Calculus of Variations, PDE, and Geometric Measure Theory

Titles and abstracts

Monday 7 September 2015

Morning session

Chair: Irene Fonseca

9:25-9:30 *Welcome: Peter Coles, Head of School*

9:30-10:30 Michael Struwe

An optimal local well-posedness result for the super-critical Lane-Emden heat flow.

Abstract: In joint work with Simon Blatt we investigate the existence of solutions to the Cauchy problem for the super-critical Lane-Emden heat flow with data in a suitable Morrey space.

10:30-11:00 Coffee Break

11:00-12:00 Gianni Dal Maso

Fracture in elasto-plastic materials.

Abstract: We study the asymptotic behaviour of a variational model for damaged elasto-plastic materials in the case of antiplane shear. The energy functionals we consider depend on a small parameter ϵ, which forces damage concentration on regions of codimension one. We determine the Γ-limit as ϵ tends to zero and show that it contains an energy term involving the crack opening.

12:00-1:00 Giuseppe Mingione

Double phase functionals: Regularity, Irregularity, and Calderón-Zygmund estimates.

Abstract: Double phase functionals are integral functionals of the Calculus of Variations characterized by the fact that their integrand changes its degree of ellipticity/coercivity accordingly to the space variable. These for instance emerge when providing models for very anisoptropic media in elasticity and homogenization. They fall in the realm of functionals with non-standard growth conditions for which regularity of minimizers do not always holds and Lavrentiev phenomenon appears. I will try to give an overview of the regularity results available in terms of maximal regularity for the gradient of minima and Calderón-Zygmund theory for related operators.

1:00-2:30 Lunch

Afternoon session

Chair: Michael Struwe

2:30-3:30 Bernd Kirchheim

Square Functions and the Muckenhoupt Weight Classes of Elliptic Measures.

Abstract: We give a new chacharacterization of the property that the elliptic measure belongs to the infinity weight Muckenhoupt class in terms of a Carleson measure property of bounded solutions. This is joint work with C. Kenig, J. Pipher and T. Toro.

3:30-4:00 Coffee Break

4:00-5:00 Maria Giovanna Mora

Homogenization of time-dependent systems of dislocations.

Abstract: It is well known that plastic, or permanent, deformation in metals is caused by the concerted movement of many curve-like defects in the crystal lattice, called dislocations. What is not yet known is how to use this insight to predict behaviour at continuum scales. In this talk I will present a rigorous upscaling result for a discrete system of moving edge dislocations in two dimensions with slip-plane confinement. In the continuum limit we obtain an evolution law for the dislocation density. This is a joint work with Mark A. Peletier (Eindhoven) and Lucia Scardia (Bath).

5:00-6:00 Giovanni Leoni

On slow motion for phase transitions.

Abstract: We discuss slow motion for phase transitions in higher dimensions through asymptotic development by Gamma convergence.

8:00-10:30 Social Dinner

Tuesday 8 September 2015

Morning session

Chair: Gianni Dal Maso

9:30-10:30 Irene Fonseca

Quantum Dots and Dislocations: Dynamics of Materials Defects

Abstract: The formation and assembly patterns of quantum dots have a significant impact on the optoelectronic properties of semiconductors. We will address short time existence for a surface diffusion evolution equation with curvature regularization in the context of epitaxially strained three-dimensional films. We will discuss optimal faceted shapes of quantum dots in epitaxially deposited films in the case in which there are a non-vanishing crystallographic miscut and a lattice incompatibility between the film and the substrate. Existence of faceted minimizers for every volume of the deposited film will be established. It will be shown that there is no wetting effect for small volumes, geometric properties including a faceted version of the zero contact angle will be derived, and the explicit shapes of minimizers for small volumes will be identified. Further, the nucleation of misfit dislocations will be analyzed.

10:30-11:00 Coffee Break

11:00-12:00 Jan Kristensen

The Morse-Sard theorem, generalized Luzin property
and level sets for Sobolev functions.

Abstract: Many classical results from multivariate calculus can be generalized to suitable Sobolev functions that need not even be everywhere differentiable. In this talk we discuss some new results that have been obtained in joint work with Jean Bourgain (Princeton) and Mikhail Korobkov (Novosibirsk).

12:00-1:00 Massimiliano Morini

Nonlocal geometric flows

Abstract: We consider a class of geometric, possibly nonlocal, motions that may be regarded as gradient flows (with respect to a suitable L^2-Riemannian structure) of some "generalized" sufficiently smooth perimeter functionals. We develop a general viscosity framework, implement a generalized minimizing movement scheme à la Almgren-Taylor-Wang, and discuss the consistency of the two approaches. Our treatment allows us to unify several results scattered in the literature as well as to establish new ones. However, it does not apply to the important case of a crystalline anisotropy.

Regarding the latter, we discuss some very recent developments, namely a new distributional formulation in terms of the distance function, which leads to existence and uniqueness in all dimensions for a relevant class of crystalline mean curvature flows. All the results are obtained in collaboration with A. Chambolle and M. Ponsilgione.

1:00-2:30 Lunch

Afternoon session

Chair: Jan Kristensen

2:30-3:30 Neshan Wickramasekera

TBA

3:30-4:00 Coffee Break

4:00-5:00 Jan Malý

Jacobians and Hessians of Sobolev mappings.

Abstract: The signum of Jacobian of a Sobolev mapping has been recently studied to compare its topological and analytical orientation. If a Sobolev mapping is (topologically) sense preserving, then it sounds reasonable that its Jacobian should be nonnegative a.e. However, this is true only if the dimension of the space is ≤ 3 or if the Sobolev exponent is not too low. On the other hand, it can happen that the Jacobian of a Sobolev homeomorphism vanishes a.e., so that we cannot obtain any indication towards its topological orientation. Similarly, we present a Sobolev function whose Hessian (determinant) vanishes a.e. and still the function itself is strictly convex. This example is related to the Monge-Ampère equation. Some more related results are also mentioned. The talk is based on joint works with Stanislav Hencl, Zhuomin Liu and Reza Pakzad.