi_i(z) = e^{-i\theta_i}\sqrt{\rho_{0,i}}$ for the two polaritons. From the Luttinger theory we find the velocities v_i and the parameters K_i defined as

$$v_i = \sqrt{\rho_{0,i}U_i/m_i} \qquad K_i = \pi\sqrt{\rho_{0,i}/m_iU_i}$$

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Spin-charge separation

With $v_1=v_2=v$ and $K_1=K_2=K$ the Hamiltonian becomes

$$H_c = \int\frac{dz}{2\pi}[u_cK_c(\partial_x\theta_c)^2 + \frac{u_c}{K_c}(\partial_x\phi_c)^2]$$

$$\begin{aligned}H_s = & \int\frac{dz}{2\pi}[u_sK_s(\partial_x\theta_s)^2 + \frac{u_s}{K_s}(\partial_x\phi_s)^2] \\ & + 2(V_1+V_2)\rho_0^2\cos\left(\sqrt{8}\phi_s\right)\end{aligned}$$

Where $\theta_{c,s} = (\theta_1 \pm \theta_1)/\sqrt{2}$, $\phi_{c,s} = (\phi_1 \pm \phi_1)/\sqrt{2}$ and

$K_{c,s} = \frac{K}{\sqrt{1 \pm (V_1+V_2)K/\pi u}}$. The spin and charge velocities are $u_{c,s} = u\sqrt{1 \pm \frac{(V_1+V_2)K}{\pi u}}$ with

$n_{c,s} = n_1 \pm n_2$ and $n_i = \langle\Psi_i^\dagger\Psi_i\rangle$.

The resulting difference in those velocities is known as spin-charge separation.

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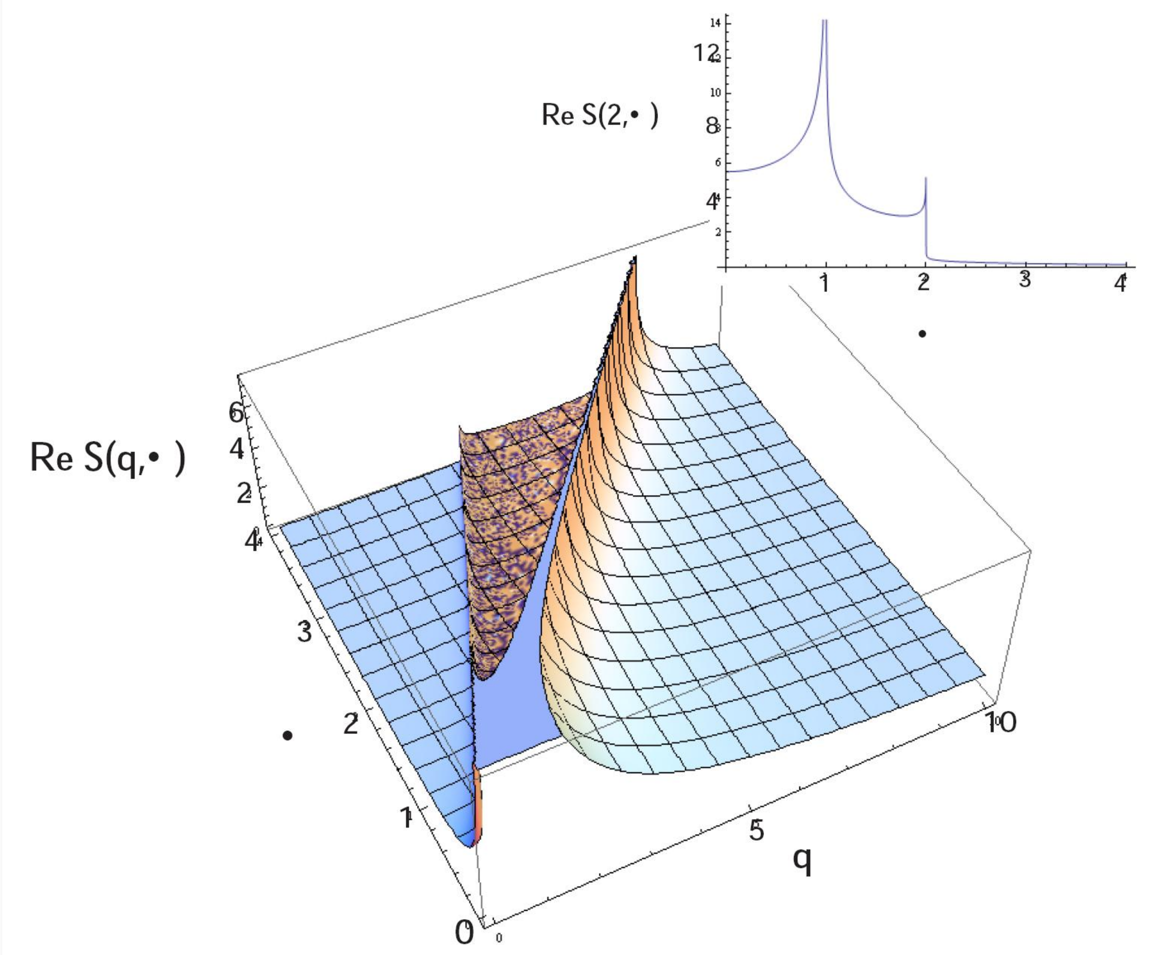
Measurement

By switching off one of the counter propagating classical fields, the correlations will be transferred from the polaritons to the propagating photon pulses. The corresponding charge (spin) density waves will transfer to time dependent photon intensities $n_i = \langle E_{i,+}^\dagger E_{i;+}\rangle$ which can be measured by standard optical techniques probing cross-correlations. The spectral function for momentum q has two peaks – one at $w=v_sq$ and one at $w=v_cq$ for OD = 3000, η = 0.4 and 10 photons in each pulse.



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