M.Sc. Projects

 Absolutel Minimizing Lipschitz Extensions

2. Metric Dimension

 Arcs with Increasing Chords

# M.Sc. Projects

### Miroslav Chlebik m.chlebik@sussex.ac.uk

School of Mathematical and Physical Sciences Department of Mathematics University of Sussex Pevensey3 5C3

2013/2014 office hours: Friday 13:00-15:00

< □ > < 同 > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

 Arcs with Increasing Chords

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

 Arcs with Increasing Chords

### Key words

• Lipschitz mappings

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

### Key words

• Lipschitz mappings  $||f(x) - f(y)|| \le L||x - t||x|$ 

 $||f(x) - f(y)|| \le L||x - y||$  for all x, y

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

### Key words

• Lipschitz mappings

- $||f(x) f(y)|| \le L||x y||$  for all x, y
- minimizing Lipschitz extensions

M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

 Arcs with Increasing Chords

### Key words

• Lipschitz mappings

 $||f(x) - f(y)|| \le L||x - y||$  for all x, y

- minimizing Lipschitz extensions
- absolutely minimizing functions

M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

- Lipschitz mappings
- $||f(x) f(y)|| \le L||x y||$  for all x, y
- minimizing Lipschitz extensions
- absolutely minimizing functions
- infinity harmonic functions

M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

Arcs with Increasing Chords

- Lipschitz mappings
- $||f(x) f(y)|| \le L||x y||$  for all x, y
- minimizing Lipschitz extensions
- absolutely minimizing functions
- infinity harmonic functions
- comparison with cones

	Main open questions
M.Sc. Projects	
<ol> <li>Absolutely Minimizing Lipschitz Extensions</li> </ol>	
2. Metric Dimension	
3. Arcs with Increasing Chords	
	· · · · · · · · · · · · · · · · · · ·

M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

 Arcs with Increasing Chords

### Questions

• Problem of regularity of these functions

▲ロト ▲周ト ▲ヨト ▲ヨト ヨー のなべ

#### M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

### Questions

• Problem of regularity of these functions

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

• Are all these functions differentiable?

#### M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

### Questions

• Problem of regularity of these functions

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

• Are all these functions differentiable?

### Recommended courses

#### M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

 Arcs with Increasing Chords

### Questions

• Problem of regularity of these functions

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

• Are all these functions differentiable?

### Recommended courses

• Functional Analysis

#### M.Sc. Projects

1. Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

 Arcs with Increasing Chords

### Questions

• Problem of regularity of these functions

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

• Are all these functions differentiable?

### Recommended courses

- Functional Analysis
- Partial Differential Equations

NA	<b>D</b> '	
Metric	Dim	ension
IVICUIC		choin





◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 三臣 - のへぐ

#### M.Sc. Projects

1. Absolutel Minimizing Lipschitz Extensions

2. Metric Dimension

 Arcs with Increasing Chords

### Key words

• Hausdorff measures and Hausdorff dimension

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 三臣 - のへぐ

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

#### 2. Metric Dimension

3. Arcs with Increasing Chords

### Key words

• Hausdorff measures and Hausdorff dimension

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

• packing measures and packing dimension

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

#### 2. Metric Dimension

3. Arcs with Increasing Chords

### Key words

• Hausdorff measures and Hausdorff dimension

- packing measures and packing dimension
- upper and lower box-counting dimension

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

#### 2. Metric Dimension

3. Arcs with Increasing Chords

### Key words

• Hausdorff measures and Hausdorff dimension

- packing measures and packing dimension
- upper and lower box-counting dimension
- upper and lower Minkowski dimension

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

#### 2. Metric Dimension

3. Arcs with Increasing Chords

### Key words

• Hausdorff measures and Hausdorff dimension

- packing measures and packing dimension
- upper and lower box-counting dimension
- upper and lower Minkowski dimension
- Lipschitz mappings

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

#### 2. Metric Dimension

3. Arcs with Increasing Chords

### Key words

• Hausdorff measures and Hausdorff dimension

- packing measures and packing dimension
- upper and lower box-counting dimension
- upper and lower Minkowski dimension
- Lipschitz mappings
- rectifiable sets

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

#### 2. Metric Dimension

3. Arcs with Increasing Chords

### Key words

- Hausdorff measures and Hausdorff dimension
- packing measures and packing dimension
- upper and lower box-counting dimension
- upper and lower Minkowski dimension
- Lipschitz mappings
- rectifiable sets
- sets of fractional dimension and fractals



◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 三臣 - のへぐ

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

#### 2. Metric Dimension

3. Arcs with Increasing Chords

### Question

• To examine in detail these dimensions and their inter-relationship.

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQ@

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

 Arcs with Increasing Chords

### Question

• To examine in detail these dimensions and their inter-relationship.

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

Recommended courses

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

#### 2. Metric Dimension

 Arcs with Increasing Chords

### Question

• To examine in detail these dimensions and their inter-relationship.

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

### Recommended courses

• Measure and Integration

#### M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

#### 2. Metric Dimension

 Arcs with Increasing Chords

### Question

• To examine in detail these dimensions and their inter-relationship.

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ

### Recommended courses

- Measure and Integration
- Functional Analysis

M.Sc. Projects
3 Arcs with
J. Arcs With
Increasing Chords

◆□ ▶ < 圖 ▶ < 圖 ▶ < 圖 ▶ < 圖 • 의 Q @</p>



◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 三臣 - のへぐ



▲ロト ▲周ト ▲ヨト ▲ヨト ヨー のなべ

▲ロト ▲帰 ト ▲ ヨ ト ▲ ヨ ト ● の Q ()



 Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

- curve length
- rectifiable curve

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ



 Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

- curve length
- rectifiable curve
- chord of a curve

▲ロト ▲帰 ト ▲ヨト ▲ヨト - ヨ - の々ぐ



 Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

- curve length
- rectifiable curve
- chord of a curve
- calculus of variations

M.Sc. Projects
3 Arcs with
Increasing Chords
mereasing chords

◆□ ▶ < 圖 ▶ < 圖 ▶ < 圖 ▶ < 圖 • 의 Q @</p>

M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords Question

• A curve *C* in the plane has the increasing chord property if  $||x_2 - x_3|| \le ||x_1 - x_4||$  whenever  $x_1, x_2, x_3$  and  $x_4$  lie in that order on *C*.

It is known that for a plane curve with the increasing chord property and endpoints a and b its length L satisfies

 $L \le 2\sqrt{3}||a-b||.$ 

M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

## Question

• A curve *C* in the plane has the increasing chord property if  $||x_2 - x_3|| \le ||x_1 - x_4||$  whenever  $x_1, x_2, x_3$  and  $x_4$  lie in that order on *C*.

It is known that for a plane curve with the increasing chord property and endpoints a and b its length L satisfies

$$L \le 2\sqrt{3}||a-b||.$$

• Can you improve the above constant " $2\sqrt{3}$ "?

M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

## Question

• A curve *C* in the plane has the increasing chord property if  $||x_2 - x_3|| \le ||x_1 - x_4||$  whenever  $x_1, x_2, x_3$  and  $x_4$  lie in that order on *C*.

It is known that for a plane curve with the increasing chord property and endpoints a and b its length L satisfies

$$L \le 2\sqrt{3}||a-b||.$$

・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・

• Can you improve the above constant " $2\sqrt{3}$ "?

Recommended courses

M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

## Question

• A curve *C* in the plane has the increasing chord property if  $||x_2 - x_3|| \le ||x_1 - x_4||$  whenever  $x_1, x_2, x_3$  and  $x_4$  lie in that order on *C*.

It is known that for a plane curve with the increasing chord property and endpoints a and b its length L satisfies

$$L \le 2\sqrt{3}||a-b||.$$

・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・

• Can you improve the above constant " $2\sqrt{3}$ "?

### Recommended courses

• Functional Analysis

M.Sc. Projects

 Absolutely Minimizing Lipschitz Extensions

2. Metric Dimension

3. Arcs with Increasing Chords

## Question

• A curve *C* in the plane has the increasing chord property if  $||x_2 - x_3|| \le ||x_1 - x_4||$  whenever  $x_1, x_2, x_3$  and  $x_4$  lie in that order on *C*.

It is known that for a plane curve with the increasing chord property and endpoints a and b its length L satisfies

$$L \le 2\sqrt{3}||a-b||.$$

・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・
 ・

• Can you improve the above constant " $2\sqrt{3}$ "?

### Recommended courses

- Functional Analysis
- Partial Differential Equations