

Computational analysis of periodic orbits in nonsmooth differential equations

Ordinary differential equation $\dot{x} = f(t, x)$ with a nonsmooth right-hand side $f(t, x)$ arise in numerous applications, for example mechanical systems with dry friction or with switches.

We assume that f is periodic in t and $x \in \mathbb{R}$. There is a criterion that shows the existence, uniqueness and stability of a periodic orbit, and determines a subset of its basin of attraction. The criterion is based on finding a scalar-valued, nonsmooth function $W(t, x)$, satisfying a partial differential equation as well as additional conditions where f and W are nonsmooth.

In this project, we develop a numerical method to compute a suitable function W for a given nonsmooth differential equation. A discretized version of the conditions will be solved using quadratic programming. The project will consist of understanding the theory as well as programming the method for several examples in MATLAB.

References:

M. di Bernardo, C.J. Budd, A.R. Champneys and P. Kowalczyk: *Piecewise-smooth Dynamical Systems*, Springer, London, 2008.

P. Giesl: The basin of attraction of periodic orbits in nonsmooth differential equations. *ZAMM Z. Angew. Math. Mech.* 85 No. 2 (2005), 89-104.

P. Giesl, C. Arguez, S. Hafstein and H. Wendland: Construction of a complete Lyapunov function using quadratic programming. In: *Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotics - Volume 1: CTDE*, pages 560-568 (2018).

Keywords: nonsmooth differential equations, basin of attraction, approximation, MATLAB, quadratic programming.

Recommended modules: Dynamical systems, Introduction to Mathematical Biology.