Workshop on Analysis and PDEs

Titles and abstracts

Monday 13 June 2022

Morning session

9:55-10:00 *Welcome: David Edmunds*

10:00-11:00 Dmitri Vassiliev

Beyond the Hodge Theorem: curl and asymmetric pseudodifferential projections

Abstract: Consider the operator $curl := *d$ acting on 1-forms over a connected oriented closed Riemannian 3-manifold. Put $P_\pm := \theta(\pm curl)$, θ being the Heaviside step function. The operators $P_\pm$ are completely determined by the Riemannian manifold and its orientation, and they constitute an orthonormal pair of projections which decompose the Hilbert space of real-valued coexact 1-forms into two orthogonal subspaces. We prove that the operators $P_\pm$ are pseudodifferential, write down their principal and subprincipal symbols and provide an algorithm for the explicit computation of their full symbols. We then consider the operator $P_+ - P_-$ and take its pointwise matrix trace. This gives us a scalar pseudodifferential operator A which we call the asymmetry operator. We prove that A is an operator of order -3 and define its regularised operator trace. This trace is a differential geometric invariant, a measure of the asymmetry of our Riemannian manifold under change of orientation.

The talk is based on the use of pseudodifferential techniques developed in a series of recent joint papers by Matteo Capoferri (Cardiff) and myself.

11:00-11:30 Coffee Break

11:30-12:30 Luboš Pick

Positioning of Orlicz space and optimality

Abstract: One of the basic questions studied in functional analysis is when a given operator, say T, is bounded from X to Y, written $T\colon X \to Y$, in which X and Y are some suitable structures. In certain disciplines such as the regularity theory of PDEs, T often takes the form either of some integral operator, or of the identity operator, and X, Y are certain appropriate function spaces. Among these, Lebesgue spaces play a primary role, but they do not hold all the answers and there are other function spaces that are also of interest. For applications, it is usually good to know that the result is in some sense the best possible. In practice this means that we want X as large as possible and Y as small as possible. To this end, we need some reasonable pool of competing spaces. In the talk we focus on the specific situation when the role of such a pool is played by the class of all Orlicz spaces.

We shall consider the highly nontrivial question of the very existence of the optimal Orlicz space. We shall point out that one of the crucial parameters is the positioning of the Orlicz space within its fundamental segment. Every Lebesgue space L^p is sandwiched between the Lorentz space $L^{p,1}$ and the Marcinkiewicz space $L^{p,\infty}$. A similar situation holds for each Orlicz space. We shall use this fact to prove certain principal alternative concerning the relation of a Lorentz space and the union of its Orlicz subspaces. We apply our result to solve a long-standing open problem whether the celebrated limiting Sobolev embedding due to Brézis and Wainger has optimal Orlicz domain. Our approach also brings an alternative and surprisingly simple argument that can be used to obtain several optimality results that had been known for long time. The talk is based on a joint work with Vít Musil (Brno) and Jakub Takáč (Warwick).

12:30-2:00 Lunch

Afternoon session

2:00-3:00 Amiran Gogatishvili

Compact embeddings in variable exponent spaces

Abstract: Let Ω be an open subset of $\mathbb{R}^N$, and let p, $q : \Omega \to [1, \infty]$ be measurable functions. We give a necessary and sufficient condition for the embedding of the variable exponent space $L^{p(\cdot)}(\Omega)$ in $L^{q(\cdot)}(\Omega)$ to be almost compact. This leads to a condition on Ω, p and q sufficient to ensure that the Sobolev space $W^{1,p(\cdot)}(\Omega)$ based on $L^{p(\cdot)}(\Omega)$ is compactly embedded in $L^{q(\cdot)}(\Omega)$; compact embedding results of this type already in the literature are included as special cases.

3:00-4:00 Eugene Shargorodsky

Interpolation of variable Lebesgue spaces

Abstract: I will discuss estimates for the interpolation constant in the Riesz-Thorin type theorem for variable Lebesgue spaces. In many cases, our upper estimates are sharp. This part of the talk is based on a joint paper with O. Karlovych.

Let B_M the class of reflexive variable Lebesgue spaces on which the Hardy-Littlewood maximal operator is bounded. The main result of the second part of the talk is that any space in B_M is an interpolation space between any standard reflexive Lebesgue space and another space in B_M. This part of the talk is based on a joint paper with L. Diening and O. Karlovych.

4:00-4:30 Coffee Break

4:30-5:30 Dorothee Haroske

Nuclear embeddings in function spaces – some recent results

Abstract: We study nuclear embeddings for spaces of Besov and Triebel-Lizorkin type. We survey some recent results in weighted spaces, smoothness Morrey spaces, spaces on quasi-bounded domains and, finally, extending to the behaviour of the Fourier transform in such spaces. Here we can extend our previous results on the compactness of corresponding embeddings. The concept of nuclearity goes back to Grothendieck (1955) and was the basis for many fundamental developments in functional analysis. Essential tools are a discretisation in terms of wavelet bases, operator ideal techniques, as well as a very useful result of Tong (1969) about the nuclearity of diagonal operators acting in sequence spaces.

This is joint work with Leszek Skrzypczak (Adam Mickiewicz University Poznań, Poland), Hans-Gerd Leopold (Friedrich Schiller University Jena, Germany), and Hans Triebel (Friedrich Schiller University Jena, Germany).

Tuesday 14 June 2022

Morning session

10:00-11:00 Andrea Cianchi

Second-order regularity of solutions to nonlinear elliptic problems

Abstract: Second-order regularity results are established for solutions to elliptic equations and systems with principal part having Uhlenbeck structure and square integrable right-hand sides. Both local and global estimates are obtained. The latter apply to solutions to homogeneous Dirichlet problems under minimal regularity assumptions on the boundary of the domain. In particular, if the domain is convex, no regularity of its boundary is needed. A key step in the approach is a sharp pointwise inequality for the involved elliptic operator. This talk is based on joint investigations with A. Balci, L. Diening and V. Maz'ya.

11:00-11:30 Coffee Break

11:30-12:30 David Bourne

Optimal transport theory and weather modelling

Abstract: I will present an application of optimal transport theory to simplified models of large-scale rotational flows (weather).

The semi-geostrophic equation is used by researchers at the Met Office to diagnose problems in simulations of more complicated weather models. It has also attracted a lot of attention in the applied analysis community, e.g., Alessio Figalli's work on the semi-geostrophic equation is listed in his Fields Medal citation. In this talk I will discuss the semi-geostrophic equation in geostrophic coordinates (SG), which is a nonlocal transport equation, where the transport velocity is defined via an optimal transport problem, or equivalently a Monge-Ampère equation. Using recent results from semi-discrete optimal transport theory, we give a new proof of the existence of weak solutions of SG. The proof is constructive and leads to an efficient numerical method. I will conclude talk by showing some simulations of weather fronts.

This is joint work with Charlie Egan and Beatrice Pelloni (Heriot-Watt University and the Maxwell Institute for Mathematical Sciences), Mark Wilkinson (Nottingham Trent University), Steven Roper (University of Glasgow), Colin Cotter (Imperial College London) and Mike Cullen (Met Office - retired).

12:30-2:00 Lunch

Afternoon session

2:00-3:00 Claudia Garetto

Well–posedness of hyperbolic Cauchy problems with multiplicities

Abstract: In this survey talk I will discuss well-posedness of hyperbolic Cauchy problems with multiplicities and the role played by the lower order terms (Levi conditions). I will summarise the results obtained in the last ten years in collaboration with Michael Ruzhansky and discuss some open problems for higher order hyperbolic equations.

3:00-4:00 Stefano Spirito

The Inviscid Limit for 2D Incompressible Fluid with Unbounded Vorticity

Abstract: In this talk we review some recent results concerning the inviscid limit for the 2D Euler equations with unbounded vorticity. In particular, by using techniques from the theory transport equations with no smooth vector fields, we show that the solutions obtained in the vanishing viscosity limit satisfiy a representation formula in terms of the flow of the velocity and that the strong convergence of the vorticity has a log rate in term on the viscosity. The talk is based on results obtained in collaboration with Gianluca Crippa (Univ. Basel) and Gennaro Ciampa (BCAM - Bilbao).

7:30-10:30 Drinks and buffet supper at Rose and David Edmunds' place

Wednesday 15 June 2022

Morning session

10:00-11:00 Lucia Scardia

Equilibrium measures for nonlocal energies: the effect of anisotropy

Abstract: Nonlocal energies are continuum models for large systems of particles with long-range interactions. Under the assumption that the interaction potential is radially symmetric, several authors have investigated qualitative properties of energy minimisers. But what can be said in the case of anisotropic kernels?

I will present some results and partial answers in this direction obtained in a long-standing collaboration with Maria Giovanna Mora and Luca Rondi, and with José Antonio Carrillo, Joan Mateu and Joan Verdera.

11:00-11:30 Coffee Break

11:30-12:30 Leonid Parnovski

Classical wave methods and modern gauge transforms: spectral asymptotics in the one-dimensional case

Abstract: The question of high energy asymptotics for the kernel of the spectral projector of the Laplacian in the context of compact manifolds is one of the most well studied areas of spectral theory since the early 1900s. In this talk, I discuss the analogous question for Schrödinger operators on the real line: What is the asymptotic behaviour of the spectral projector of a Schrödinger operator on the real axis? By combining the classical wave method, originally introduced by B. Levitan in the 1950s, with the periodic gauge transform technique, we are able to show that when the potential is bounded with all derivatives, the diagonal values of the kernel of the spectral projection (aka the local density of states), has a full asymptotic expansion in powers of the spectral parameter. This settles the one-dimensional case of a conjecture that we have formulated together with R. Shterenberg in 2014. This is a joint work with J. Galkowski and R. Shterenberg.

12:30-2:00 Lunch

Afternoon session

2:00-3:00 Alexander V. Sobolev (University College London)

Regularity of solutions for the Coulomb multi-particle Schrödinger equation, and the one-particle density matrix

Abstract: Regularity properties for multi-particle systems have been the focus of mathematical research since 1950's. Such results determine spectral properties of the one-particle density matrix which is the key objects in the quantum-mechanical approximation schemes. In this talk I will give a short survey of recent regularity results with emphasis on sharp bounds for the eigenfunctions, and show how these bounds lead to the asymptotic formula for the eigenvalues of the one-particle density matrix.