Random Matrices and Applications

Titles and abstracts

Monday 16$^{\text{th}}$ September 2019

10:20-10:50 *Coffee and greetings*

10:50-11:00 *Welcome*

11:00-12:00 Yan Fyodorov (Kings College London)

On statistics of bi-orthogonal eigenvectors in non-selfadjoint Gaussian random matrices

Abstract: I will discuss a method of studying the joint probability density (JPD) of an eigenvalue and the associated 'non-orthogonality overlap factor' (also known as the 'eigenvalue condition number') of the left and right eigenvectors for non-selfadjoint Gaussian random matrices. I will sketch derivation of the general finite expression for the JPD of a real eigenvalue and the associated non-orthogonality factor in the real Ginibre ensemble, and then analyze its 'bulk' and 'edge' scaling limits. I will also discuss ongoing work on real elliptic ensembles. The ensuing distribution is maximally heavy-tailed, so that all integer moments beyond normalization are divergent. This work complements the studies of P. Bourgade and G. Dubach. The presentation will be mainly based on: Y.V. Fyodorov, Commun. Math. Phys. 363 (2), 579–603 (2018)

12:00-13:30 Lunch Break

13:30-14:30 Nina Snaith (University of Bristol)

Zeros, moments and determinants

Abstract: For 20 years we have known that average values of characteristic polynomials of random unitary matrices provide a good model for moments of the Riemann zeta function. Now we consider mixed moments of characteristic polynomials and their derivatives, calculations which are motivated by questions on the distribution of zeros of the derivative of the Riemann zeta function.

14:30-15:00 Coffee break

15:00-16:00 Francesco Mezzadri (University of Bristol)

Moments of Random Matrices and Hypergeometric Orthogonal Polynomials

Abstract: We establish a new connection between spectral moments of $n \times n$ random matrices X_n and hypergeometric orthogonal polynomials. Specifically, we consider moments $\mathbb{E}[\mathrm{Tr}(X_n^{-s})]$ as a function of the complex variable s, whose analytic structure we describe completely. We discover several remarkable features, including a reflection symmetry (or functional equation), zeros on a critical line in the complex plane, and orthogonality relations. We characterise the moments in terms of the Askey scheme of hypergeometric orthogonal polynomials. We also calculate the leading order $n \to \infty$ asymptotics of the moments and discuss their symmetries and zeroes. We discuss aspects of these phenomena beyond the random matrix setting, including the Mellin transform of products and Wronskians of pairs of classical orthogonal polynomials. When the random matrix model has orthogonal or symplectic symmetry, we obtain a new duality formula relating their moments to hypergeometric orthogonal polynomials. This work is in collaboration with Fabio Cunden, Neil O'Connell and Nick Simm.

16:00-16:30 Coffee Break

16:30-17:30 Nick Simm (University of Sussex)

Characteristic polynomials of complex random matrices and Painlevé transcendents

Abstract: I will discuss correlation functions of characteristic polynomials for some $N \times N$ non-Hermitian random matrix models, both for finite-N and as $N \to \infty$. We reveal several instances where the correlation functions (or moments) can be expressed in terms of Painlevé transcendents: solutions of a special class of non-linear second order differential equations. The approach is based on finding appropriate *duality identities* which relate non-Hermitian matrices to their Hermitian or unitary counterparts. This is joint work with Alfredo Deaño (University of Kent).

7:30 Dinner in Brighton