Statistical and empirical properties of Factor Model Quantile Simulation (FMQS)

Yang Han (Technische Universität München)

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Simulation of Stock Returns through FMQS

Quantile Regression on Factor Model and Simulation of Market Returns

Factor Model

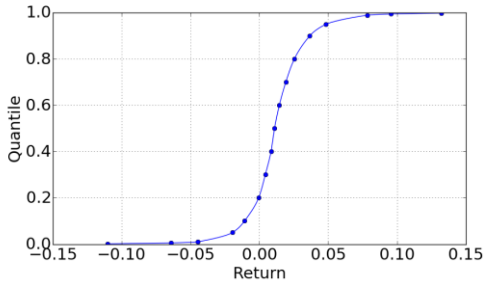
$$R_i = \alpha_i + \beta_i R_M + \gamma_i R_M^2 + \varepsilon_i$$

Model for Market Return

$$dY_t = \mu dt + \sqrt{V_{t-}} dW_t^y + dJ_t^y$$

FMQS

Distribution of each Stock Return



Potential Applications

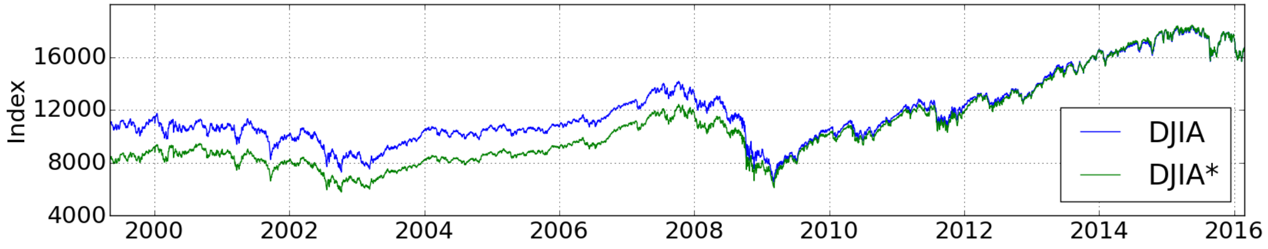
- Risk Management
- Importance Sampling
- Portfolio Optimization
- ...

Outline of FMQS Methodology (1/6)

Adjustment of the historical Data

- **Time Series Data**
- Quantile Regression
- Market Simulation
- Quantile Calculation
- Interpolation
- Inversion Sampling

Investment Universe	All 30 stocks currently in DJIA
Market Return	Estimated through index DJIA*
Time Horizon	4 Mai 1999 to 25 February 2016
Adjustments	Visa stock prices prior 18 March 2008 were reconstructed by PCA



Outline of FMQS Methodology (2/6)

Quantile Regression estimates the Parameters of the Factor Model

- Time Series Data
- **Quantile Regression**
- Market Simulation
- Quantile Calculation
- Interpolation
- Inversion Sampling

Quadratic CAPM	$R_i = \alpha_i + \beta_i R_M + \gamma_i R_M^2 + \varepsilon_i$
Quantile Regression	$Q^q(R_i \mid X) = \alpha_i^q + \beta_i^q R_M + \gamma_i^q R_M^2 + \varepsilon_i^q$

Results	Estimation of $\alpha_i^q, \beta_i^q, \gamma_i^q \quad \forall i, q$

Outline of FMQS Methodology (3/6)

Market Return Simulation through a Stochastic Volatility Jump Diffusion Model

- Time Series Data
- Quantile Regression
- **Market Simulation**
- Quantile Calculation
- Interpolation
- Inversion Sampling

Stock Process	$dY_t = \mu dt + \sqrt{V_{t-}} dW_t^y + dJ_t^y$
Volatility Process	$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_{t-}} dW_t^\nu + dJ_t^v$
Jump Process	$dJ_t^y = \xi_t^y dN_t$

Results	Simulation of R_M

Outline of FMQS Methodology (4/6)

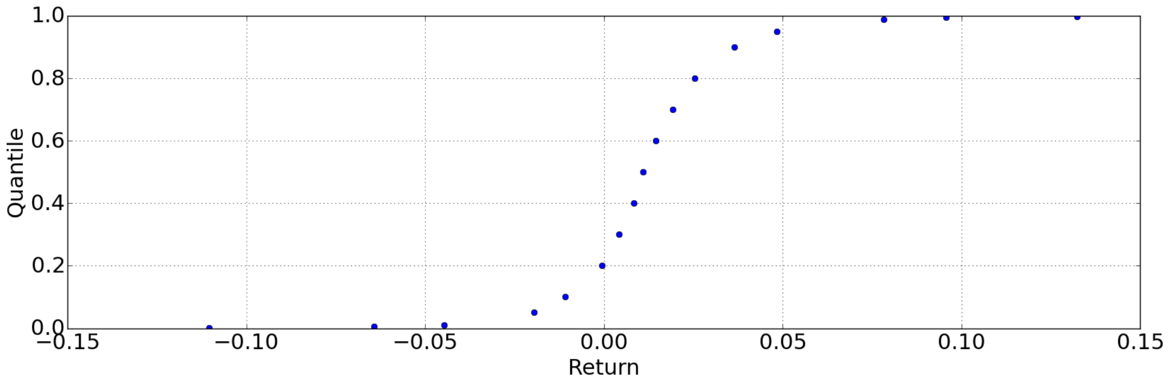
Quantiles can be calculated using the previous Results

- Time Series Data
- Quantile Regression
- Market Simulation
- **Quantile Calculation**
- Interpolation
- Inversion Sampling

$\alpha_i^q, \beta_i^q, \gamma_i^q \quad \forall i, q$

R_M

Quadratic CAPM $R_i = \alpha_i + \beta_i R_M + \gamma_i R_M^2 + \varepsilon_i$

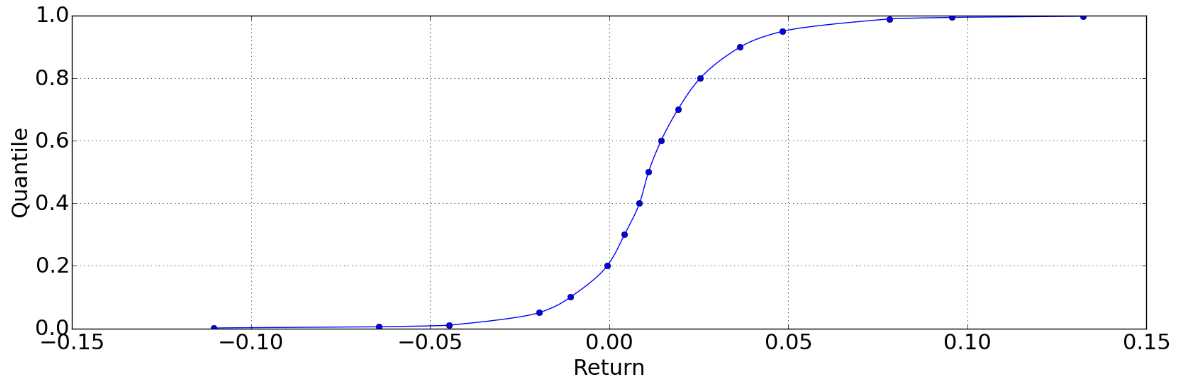


Outline of FMQS Methodology (5/6)

Through Interpolation we generate the Distribution from the Quantiles

- Time Series Data
- Quantile Regression
- Market Simulation
- Quantile Calculation
- **Interpolation**
- Inversion Sampling

Monotonicity	Interpolation maintains monotonicity
Preservation of shape	Interpolation is continuously differentiable

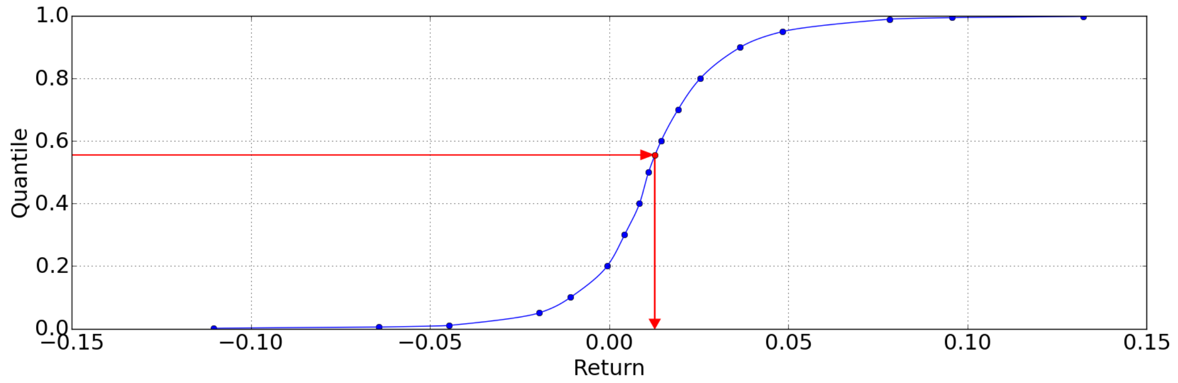


Outline of FMQS Methodology (6/6)

Inversion Sampling can be used to sample from the calculated Distribution

- Time Series Data
- Quantile Regression
- Market Simulation
- Quantile Calculation
- Interpolation
- **Inversion Sampling**

Let U be an uniform random variable on $(0,1)$ and let F^{-1} be the inverse of some distribution F. Then $Y := F^{-1}(U)$ follows the distribution F.



Empirical Results

Parameter estimation for Quantile Regression and SVJD Model

Quantile Regression

$$R_i = \alpha_i + \beta_i R_M + \gamma_i R_M^2 + \varepsilon_i$$

	q	Quantiles	alpha	beta	gamma
0	0.1	-0.010812	-2.158028e-02	1.138099	-6.127396
1	0.2	-0.000578	-1.172315e-02	1.170988	-5.646079
2	0.3	0.004271	-6.708940e-03	1.118705	-2.067595
3	0.4	0.008439	-2.523657e-03	1.107437	-1.118226
4	0.5	0.010959	3.688283e-07	1.094858	0.099319
5	0.6	0.014461	3.262038e-03	1.091171	2.868163
6	0.7	0.019166	7.812528e-03	1.094618	4.069515
7	0.8	0.025386	1.370194e-02	1.113056	5.531105
8	0.9	0.036554	2.350128e-02	1.210358	9.495999

SVJD Model

$$dY_t = \mu dt + \sqrt{V_{t-}} dW_t^y + dJ_t^y$$
$$dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_{t-}} dW_t^\nu + dJ_t^v$$
$$dJ_t^y = \xi_t^y dN_t$$

	mu	kappa	theta	sigma
0	0.128212	0.790635	0.280707	0.087334

	rho	lambda	muJ	sigmaJ
0	-0.840489	24.900211	-0.005126	0.007576

References

Summary of the most relevant References

Carol Alexander. *Market Models: A Guide to Financial Data Analysis*. John Wiley & Sons, 2001.

Carol Alexander. *Market Risk Analysis, Quantitative Methods in Finance*. Market Risk Analysis. John Wiley & Sons, 2008a.

Carol Alexander. *Market Risk Analysis, Practical Financial Econometrics*. Market Risk Analysis. John Wiley & Sons, 2008b.

Michael Johannes and Nicholas Polson. MCMC methods for continuous-time financial econometrics. In Yacine Aït-Sahalia and Lars Hansen, editors, *Handbook of Financial Econometrics*. Elsevier, 2002.

Andreas Kaeck and Carol Alexander. Stochastic volatility jump-diffusions for european equity index dynamics. *European Financial Management*, 19(3):470–496, 2013.

Roger Koenker and Gilbert Bassett Jr. Regression quantiles. *Econometrica: journal of the Econometric Society*, pages 33–50, 1978.

Thank you for your attention

Backup slides

Analytical results

Assuming a normal distributed market return

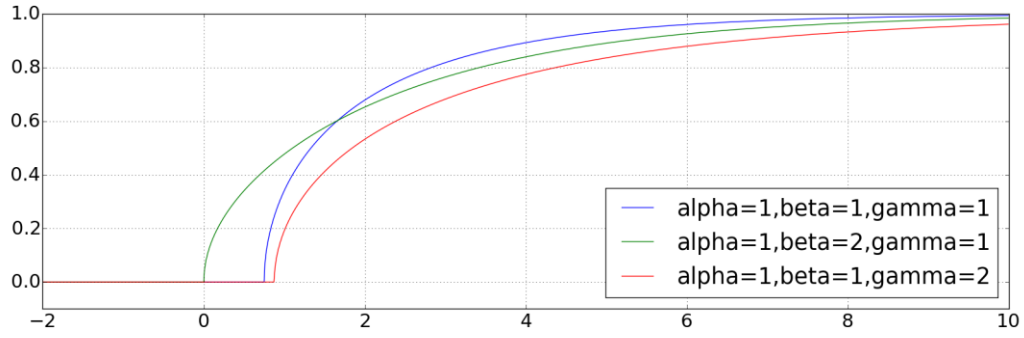
Distribution

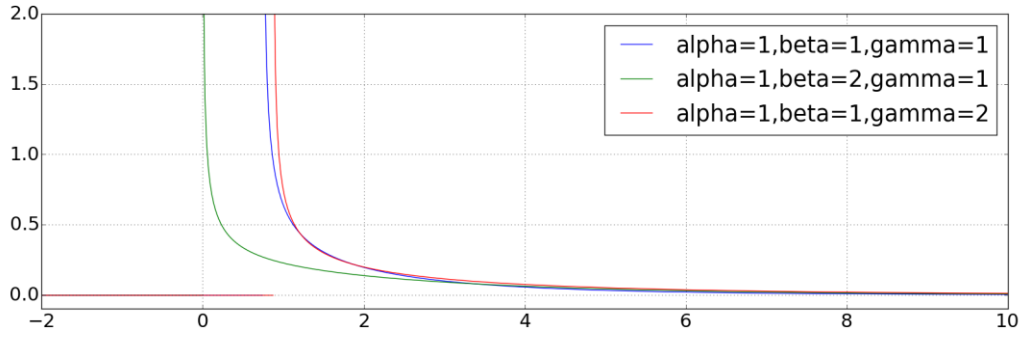
$$F(r) = \frac{1}{2}\left(\mathrm{Erfc}\left(\frac{\beta - \sqrt{\beta^2 + 4r\gamma - 4\alpha\gamma} + 2\gamma\mu}{2\sqrt{2}\gamma\sigma}\right) - \mathrm{Erfc}\left(\frac{\beta + \sqrt{\beta^2 + 4r\gamma - 4\alpha\gamma} + 2\gamma\mu}{2\sqrt{2}\gamma\sigma}\right)\right)\mathbb{1}_{\{r \geq \alpha - \frac{\beta^2}{4\gamma}\}}$$

Density

$$f(r) = \frac{\mathrm{Exp}\left(-\frac{\beta^2+2\beta\gamma\mu+2\gamma(r-\alpha+\gamma\mu^2)}{2\gamma^2\sigma^2}\right)\left(\mathrm{Exp}\left(\frac{(\beta-\sqrt{\beta^2+4(r-\alpha)\gamma}+2\gamma\mu)^2}{8\gamma^2\sigma^2}\right) + \mathrm{Exp}\left(\frac{(\beta+\sqrt{\beta^2+4(r-\alpha)\gamma}+2\gamma\mu)^2}{8\gamma^2\sigma^2}\right)\right)}{\sqrt{2\pi}\sqrt{\beta^2+4(r-\alpha)\gamma}\sigma}\mathbb{1}_{\{r \geq \alpha - \frac{\beta^2}{4\gamma}\}}$$

Example: Standard normal market return





$$\mathrm{Cor}(R_1, R_2) = \frac{(\beta_1 + 2\gamma_1\mu)(\beta_2 + 2\gamma_2\mu)\sigma^2 + 2\gamma_1\gamma_2\sigma^4}{\sqrt{\sigma^4((\beta_1 + 2\gamma_1\mu)^2 + 2\gamma_1^2\sigma^2)((\beta_2 + 2\gamma_2\mu)^2 + 2\gamma_2^2\sigma^2)}}$$

Moments

Correlation

$$\text{mean} = \alpha + \beta\mu + \gamma(\mu^2 + \sigma^2)$$
$$\text{variance} = (\beta + 2\gamma\mu)^2\sigma^2 + 2\gamma^2\sigma^4$$