Computing Lyapunov functions using quadratic programming

Stability of an equilibrium x_0 in a dynamical system given by an ODE of the form $\dot{x} = f(x)$, $x \in \mathbb{R}^n$, can be studied by a Lyapunov function. A Lyapunov function is a function $V: \mathbb{R}^n \to \mathbb{R}$ which is strictly decreasing along solutions of the ODE. This can be expressed through the orbital derivative $V'(x)$, the derivative along solutions of the ODE, as

$$V'(x) := \nabla V(x) \cdot f(x) < 0 \tag{1}$$

for all $x \neq x_0$. There exists a method to compute Lyapunov functions by approximately solving an equation for $V'(x)$ using meshfree collocation, in particular Radial Basis Functions.
In this project, we want to solve the inequality (1) rather than an equation, using quadratic programming. The project will consist of understanding the theory, developing as well as programming the new method, and comparing it to the previous method by testing it for several examples in MATLAB.
References:
P. Giesl: Construction of Global Lyapunov Functions Using Radial Basis Functions. Lecture Notes in Math. 1904, Springer, 2007.
P. Giesl, C. Argaez, S. Hafstein and H. Wendland: Construction of a complete Lyapunov function using quadratic programming. In: Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotics - Volume 1: CTDE, pages 560-568 (2018).

Keywords: differential equations, Lyapunov function, basin of attraction, MATLAB, quadratic programming.
Recommended modules: Dynamical systems, Introduction to Mathematical Biology.