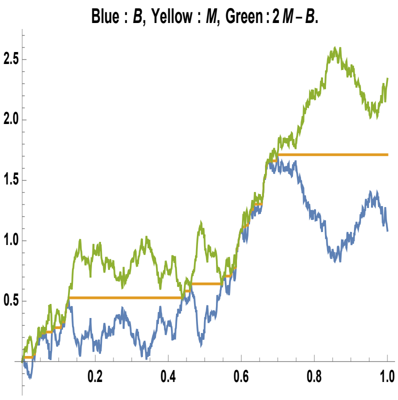
SYMMETRIES OF THE BROWNIAN PATH: FROM M/M/1 TO BROWNIAN QUEUES,

PROJECT PROPOSED BY ANTOINE DAHLQVIST

The aim of this project will be to understand and relate two classical results in probability.

1. When a Markovian queue with one server is at equilibrium, a beautiful result due to Burke shows that the arrival and the departure processes are independent Poisson processes with the same rate. On the one hand, the fact that the rates are identical confirms the intuition: there are as many people coming in as people going out of the queue. On the other hand, the independence seems surprising: if you see a lot of people arriving in a shop, you might think first that more people should then leave as a result.



2. When $(B_t)_{t\geq 0}$ is a standard Brownian motion[1] and M_t is the running maximum at time t, that is $M_t = \sup\{B_s : 0 \leq s \leq t\}$, the processes

$$kM_t - B_t, t \geq 0,$$

never satisfy the Markov property but for 3 values of k,

$$k \in \{0, 1, 2\}.$$

For $k = 0$, this is the Markov property of a Brownian motion. For $k = 1$, it is Paul Lévy's Theorem, saying that $(M_t - B_t)_{t\geq 0}$ has the same law as a reflected Brownian motion $(|B_t|)_{t\geq 0}$. When $k = 2$, it is due to Jim Pitman, $(2M_t - B_t)_{t\geq 0}$ has the same law as the norm of Brownian motion in three dimensions but also

[1] a sample of this random function is plotted above in blue.

the same law as a one-dimensional Brownian motion conditioned to stay positive forever!

Project:

(1) A first goal of the project will be to understand and prove Pitman's result both in a discrete[2] and continuous setting and to prove the Burke property for M/M/1 queues.
(2) The second part will investigate the interactions between these two results, studying the notion of Brownian queues.

References

[1] Brownian analogues of Burke's theorem, Neil O'Connell; Marc Yor., Stochastic processes and their applications
[2] Pitman's 2M -X Theorem for skip-free random walks with Markovian increments, Hambly, Martin, O'Connell, Elect. Comm. in Probab. 6 (2001) 7–77
[3] Brownian motion, Peter Mörters, Yuval Peres

Related Modules: Probability and Statistics, Probability models, Random processes.

[2] involving random walks.