New Perspectives in Analysis, Probability and Applications

A three-day long workshop on *New Perspectives in Analysis, Probability and Applications* will take place at the Department of Mathematics, University of Sussex from Wednesday 24 to Friday 26 January 2018. The purpose of this workshop is to discuss recent developments in these areas, and to explore future directions for common interaction. This is also the occasion to celebrate the new appointments of several lecturers at Sussex.

Speakers include:

- John Ball (Oxford)
- Denis Denisov (University of Manchester)
- Andrew Duncan (Sussex)
- Dmitry Korshunov (University of Lancaster)
- Konstantinos Koumatos (Sussex)
- Grigorios Pavliotis (Imperial College London)
- Michela Ottobre (Heriot-Watt)
- Lucia Scardia (University of Bath)
- Vladislav Vysotsky (Sussex)

The workshop consists of three one-day mini-conferences, each devoted to a special topic:

Wednesday 24th January: Day on Mathematics of Material Science

10:30 - 11:00 Coffee
11:00 - 12:00 John Ball (University of Oxford and Heriot-Watt University): Generalized Hadamard jump conditions and polycrystal microstructure
12:00 - 13:30 Lunch
13:30 - 14:30 Lucia Scardia (University of Bath): The equilibrium measure for a nonlocal dislocation energy
14:30 - 15:00 Coffee
15:00 - 16:00 Konstantinos Koumatos (University of Sussex): Programming of shape in narrow strips of liquid crystal elastomers

Thursday 25th January: Day on Diffusions: Theory and Applications

10:30 – 11:00 Coffee Break
11:00 – 12:00 Michela Ottobre (Heriot Watt): A one-dimensional model for self-propelled diffusions
12:00 – 13:30 Lunch
13:30 – 14:30 Grigorios Pavliotis (Imperial College London): Long time behaviour and phase transitions for the McKean-Vlasov equation
14:30 - 15:00 Coffee Break
14:00 – 15:00 Andrew Duncan (University of Sussex): Tuning Monte Carlo Samplers with Diffusions

Friday 26th January: Day on Markov Chains

10:00 – 11:00 Dmitry Korshunov (University of Lancaster): Change of measure technique for Markov chains
11:00 – 11:30 Coffee break
11:30 – 12:30 Denis Denisov (University of Manchester): Key renewal theorem for asymptotically homogeneous Markov chains
12:30 – 14:10 Lunch
14:10 – 15:10 Vladislav Vysotsky (University of Sussex): Stationarity of overshoots of random walks via induced Markov chains

Abstracts

John Ball (University of Oxford and Heriot-Watt University)
Title: Generalized Hadamard jump conditions and polycrystal microstructure
Abstract: The talk will describe various generalizations of the Hadamard jump condition, and how they can lead to information about polycrystal microstructure arising from martensitic phase transformations. Joint work with Carsten Carstensen (Humboldt University, Berlin).

Lucia Scardia (University of Bath)
Title: The equilibrium measure for a nonlocal dislocation energy
Abstract: In this talk I will present a recent result on the characterisation of the equilibrium measure for a nonlocal and anisotropic energy arising as the Gamma-limit of discrete interacting dislocations, and an extension to more general anisotropies.

This is joint work with J.A. Carrillo, J. Mateu, M.G. Mora, L. Rondi and J. Verdera.

Konstantinos Koumatos (University of Sussex)
Title: Programming of shape in narrow strips of liquid crystal elastomers
Abstract: Using the theory of Gamma-convergence, we derive from three-dimensional elasticity new one-dimensional models for ribbons exhibiting spontaneous curvature and twist. We apply the models to shape-selection problems for thin films of nematic elastomers with the twist and splay-bend geometries for the nematic director. For the former, we discuss the possibility of helicoid-like shapes as an alternative to spiral ribbons.

This is joint work with V. Agostiniani and A. DeSimone

Michela Ottobre (Heriot Watt University)
Title: A one-dimensional model for self-propelled diffusions
Abstract: One of the new challenges of statistical mechanics arises from the study of interacting particle systems of self-propelled particles. Such models are at the root of many biological phenomena, such as bacterial migration, flocking of birds etc. In this talk we will consider a non-linear PDE for a Viksek-type model (i.e. a kinetic PDE with a Vlasov-type nonlinearity). The PDE at hand is i) not in gradient form and ii) it is non-uniformly elliptic (but hypoelliptic instead). Moreover, as typical in this framework, the dynamics exhibits multiple equilibria (stationary states). This is a joint work with P.Butta (La Sapienza, Rome), F. Flandoli (Scuola Normale, Pisa) and B. Zegarlinski (Imperial College).

Greg Pavliotis (Imperial College London)
Title: Long time behaviour and phase transitions for the McKean-Vlasov equation
Abstract: We study the long-time behaviour and the number and structure of stationary solutions for the McKean-Vlasov equation, a nonlinear nonlocal Fokker-Planck type equation that describes the mean field limit of a system of weakly interacting diffusions. We consider two cases: the McKean-Vlasov equation in a multiscale confining potential with quadratic, Curie-Weiss, interaction (the so-called Dasai-Zwanzig model), and the McKean-Vlasov dynamics on the torus with periodic boundary conditions and with a localized interaction. Our main objectives are the study of convergence to a stationary state and the construction of the bifurcation diagram for the stationary problem. The application of our work to the study of models for opinion formation is also discussed.

Andrew Duncan (University of Sussex)
Title: Tuning Monte Carlo Samplers with Diffusions
Abstract: As increasingly sophisticated enhanced sampling methods are developed, a challenge that arises is how to tune the simulation parameters appropriately to achieve good sampler performance. This is particularly pertinent for biased sampling methods, which trade off asymptotic exactness for computational speed. While a reduction in variance due to more rapid sampling can outweigh any bias introduced, the inexactness creates new challenges for hyper-parameter selection. The natural metric in which to quantify this discrepancy is the Wasserstein or Kantorovich metric. However, the computational difficulties in computing this quantity has typically dissuaded practitioners. To address this, we introduce a new computable quality measure using Stein operators constructed from diffusions which quantify the maximum discrepancy between sample and target expectations over a large class of test functions. We will also illustrate applications to hyper-parameter selection, convergence rate assessment, and quantifying bias-variance trade-offs in sampling. This is joint work with Sebastian Vollmer (University of Warwick), Jackson Gorham (Stanford University) and Lester Mackey (Microsoft Research).

Dmitry Koshunov (University of Lancaster)
Title: Change of measure technique for Markov chains

Abstract: We discuss general methodology of Doob's h-transform for Markov chains, and particularly for Markov chains with asymptotically zero drift. We show how Doob's h-transforms allow us to derive sharp tail asymptotics of invariant measures and return probabilities of recurrent Markov chains. It is a good compliment to the approach based on Lyapunov functions, which leads to certain super- or submartingales and whose main disadvantage is that it is very hard (or even impossible) to derive exact asymtotics via such test functions.

Denis Denisov (University of Manchester)
Title: Key renewal theorem for asymptotically homogeneous Markov chains
Abstract: We consider a one-dimensional transient Markov chains X_n on a positive half line. Let $H_y(B) = \Sigma_{n=0}^{\infty} P_y(X_n \in B)$ be the renewal measure of X_n. In this talk I will discuss renewal theorems for H_y .First I will consider renewal theorems on growing intervals $H_y(x, x + t(x))$, where *t(x)* is a monotone increasing to infinity function. Next, assuming additionally that X_n is asymptotically homogeneous, I will discuss a local (and hence key) renewal theorem for $H_y(x, x + t]$ as *x* tends to infinity, where *t* is fixed.

The talk is based on the following joint work with D. Korshunov and V.Wachtel. The key renewal theorem will appear in the next revision of this preprint.
D. Denisov, D. Korshunov and V. Wachtel (2016). *At the Edge of Criticality: Markov Chains with Asymptotically Zero Drift*, arXiv:1612.01592, 1-213.

Vladislav Vysotsky (University of Sussex)
Title: Stationarity of overshoots of random walks via induced Markov chains
Abstract: Consider the Markov chain formed by the sizes of consequent overshoots over a fixed level of a recurrent one-dimensional random walk. We show that this chain is ergodic and has a unique invariant distribution of a simple explicit form. We will present a natural approach to obtain these properties using general results of ergodic theory with infinite invariant measure. This approach is universal: it allows us to find invariant measure and prove its uniqueness and ergodicity for general recurrent Markov chains sampled at the moments of returns to a fixed set. This is a joint work with Alex Mijatovic (Kings College London).