Modeling Dynamic Redemption and Default Risk for LBO Evaluation: A Boundary Crossing Approach

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Abstract

We analyze corporate financial policies in leveraged buyouts (LBOs) in the presence of default risk. Our model captures the LBO-specific stepwise debt reduction, either with predetermined or cash-flow dependent (cash sweep) principal payments, and thus allows for dynamic redemption. These dynamics imply stochastic, discontinuous default boundaries. Our framework enables us to derive explicit-form solutions for the net present value (NPV) and the internal rate of return (IRR) of an LBO investment. We show that in scenarios with high entry debt and high redemption payments, the flexibility associated with dynamic redemptions creates value for investors, while fixed redemptions yield higher NPV and IRR values for moderate redemption due to lower debt yields. Moreover, we discuss optimal corporate financial policies implied by NPV or IRR maximization and find that the latter always results in increased leverage with higher default probability. The model of piecewise linear boundaries developed in this article is sufficiently flexible to be applied to a wide range of problems in corporate finance.

Keywords: Default at first passage, Dynamic redemption, Barrier options, Brownian motion, Numerical integration, Leveraged buyouts

JEL classification: C61, C63, G12, G13, G32, G33

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1. Introduction

Leveraged buyouts (LBOs) are a specific type of corporate transaction in which the buyer, often private equity (PE) funds, acquires a target company for a limited holding period (on average, three to five years, see, e.g., Kaplan and Strömberg, 2008). In particular, after entry, the buyer imposes a new capital structure onto the target; this structure is characterized by a small portion of equity and a significant portion of debt (Axelson et al., 2013, find that LBO-backed firms have a debt-to-enterprise value ratio that is twice as high as that of their industry peers). The target firm reduces this initial debt level in a stepwise manner over the holding period, either by contractually fixed principal payments or by a redemption schedule that depends on the target firms' generated cash flows (the so called "cash sweep" redemption). As these observations suggest, LBOs are characterized by a different capital structure and redemption policy than their industry peers (e.g., Axelson et al., 2013). Our article examines the dynamics of financial policies in an LBO setting and their link to investment decisions. By accounting for the aforementioned characteristics, we provide explicit-form solutions for pricing LBO investments and quantitative explanations for those solutions.

Discussions between critics and proponents of the role of debt in LBO transactions are heated. On the one hand, proponents note that the extensive use of debt creates interest tax shields, efficiency gains (e.g., Berg and Gottschalg, 2005) and lower agency costs due to the disciplining effect of debt (Jensen and Meckling, 1976). On the other hand, critics claim that extensive debt usage in LBO transactions exposes the target firm to high bankruptcy risks[1] while the PE fund reaps unjustifiably high tax savings (e.g., Rasmussen, 2009). Although the subject of several empirical studies, this discrepancy puzzles theorists concerned with the effect of a firm's financing policy.

As the transaction volume of LBOs in 2007 of approximately \$1.6 trillion demonstrates (Kaplan and Strömberg, 2008), it is highly economically relevant

[1] Tykvová and Borell (2012) find no evidence that the bankruptcy rates of PE-owned firms differ from those of their peers. In contrast, Hotchkiss et al. (2014) find a higher bankruptcy probability. Strömberg (2007) finds that approximately 6% of the PE target firms in his sample default; however, his study does not cover the effects of the financial crisis.

to identify a pricing mechanism that captures the specifics of an LBO. Therefore, the aim of this paper is to address the aforementioned challenges. We develop a model to analyze the financial effects of an LBO based upon a boundary-crossing approach. Our model explicitly reflects upon default risk and captures the particular feature of dynamic (cash-flow dependent) debt redemption. By introducing piecewise discontinuous boundaries into this classical real options application, we are able to provide explicit-form solutions that allow us to assess an LBO based on the NPV and IRR investment criteria. This approach provides the opportunity to determine important value drivers, such as the tax shield. Moreover, by maximizing either NPV or IRR, we find the optimal capital structure and redemption policy of an LBO. Thus, we are able to compare and critically discuss the impact of the investment criteria on optimal debt levels in a dynamic framework.

We may know the economic rationales motivating PE investors to impose an LBO-specific debt structure (e.g., tax shield generation, IRR maximization), but existing models quantifying the value impact of debt do not fully capture all LBO-related aspects. A well established body of literature discusses the impact of different "financing policies", i.e., strategies of redeeming debt, taking on new debt and adapting the level of debt to changes in economic conditions (e.g., Myers, 1974; Miles and Ezzell, 1980; Cooper and Nyborg, 2010). These financing policies drive the risk properties of future debt levels and, by doing so, the risk of the tax savings attached to them. None of the established models account for the debt dynamics in LBOs: On the one hand, as first proposed by Myers (1974), a financing policy with state-independent absolute debt levels does not properly capture the "cash sweep" (path-dependent) redemption dynamics. On the other hand, the policy of Miles and Ezzell (1980) assuming that firms regularly adjust the level of outstanding debt to changes in firm value by adopting a state-independent leverage ratio is unable to model a stepwise redemption.

Moreover, the financing policy of LBOs also stands in stark contrast to established models of the optimal capital structure, "trading-off" the benefits and costs of debt. Most trade-off models assume a perpetual setting and derive an optimal debt level to be permanently maintained[2]. Instead, the financing policy of LBOs is characterized by high entry debt levels and flexible redemption over the course of a limited holding period. Existing capital structure models (e.g., Leland, 1994; Goldstein et al., 2001) consider a console bond or dynamically adjust the debt issue proportional to the asset value. To find the optimal debt level, a certain continuous threshold triggering bankruptcy is determined endogenously. The critical implication of this approach is that redemption payments are either not directly mapped or move proportional to the asset value. Thus, existing dynamic models of the optimal capital structure choice are unable to capture the empirically observed financing policies in LBOs and need to be extended (Axelson et al., 2013).

Some models, however, try to overcome the described shortcomings and to account for the LBO-specific debt dynamics: Arzac (1996) provides two potential solutions, a recursive APV and a European call option approach. The recursive APV approach still yields significant valuation errors as the valuation of the debt-related tax benefits requires a known discount rate. The option approach addresses this challenge but requires another simplifying assumption: The firm can only default on its debt at the due date which is equal to the end of the holding period. Other models allow for potential default over the entire lifetime of the debt contract: Couch et al. (2012) use a barrier option approach to value debt-related tax savings in an LBO setting. Default is triggered by the EBIT hitting a lower constant barrier reflecting an interest coverage ratio. An extended version allows for one-time refinancing over the infinite lifetime of the firm. Braun et al. (2011) also use a barrier option approach allowing for default over the entire lifetime of the debt contract in LBOs. Default occurs when the firm value drops below the face value of debt. In this approach, the future debt levels serving as lower barrier are assumed to be certain and described by an exponentially declining function. While both models allow for default over the entire lifetime of the debt contract, they still do not fully reflect upon the redemption policies typically employed in LBOs: First, a fixed

[2] As empirical studies on the speed of adjustment (e.g., Flannery and Rangan, 2006) suggest, the capital structure of public firms fluctuates around a certain (optimal) target level.

and stepwise redemption of debt requires a stepwise adjustment of the default barrier, imposing technical problems due to the non-differentiable nature of the barrier. Second, the "cash sweep" (dynamic) redemption policy necessitates multiple path-dependent adjustments of the default barrier.

We contribute to this literature stream by developing a model flexible enough to incorporate redemption policies with either fixed, stepwise repayments or dynamic, path-dependent repayments. In addition, high debt levels are an obvious characteristic of LBOs, consequently, the risk of default is specifically important. We introduce a boundary-crossing approach to map complex default triggers implied by the LBO-specific redemption policies. The mechanics are equivalent to a down-and-out barrier option with rebate. Default occurs either if a cash obligation consisting of repayment plus interest (fixed redemption) or a cash-flow-dependent covenant, e.g., a given maximum interest coverage or debt-to-EBITDA ratio, is hit within the holding period. Existing barrier option models (e.g., Merton, 1973; Cox and Rubinstein, 1985; Kunitomo and Ikeda, 1992; Roberts and Shortland, 1997; Lo et al., 2003) require boundaries that follow a certain differentiable function. Due to the dynamic, path-dependent redemption policy, debt becomes a path-dependent variable, and in turn, the default trigger is also path-dependent. This type of trigger is difficult to implement because the stochastic boundary is subject to stepwise changes, i.e., the boundary does not have a differentiable functional form. In brief, the existing barrier option models cannot be used to capture the aforementioned dynamics. Hence, we apply the basic idea of Wang and Pötzelberger (1997) by using piecewise linear boundaries. This approach offers the opportunity to model any type of boundary, including discontinuous ones. Wang and Pötzelberger (2007) extend their early approach to allow for an application to geometric Brownian motions (gBm). Based on this work, our model equations are in explicit form: To solve for complex default boundaries, numerical integration is required. (3.) Additionally, our analysis also endogenizes the pricing of debt. Based on the boundary-crossing approach, we derive the promised yield of debt resulting in NPV-neutral debt contracts. As credit risk premia of leveraged loans have been shown to be an important factor in LBO leverage choices (Axelson et al., 2013) our model captures this important feature.

Beyond the aforementioned contribution to pricing techniques, our model accounts for two additional important characteristics of LBOs. First, Colla et al. (2012) demonstrate that firm-specific drivers such as operating performance (EBIT) and volatility are important determinants of leverage choices in LBOs. We include these variables by assuming a stochastic EBIT process following a gBm and allowing for changes in drift and standard deviation. Second, PE sponsors in particular steer target companies based on the IRR of their investment rather than on its net present value (NPV) (see, e.g., Kaplan and Schoar, 2005). In other words, PE investors may choose an LBO capital structure based on a completely different rationale, as standard trade-off frameworks suggest. To provide an example, consider the optimal capital structure choice in the prominent trade-off model of Leland (1994) where the objective is to maximize the equity value. The equityholders' endogenous choice of the bankruptcy trigger directly implies the optimal debt level. In the most simple terms, given an already fixed investment outlay, the optimal capital structure choice in these models rests upon the NPV criterion. As our model framework allows for inversion, we are able to determine the IRR for any capital structure chosen by the investor and, thus, to identify the one maximizing the IRR. Using both investment criteria in a dynamic model enables critical reflection on the IRR beyond the well-known arguments.[3]

We find that optimizing an LBO debt structure based on the IRR in general results in higher leverage, increased default risk, and a lower value creation than optimization based on the NPV. Moreover, we provide an economic rationale for the existence of fixed and cash sweep debt contracts in LBOs. Cash sweep debt redemption generates equity-like payoffs to debtholders, because redemption varies with interest-exceeding stochastic cash flows. Thus, debtholders demand a risk premium within their promised yield. On the opposite side, flexible debt repayments reduce the risk to technically default which increases the

[3]Lorie and Savage (1949) and Hirshleifer (1958) revealed a number of difficulties of the IRR, thereby shaping subsequent academic opinion. The very special reinvestment assumption, the possibility of multiple results, the possibility of making an incorrect investment decision for mutually exclusive projects, and the difficulty of applying the IRR rule when the cost of capital varies over time are the four main pitfalls in the classical literature. Brealey and Myers (and subsequently Allen) summarized these issues in their famous textbook "Principles of Corporate Finance." The first edition was released in 1980, and their arguments have remained unchanged through the most recent version (Brealey et al., 2013).

expected future free cash flows to firm. This trade-off determines which of the two redemption policies is favorable. Given a moderate financing policy with low default probability, fixed debt redemption is value-creating for equityholders caused by a lower promised yield of debt. Cash sweep debt redemption creates value under riskier financing policies, because a comparable fixed redemption, forcing high cash obligations, implies a significantly higher risk of default.

The remainder of the paper is organized as follows. Section 2 introduces the model, with Section 2.1 stating the basic assumptions, Section 2.2 illustrating the specific debt structure, Section 2.3 defining the default triggers, Section 2.4 deriving the payoff components, Section 2.5 introducing the investment criteria, and Section 2.6 elaborating on the pricing of debt. Section 3 presents the stochastics of the model and shows the explicit analytic-form solution for some special cases and the explicit integral form solution for the general case. In Section 4, we use the stochastic attributes derived to develop solution formulae for all NPV and IRR components. Section 5 illustrates the results through a numerical application and provides comparative statics. Section 6 concludes the paper.

2. The setting

2.1. Basics of the model

Table 1 provides a notation index. Our assumptions concerning the nature of uncertainty are standard. Let $(\Omega,\ \mathcal{F},\ \mathbb{P}, (\mathcal{F}_t)_{t\geq 0})$ be a complete probability space supporting a standard Brownian motion W_t and $[0, T]$ a time interval, where $T \to \infty$ is possible. We denote the available information at time t, with $t \in [0, T]$, by the filtration $\mathcal{F}_t \subset \mathcal{F}$ where $\mathcal{F}_t$ describes the augmented σ-algebra generated by the standard Brownian motion. We assume a market without arbitrage opportunities. For each subjective probability measure $\mathbb{P}$, there exists an equivalent measure $\mathbb{Q}$ called the risk-neutral probability measure. We denote the expected value operator by $\mathbb{E}_{(.)}$ and use the subscript to indicate the respective probability measure. In the subsequent analysis, we pursue a risk-neutral pricing approach.

Consider a levered firm, the value of which at time t is given by V_t^L. According to Modigliani and Miller (1963), the value of the levered firm can be

Table 1: Notation index. This table summarizes all notations applied within the paper categorized by input parameters (upper-left), model output (upper-right) and stochastic notation (lower-left).

Input parameters		**Model output**	
Variable	**Description**	**Variable**	**Description**
T	holding period in years	co_t	cash obligation against debtholders in t
$EBIT_t$	earnings before interests and taxes in t	ds_t	total debt service in t
τ_c	corporate tax rate	db_t	default boundary in t
X_t	unlevered after-tax cash flow in t	XC_t	excess cash in t
$\mu_{\mathbb{P}}$	drift rate of EBIT	I_0	initial investment in $t = 0$
μ	risk-neutral drift rate of EBIT	PO_t	payoff in t
σ	volatility of EBIT	PV_{PO}	present value of a payoff
r	risk-free rate	V_t^L	value of the levered firm in t
r_A	asset rate	V_t^U	value of the unlevered firm in t
λ	market price of risk	V_t^{TS}	value of the tax shield in t
b_p	input parameter for P_H	P_H	contingent present value factor for V_t^{TS}
a	input parameter for P_H	$pd_{t,\mathbb{Q}}$	probability of default in t
y_D	promised yield of debt	$cd_{t,\mathbb{Q}}$	cumulative probability of default up to t
D_t	debt level of the target company in t	NPV	net present value
l^*	industry avg. multiple for debt after exit	IRR	internal rate of return
γ	cash sweep redemption ratio	C_t	interest payment in t
θ	dividend ratio	R_t	debt redemption in t
β	debt-to-EBIT covenant	PO_t^{Dh}	payoff to debtholders in t
ρ	bankruptcy cost ratio	NPV^{Dh}	net present value for debtholders

Stochastic notation	
Variable	**Description**
t	arbitrary point in time
d	point in time of default
$\mathbb{P}$	subjective real-world probability measure
$\mathbb{Q}$	risk-neutral probability measure
$\mathbb{E}_P$	expected value under $\mathbb{P}$
$\mathbb{E}_Q$	expected value under $\mathbb{Q}$
$\mathcal{F}$	filtration
Ω	probability space
$\mathbf{1}$	indicator function
$\mathbf{N}$	cumulative normal distribution function
W_t	standard Brownian motion in period t
M_t	minimum of standard Brownian motion up to t
α	drift rate of Brownian motion
m	lower constant boundary to Brownian motion

determined by adding the present value of the tax savings from interest payments on debt V_t^{TS} to the value of an otherwise identical but unlevered firm V_t^U. The corporate tax rate τ_c, and the risk-free rate r are assumed to be deterministic and constant. We do not consider personal taxes.

The operations of the firm generate an uncertain income. Similar to several representatives of the corporate finance theory literature (e.g., Hackbarth et al., 2007), our measure of income is the earnings before interest and taxes $EBIT$ of a nondepreciating machine with a mean return $\mu_{\mathbb{P}}$. Neither an income metric nor a cash flow are typically traded assets. Thus, we do not assume $EBIT$ to be spanned. Instead, we introduce a spanning portfolio Y with a mean return r_A and a volatility σ that is equal to the volatility of $EBIT$. As the risk of $EBIT$ contains an idiosyncratic component, $r_A > \mu_{\mathbb{P}}$ holds. The evolutions of $EBIT$ and Y are as follows:

$$\frac{dEBIT}{EBIT} = \mu_{\mathbb{P}} dt + \sigma dW_t \tag{1}$$

$$\frac{dY}{Y} = r_A dt + \sigma dW_t. \tag{2}$$

For Y, the risk-neutral mean return is r due to the spanning property. Thus, we have $r_A - \lambda\sigma = r$, where λ is the market price of risk. Rearranging this term results in:

$$\lambda = \frac{r_A - r}{\sigma}. \tag{3}$$

With (3) at hand, we find the risk-neutral drift of $EBIT$ which we denote by μ:

$$\begin{aligned} \mu =& \mu_{\mathbb{P}} - \lambda\sigma \\ =& \mu_{\mathbb{P}} - (r_A - r). \end{aligned} \tag{4}$$

We retain the typical Modigliani and Miller assumption that $EBIT$ is independent of the pursued debt policy. To conclude, $EBIT_t$ follows a geometric Brownian motion (gBm) under the risk-neutral probability measure $\mathbb{Q}$ with an initial value of $EBIT_0 > 0$ given by:

$$dEBIT_t = \mu EBIT_t dt + \sigma EBIT_t dW_t, \tag{5}$$

where μ is the constant risk-neutral drift rate, σ the constant standard deviation of the $EBIT$ and W_t a standard Brownian motion.

The income process also drives the free cash flow to firms in our model, which we denote by X. For simplicity, we assume that $EBIT$ less corporate taxes allows us to arrive at X[4]:

$$X_t = EBIT_t(1 - \tau_c). \tag{6}$$

The firm's debt is subject to the risk of default. The firm pays interest and redemption on the outstanding total amount of debt D_t. The credit risk-adjusted yield of debt is denoted by y_D. This promised yield is determined endogenously under the assumption that debtholders claim an interest rate that yields an NPV of zero for the debt contract.[5]

2.2. Debt structure of an LBO

To develop our model, we begin with the debt structure that is imposed on the target firm by the investor because several other variables are directly linked to this.

Figure 1 depicts a typical debt structure employed in LBOs throughout the holding period. Prior to the buyout in $t = Pre$ (Pre-LBO), the target firm has a certain total amount of debt outstanding, D_{Pre}, which is not limited by any assumptions. In $t = 0$, the investor buys the target firm and imposes a new debt structure upon the target by redeeming the pre-LBO debt. The new capital structure in $t = 0$ with an initial amount of debt D_0 implies an increased level of debt in most cases (but this need not be the case). During the holding period, the LBO-induced debt is usually reduced stepwise. At the end of the holding period T, the realized total amount of debt is D_T. At exit, the remaining LBO debt (D_T) is usually redeemed fully, and the new owner imposes a new debt level D_{Post} for the post-LBO phase. We assume D_{Post} to be a multiple of the uncertain exit period cash flow $D_{Post} = l^* X_T$, i.e., D_{Post} is contingent on the

[4]Note that any cash flow adjustments, such as capital expenditures, depreciations or changes in net working capital, are easily included in the model but inflate the solution formulae.

[5]This is a standard assumption in contingent-claims-based approaches to valuing corporate debt. Merton (1974) first introduced the idea when modeling interest rates under default risk using option pricing theory.

state of the firm in T. The leverage ratio l^*, with $l^* \geq 0$, can be, e.g., regarded as a sustainable industry average. While we introduce this assumption to ensure a state-dependent exit price, it is not critical to the model. Our approach also works with any other assumption concerning D_{Post}.

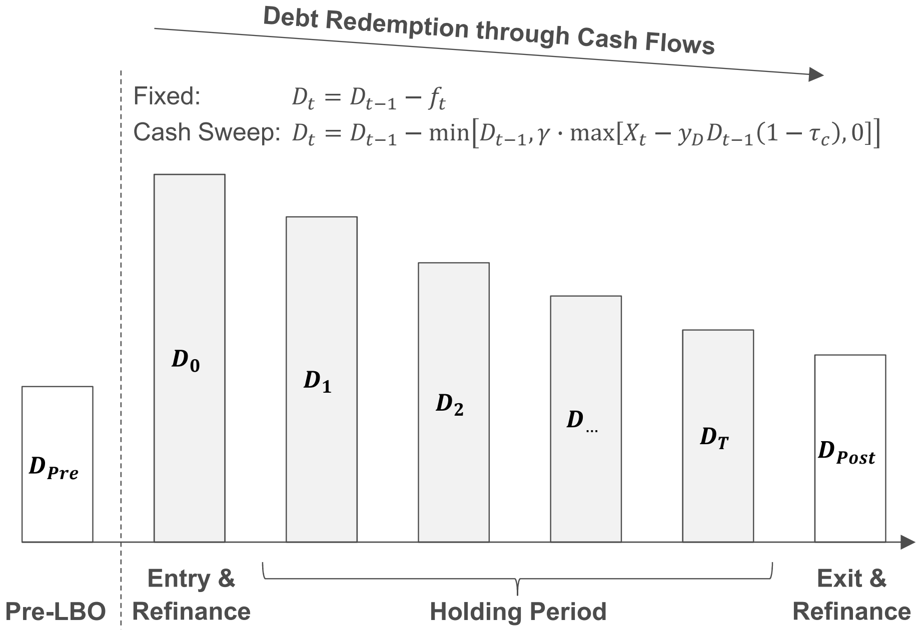

Figure 1: Basic structure of debt redemption in an LBO. The investor imposes a new, increased debt level D_0 at entry ($t = 0$). During the holding period, the acquired firm partially repays the increased debt to arrive at D_T, which may be higher than, lower than or equal to the new post-exit debt level D_{Post}. D_{Post} depends on the state of the firm in T.

We analyze two major redemption policies popular in LBOs: fixed and cash sweep repayment. Irrespective of the case, we denote debt redemption with $R_t^{(.)}$. In the fixed case, there is a predetermined redemption f_t at each time point t during the holding period. In contrast, cash sweep redemption describes is flexible: A proportion γ, with $\gamma \in [0, 1]$, of the firm's realized free cash flow X_t increased by the tax savings, $y_D \tau_c D_{t-1}$, and reduced by interest payments, $y_D D_{t-1}$ is repayed.[6] Considering the specific structures of the redemption cases

[6]For simplicity, we assume γ to be a constant parameter. Note that a time-dependent γ_t can easily be incorporated into the model.

yields:

$$R_t^{fixed} = f_t, \tag{7}$$

$$R_t^{sweep} = \min\left(D_{t-1}, \gamma \max\left(X_t - y_D D_{t-1}(1-\tau_c), 0\right)\right). \tag{8}$$

The min-max combination in Equation (8) is necessary to account for the specific flexibility of cash sweep debt redemption. The max condition prevents new debt from being added if X_t is not sufficient to serve the after-tax interest payments $y_D D_{t-1}(1-\tau_c)$. The min condition restricts D_{t-1} to be positive ($D_{t-1} \geq 0$), i.e., if $\gamma(X_t - y_D D_{t-1}(1-\tau_c))$ exceeds the outstanding debt, only D_{t-1} will be redeemed (non-negative condition).

Hence, the firm's future debt level at an arbitrary time t is determined by:

$$D_t^{(.)} = D_0 - \sum_{s=1}^{t} R_s^{(.)} \tag{9}$$

for both redemption policies.

The total debt service $ds_t^{(.)}$ at an arbitrary point in time t equals the sum over redemption $R_t^{(.)}$ and after-tax interest payments $NC_t^{(.)}$. Therefore, we obtain the following congruent definition:

$$ds_t^{(.)} = NC_t^{(.)} + R_t^{(.)}, \tag{10}$$

where Equations (7) to (8) specify $R_t^{(.)}$, and where $NC_t^{(.)}$ is described by:

$$NC_t^{(.)} = y_D D_{t-1}(1-\tau_c). \tag{11}$$

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2.3. Default in an LBO

In our model, default is triggered if our unlevered after-tax cash flow X becomes sufficiently low and hits the default boundary from above. In the literature regarding (the optimal choice of) debt financing two contrasting economic mechanisms are usually considered to determine the default boundary (Strebulaev and Whited, 2011). Most existing models endogenously determine the optimal default threshold by maximizing the equity value (e.g., Leland, 1994; Goldstein et al., 2001). Usually, the aforementioned boundary is a certain asset value (e.g., Leland, 1994). Such an endogenously chosen default trigger implicitly assumes equityholders to have deep pockets, i.e., they always prevent illiquidity by covering coupon payments if needed (Strebulaev and Whited, 2011). However, the implied "deep pocket" assumption does not generally hold for equityholders in LBO transactions as they are often closed PE funds with a fixed fund size which is fully distributed to promising investments. Therefore, as discussed by Achleitner et al. (2012), debtholders should not expect PE investors to prevent a default by injecting additional equity.

Others consider a flow-based, exogenous threshold (e.g., Kim et al., 1993). We apply this second economic mechanism where we trigger default by illiquidity or a covenant violation. The so-called exogenous default trigger is appropriate for our model for three reasons. Firstly, as outlined in the previous section, high initial debt levels are redeemed stepwise over the holding period generating significant cash obligations. Thus, the risk of illiquidity is outstanding compared to the aforementioned excessive indebtedness argument (Achleitner et al., 2012). Secondly, debt contracts of LBOs comprise a variety of covenants. For instance, Achleitner et al. (2012) find a significantly higher number of financial covenants in PE-sponsored debt contracts in contrast to non-sponsored debt contracts. Almost every (97%) sponsored loan includes a combination of a debt-to-EBITDA and a cash flow coverage covenant. Thirdly, redemption policies in LBOs, particularly cash-sweep redemption, demand discontinuous, path-dependent default boundaries rather than one continuous, fixed default trigger (e.g., Goldstein et al., 2001).

In the absence of a covenant, default is triggered if the realized cash flows X_t plus any available excess cash accumulated before time point t, XC_{t-1}, do not cover the cash obligations co_t. Under fixed redemption, cash obligation and debt service are identical ($ds_t^{fixed} = co_t^{fixed}$), while in the cash sweep case, the cash obligation is limited to the after-tax interest payments:

$$co_t^{fixed} = NC_t^{fixed} + R_t^{fixed}, \tag{12}$$

$$co_t^{sweep} = NC_t^{sweep}. \tag{13}$$

Excess cash XC_t is the sum of previous period's excess cash up to time $t-1$, invested at the risk-free rate r for one period plus the retained share of the net cash flow $(1-\theta)(X_t - co_t)$ in t. The parameter θ, with $\theta \in [0,1]$, denotes the dividend ratio established by the investor.[7] Occasionally, θ is restricted in LBOs through debt contracts to foster the excess cash account, which creates a cushion against default and shields the debtholders. Summarizing, the excess cash XC_t is determined by:

$$XC_t = XC_{t-1}e^{r(1-\tau_c)} + (1-\theta)\left(X_t - co_t\right). \tag{14}$$

Moreover, debt contracts usually contain certain minimum requirements, called covenants, for a specific income or cash flow metric. A typical covenant is a ratio β of debt-to-EBIT or debt-to-EBITDA.[8] While we apply debt-to-EBIT ($(D_{t-1}-XC_{t-1})/EBIT_t \leq \beta$), the model allows for any other common covenant related to debt, interest and performance measures. Note that we consider net debt ($D_{t-1}-XC_{t-1}$) for the covenant condition, as excess cash can be regarded as negative debt.

Independent of the redemption policy considered, default of the target firm is triggered if cash flow plus excess cash are not sufficient to cover the firm's cash obligation or if the covenant condition is no longer fulfilled. We rearrange both conditions for X_t and formulate a general rule for default and going concern in our model:

- *Default (def):*

$$\left(X_t < co_t^{(.)} - XC_{t-1}e^{r(1-\tau_c)}\right) \cup \left(X_t < \frac{D_{t-1} - XC_{t-1}}{\beta}(1-\tau_c)\right),$$
$$\text{for } \exists\ 0 < t \leq T. \tag{15}$$

- *Going concern (gc):*

$$\left(X_t \geq co_t^{(.)} - XC_{t-1}e^{r}\right) \cup \left(X_t \geq \frac{D_{t-1} - XC_{t-1}}{\beta}(1-\tau_c)\right),$$
$$\text{for } \forall\ 0 < t \leq T. \tag{16}$$

Based on condition (15), we can derive a (lower) default boundary to the cash flow X_t. Mechanically, we determine the default boundary at time $t-1$ for the subsequent period up to t and denote it by $db_t^{(.)}$. These path-dependent

[7] For simplicity, we assume θ to be a constant parameter. Note that a time-dependent θ_t can easily be incorporated into the model.

[8] For simplicity, we assume β to be a constant parameter. Note that a time-dependent β_t can easily be incorporated into the model.

boundaries are defined for our two redemption cases in the following way:

$$db_t^{(\cdot)} = \max\left(co_t^{(\cdot)} - XC_{t-1}e^{r(1-\tau_c)}, \frac{D_{t-1} - XC_{t-1}}{\beta}(1-\tau_c)\right), \text{ for } [t-1, t]. \quad (17)$$

We denote the time at which a default occurs as d. Figure 2 illustrates possible scenarios for an LBO. The cash flow hitting the default boundary triggers default, whereas the going concern condition is satisfied as long as the cash flow remains above the default boundary.

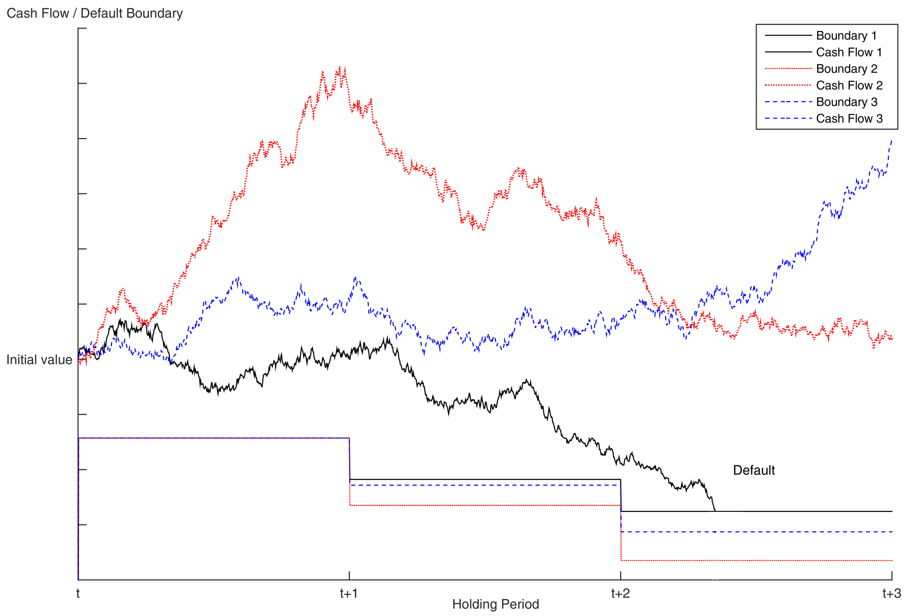

Figure 2: Potential cash flow paths and default boundaries. The figure illustrates three cash flow paths and their corresponding dynamic (stochastic), discontinuous default boundaries during the holding period, t to $t+3$. The upper two paths, cash flow 2 (red, dotted path) and cash flow 3 (blue, dashed path), are scenarios in which the going concern condition holds until exit. Due to the cash sweep redemption, i.e., higher cash flows imply a higher redemption of the initial debt level, the corresponding default boundaries 2 (red, dotted line) and 3 (blue, dashed line) decrease accordingly. Cash flow 1 (black, solid path) describes the case in which the corresponding default boundary (black, solid line) is reached in the third period. In comparison to the other cases, the firm redeems less of its cash flows and the corresponding default boundary remains at a higher level.

Subsequently, we examine the evaluation of an LBO in greater detail. We regard the classical financial decision-making principles: the NPV and the IRR. We adopt the perspective of the deal sponsor and evaluate the LBO on an equity basis.

2.4. Payoff structure of an LBO

An LBO generates three different types of payoffs distinguished by the time of their occurrence: the initial investment I_0 to purchase the target, the equity cash flows PO_{HP} at an arbitrary point in time during the holding period, and the exit equity value PO_{Exit} from selling the target company.

The initial equity investment I_0 at time $t = 0$ is equal to the enterprise deal value V_0^L minus the entry debt D_0. The enterprise deal value is the sum of the unlevered firm value, V_0^U, and the tax shield value, V_0^{TS}. We define V_0^U simply as a perpetuity depending on the current EBIT level $EBIT_0$, the existing corporate tax rate τ_c, the risk-free rate r, and the risk-neutral drift of the EBIT-process μ_{pre} prior to the LBO. To price V_0^{TS}, we follow Couch et al. (2012) as this approach resonates well with our basic idea of a default boundary defined by a covenant. The basic assumption is that the firm holds D_{Pre} constant and earns interest tax savings of $(y_{D,Pre}D_{Pre}\tau_c)$ in each period. These tax savings are subject to default risk, and the bankruptcy trigger is a constant barrier determined by a covenant, which is related to the stochastic $EBIT$ process. While Couch et al. (2012) use an interest coverage ratio, we continue to apply our debt-to-EBIT covenant. By constructing a perpetual, down-and-in, cash-at-hit-or-nothing, single-barrier option that pays one dollar when the underlying, $X = EBIT(1-\tau_c)$, hits the barrier, $(1-\tau_c)(D-XC)/\beta$, and zero otherwise, one arrives at a "contingent present value factor for the random time when the underlying hits the barrier from above" (Couch et al., 2012, p. 127). The valuation formula for such an option P_H is stated in Equation (21) with parameters b_p and a defined in Equations (22) and (23). A complete derivation of the option's valuation formula is provided in Rubinstein and Reiner (1991).

$$V_0^U = X_0 \frac{1}{r - \mu_{pre}}, \tag{18}$$

$$V_0^{TS} = \frac{y_{D,Pre}D_{Pre}\tau_c}{r}(1 - P_{H,pre}), \tag{19}$$

$$I_0 = V_0^U + V_0^{TS} - D_0, \tag{20}$$

with

$$P_{H,pre} = \left(\frac{\frac{D_{t-1} - XC_{t-1}}{\beta}(1-\tau_c)}{X_0} \right)^{b_{p,pre}}, \tag{21}$$

$$b_{p,pre} = a_{pre} + \sqrt{a_{pre}^2 + 2\frac{r}{\sigma^2}}, \tag{22}$$

$$a_{pre} = \frac{\mu_{pre}}{\sigma^2} - \frac{1}{2}. \tag{23}$$

While extensions to the derivation of I_0, e.g., a bid premium or expected costs of financial distress, can easily be incorporated into the model, our definition is sufficient to endogenize the initial investment.

The equity cash flows as payoffs over the holding period depend on whether $X_t \geq db_t^{(\cdot)}$ holds for all periods prior to t. As long as the default boundary has not been reached, the equity payoff PO_t^{gc} is determined as the difference between the cash flow to firm X_t and the total debt service ds_t, multiplied by the dividend ratio θ. After a default has occurred, at time d, no future cash flows are generated. The firm only realizes a payoff PO_d^{def} at d. PO_d^{def} is a maximum of zero and liquidation value Liq_d^{def}. The liquidation value is the sum over the period's cash flow X_d, the current excess cash account balance XC_d, the market value of assets less bankruptcy costs $(1-\rho)V_d^U$ reduced by the outstanding debt D_d and the period's cash obligation co_d. The parameter ρ, with $\rho \in [0,1]$, denotes the bankruptcy cost ratio with respect to the endogenous market value of assets V_d^U, which is determined in accordance with Equation (18). Thus, the liquidation value at time d is:

$$Liq_d^{def} = X_d + XC_d + (1-\rho)V_d^U - D_d - co_d. \tag{24}$$

The described structure translates into four state-dependent payoffs during the holding period characterized by:

$$PO_t = \begin{cases} PO_t^{gc} & = \theta(X_t - ds_t), \quad \text{if } X_t \geq db_t^{(\cdot)} \ (0 < t < d) \\ PO_t^{def,+} & = Liq_t^{def}, \quad \text{if } (X_t < db_t^{(\cdot)}) \cap (Liq_t^{def} > 0) \ (t = d \leq T) \\ PO_t^{def,0} & = 0, \quad \text{if } (X_t < db_t^{(\cdot)}) \cap (Liq_t^{def} \leq 0) \ (t = d \leq T) \\ PO_t^{0} & = 0, \quad \text{if } t > d. \end{cases} \tag{25}$$

At exit, there is an equity payoff from selling the target company. We derive the exit equity value based on the following components: the sum over the unlevered value of the firm V_T^U, the value of the tax shield V_T^{TS}, and the excess cash accumulated V_T^{XC}, reduced by the realized debt level at exit D_T. Consistent with entry valuation, V_T^U and V_T^{TS} are defined as in Equations (18) and (19). Note that V_T^{TS} is attached to the state-dependent post-LBO debt level D_{Post}. If the dividend ratio has been set below one ($\theta < 1$), the target company has accumulated excess cash, which contributes the value V_T^{XC} at exit. Finally, D_T needs to be subtracted to arrive at an equity payoff. Thus, the payoff at exit PO_{Exit} is given by:

$$PO_{Exit} = \begin{cases} PO_{Exit}^{gc} = V_T^U + V_T^{TS} + V_T^{XC} - D_T, & \text{if } X_t \geq db_t^{(\cdot)} \ (0 < t \leq T) \\ PO_{Exit}^{def} = 0, & \text{if } X_t < db_t^{(\cdot)} \ (0 < t \leq T). \end{cases} \tag{26}$$

2.5. Performance Evaluation of an LBO

Based on the conditional payoffs, we derive criteria for the combined investment and financing decisions. First, we consider the NPV as the discounted value of all payoffs from the investment over the holding period until exit.

$$NPV^{Eq} = -I_0 + PV_{HP} + PV_{Exit}, \tag{27}$$

where PV_{Exit} reflects the present value of the firm's equity at exit, PV_{HP} the present value of all payoffs during the holding period and I_0 the initial investment.

Despite the well-known pitfalls of the IRR criterion (see Section 1) many investors, particularly PE funds, identify worthwhile investment projects and evaluate their performance based on this measure. We acknowledge this and incorporate it into our model. The IRR is a function ϕ of the aforementioned variables by setting $NPV = 0$:

$$IRR^{Eq} = \phi\left(NPV^{Eq} = 0,\ I_0,\ PV_{HP},\ PV_{Exit}\right). \tag{28}$$

Analyzing investment and financing decisions based on both criteria, NPV and IRR, will allow us to draw conclusions concerning their impact on decisions in a dynamic model, an analysis that has yet to be conducted, to the best of the author's knowledge.

Following the risk-neutral pricing approach with continuously changing cash flows, we use e^{-rt} to discount the payoffs. All conditions identified in Section 2.4 for the payoffs are captured with indicator functions, $\mathbb{I}_{condition}$, that by definition are equal to one if the specified *condition* is satisfied and zero if it is not. In total, there are four relevant conditions regarding the equity payoffs, which are summarized below:

1. Going concern (gc) : $X_t \geq db_t^{(\cdot)}$ for $(0 < t < d)$, (29)
2. Default (def) : $X_t < db_t^{(\cdot)}$ for $(t = d \leq T)$, (30)
3. Non-negative condition (noneg) : $D_t \geq 0$ for $(0 < t \leq d)$, (31)
4. Liquidation value equity (liqEq) : $Liq_t^{def} > 0$ for $(t = d \leq T)$. (32)

We apply conditions (29) to (32) to explicitly highlight the value components of the NPV and the IRR. We begin with the Equations for I_0 and PV_{Exit}, as they are independent of the redemption policies:

$$I_0 = V_0^U + V_0^{TS} - D_0, \tag{33}$$

$$PV_{Exit} = e^{-rT}\mathbb{E}_Q\left((V_T^U + V_T^{TS} + V_T^{XC} - D_T)\mathbb{I}_{\{gc,0<t\leq T\}}\right). \tag{34}$$

As cash sweep redemption should not exceed the current debt level ($D_t \geq 0$), we introduce a min condition (Equation (31)) that can also be described by an indicator function. Thus, we can state PV_{HP} in explicit form but depending on the chosen redemption.

$$\begin{aligned} PV_{HP}^{fixed} = & \sum_{t=1}^{T} e^{-rt}\mathbb{E}_Q\left((\theta X_t - \theta NC_t - \theta R_t^{fixed})\mathbb{I}_{\{gc,0<t\leq d\}}\right) \\ & + e^{-rd}\mathbb{E}_Q\left(Liq_d^{def}\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqEq,0<d\leq T\}}\right), \end{aligned} \tag{35}$$

$$\begin{aligned} PV_{HP}^{sweep} = & \sum_{t=1}^{T} e^{-rt}\mathbb{E}_Q\left((\theta X_t - \theta NC_t - \theta R_t^{sweep}\mathbb{I}_{\{noneg,0<t\leq d\}})\mathbb{I}_{\{gc,0<t\leq d\}}\right) \\ & + e^{-rd}\mathbb{E}_Q\left(Liq_d^{def}\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqEq,0<d\leq T\}}\right). \end{aligned} \tag{36}$$

The first term in Equation (35) is the present value of the expected going concern payoffs under fixed redemption, where the indicator function is equal to one if the going concern condition holds ($X_t \geq db_t^{(\cdot)}$). Under cash sweep redemption, an additional indicator function exists within the sum in Equation

(36) that limits the redemption payment to the outstanding debt (non-negative condition). The final term outside the sum is equal for both redemption cases. It accounts for the present value of the expected default payoff, where the combination of the two indicator functions yields a value of one if in $t = d$ the liquidation value exceeds zero ($Liq_d^{def} > 0$) and the cash flow reaches the default boundary ($X_d < db_d^{(\cdot)}$).

2.6. Pricing of debt under default risk in an LBO

Finally, we discuss the deal's implications from a debtholder's perspective. As we have already postulated, the promised yield of debt y_D is determined endogenously in our model. y_D is a fixed rate that is constant throughout the holding period and is set such that the net present value for the debtholders is equal to zero ($NPV^{Dh} = 0$). In each period t, the debtholders receive interest payments $C_t = y_D D_{t-1}$ and redemptions $R_t^{(\cdot)}$. $R_t^{(\cdot)}$ depends on the chosen redemption policy and is defined in Equations (7) and (8).

The initial investment of debtholders is the initial debt level D_0, and at exit the remaining debt claim D_T is fully redeemed. In the event of default at $t = d$, debtholders receive the minimum of the firm's liquidation value $(1-\rho)V_d^U$ and of their total remaining claim $C_d + D_{d-1}$, where C_d represents the outstanding interest payments and D_{d-1} the outstanding face value of debt. Thus, we can write the payoffs to debtholders consistently with the methodology in Section 2.4:

$$I_0^{Dh} = D_0, \tag{37}$$

$$PO_t^{Dh} = \begin{cases} PO_t^{Dh,gc} & = C_t + R_t^{(\cdot)}, \quad \text{if } X_t \geq db_t^{(\cdot)} \ (0 < t < d) \\ PO_t^{Dh,def,1} & = (1-\rho)V_t^U, \quad \text{if } (X_t < db_t^{(\cdot)}) \cap ((1-\rho)V_t^U < C_t + D_{t-1}) \ (t = d \leq T) \\ PO_t^{Dh,def,2} & = C_t + D_{t-1}, \quad \text{if } (X_t < db_t^{(\cdot)}) \cap ((1-\rho)V_t^U \geq C_t + D_{t-1}) \ (t = d \leq T) \\ PO_t^{Dh,0} & = 0, \quad \text{if } t > d, \end{cases} \tag{38}$$

$$PO_{Exit}^{Dh} = \begin{cases} PO_{Exit}^{Dh,gc} = D_T, & \text{if } X_t \geq db_t^{(\cdot)} \ (0 < t \leq T) \\ PO_{Exit}^{Dh,def} = 0, & \text{if } X_t < db_t^{(\cdot)} \ (0 < t \leq T). \end{cases} \tag{39}$$

While the going concern, default and non-negative conditions are equivalent to the equityholder perspective from Section 2.5 (Equations (29) to (31)), there are two additional conditions regarding the default payoff for debtholders as described above. We formalize these conditions as follows:

5. Liquidation value debt 1 (liqD1):$(1-\rho)V_t^U < C_t + D_{t-1}$ for $(t = d \leq T)$, (40)
6. Liquidation value debt 2 (liqD2):$(1-\rho)V_t^U \geq C_t + D_{t-1}$ for $(t = d \leq T)$. (41)

Based on Equations (37) to (39), we establish NPV^{Dh} and specify the present values for holding period (PV_{HP}^{Dh}) and exit (PV_{Exit}^{Dh}) by applying the indicator conditions (29) to (31) and (40) and (41).

$$NPV^{Dh} = -D_0 + PV_{HP}^{Dh,(.)} + PV_{Exit}^{Dh}, \tag{42}$$

with

$$\begin{aligned}
PV_{HP}^{Dh,fixed} =& \sum_{t=1}^{T} e^{-rt}\mathbb{E}_Q\left((C_t + R_t^{fixed})\mathbb{I}_{\{gc,0<t\leq d\}}\right) \\
&+ e^{-rd}\mathbb{E}_Q\left((1-\rho)V_d^U\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqD1,0<d\leq T\}}\right) \\
&+ e^{-rd}\mathbb{E}_Q\left((C_d + D_{d-1})\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqD2,0<d\leq T\}}\right), \qquad (43) \\
PV_{HP}^{Dh,sweep} =& \sum_{t=1}^{T} e^{-rt}\mathbb{E}_Q\left((C_t + R_t^{sweep}\mathbb{I}_{\{noneg,0<t\leq d\}})\mathbb{I}_{\{gc,0<t\leq d\}}\right) \\
&+ e^{-rd}\mathbb{E}_Q\left((1-\rho)V_d^U\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqD1,0<d\leq T\}}\right) \\
&+ e^{-rd}\mathbb{E}_Q\left((C_d + D_{d-1})\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqD2,0<d\leq T\}}\right), \qquad (44) \\
PV_{Exit}^{Dh} =& e^{-rT}\mathbb{E}_Q\left(D_T\mathbb{I}_{\{gc,0<t\leq T\}}\mathbb{I}_{\{noneg,0<t\leq T\}}\right). \qquad (45)
\end{aligned}$$

To conclude, Equation (42) provides a valuation formula for debt in our framework. By applying the assumption that $NPV^{Dh} = 0$ from above, we are able to calculate the promised yield y_D iteratively because it is a function of the aforementioned variables and the default boundary $db_t^{(.)}$:

$$y_D = \eta\left(NPV^{Dh} = 0,\ D_0,\ PV_{HP}^{Dh,(.)},\ PV_{Exit}^{Dh}, db_t^{(.)}\right). \tag{46}$$

Note that $db_t^{(.)}$ is incorporated into the determination of y_D, while that was not required in the case of the IRR (Equation (28)). The reason is that changes in y_D result in changes in db_t and vice versa. As a consequence, Equation (46) endogenizes the promised yield of debt y_D in our framework.

In the next section, we develop an approach to transform the indicator functions developed in Section 2 into explicit-form solutions, thereby allowing us to evaluate the financial effects of an LBO by simple numerical integration.

3. Derivation of useful stochastic properties

A default is triggered by the unlevered after-tax cash flow X_t reaching the default barrier $db_t^{(\cdot)}$. For both redemption policies, such a structure is equivalent to a down-and-out barrier option where the default barrier is the lower boundary.

As our setting incorporates dynamic redemption schedules, we face path-dependent boundaries. Thus, the classical Merton framework requiring constant or exponential boundaries cannot be used to derive explicit analytic formulae. Roberts and Shortland (1997) and Lo et al. (2003) find valuable approximation approaches for any boundary that can be expressed as a continuous and differentiable function throughout the examined interval. However, our redemption policies require discontinuous boundaries (see Figure 2). Therefore, we follow the idea of Wang and Pötzelberger (1997) to apply piecewise linear boundaries. The equations under this approach are in explicit form and can be solved by repeated application of numerical integration.

We proceed in three steps: First, we present an explicit analytic solution for the default probability of a standard Brownian motion with drift versus a constant lower boundary. Second, we replace the standard Brownian motion with the geometric form described in Equation (5). This solution will still be in explicit analytic form. Finally, we use the results of Wang and Pötzelberger (1997) to arrive at an equation in explicit integral form for any piecewise linear lower boundaries.

3.1. Standard Brownian motion versus constant default barrier

We begin with a Brownian motion without drift, W_t, on the filtered probability space $(\Omega,\ \mathcal{F},\ \mathbb{Q}, (\mathcal{F}_t)_{t\geq 0})$ and adjust it to one with drift, $\hat{W}_t$:

$$\hat{W}_t = \alpha t + W_t. \tag{47}$$

The minimum $\hat{M}_t$ of such a process under the conditions $\hat{M}_t \leq 0$ and $\hat{W}_t \geq$

$\hat{M}_t$ is defined by:

$$\hat{M}_t = \min_{0 \leq t \leq T} \hat{W}_t. \tag{48}$$

Hence, $\hat{M}_t$ and $\hat{W}_t$ take values in the set $\{(m, w); w \geq m, m \leq 0\}$. This allows us to derive the joint density function of both under the risk-neutral probability measure $\mathbb{Q}$ (a detailed derivation can be found in Appendix A.1):

$$f_{\hat{M}_t, \hat{W}_t}(m, w) = \frac{2(w - 2m)}{t\sqrt{2\pi t}} e^{\alpha w - \frac{1}{2}\alpha^2 t - \frac{(2m-w)^2}{2t}}. \tag{49}$$

Based on this density function, we are able to derive the default probability $cd_{t,\mathbb{Q}}$:

$$\begin{aligned} cd_{t,\mathbb{Q}} &= \mathbb{Q}\left\{\hat{M}_t < m\right\} \\ &= \frac{1}{\sqrt{2\pi t}} \left(\int_{-\infty}^{m} e^{-\frac{1}{2t}(w-\alpha t)^2} dw - e^{2\alpha m} \int_{-\infty}^{m} e^{-\frac{1}{2t}(w-2m-\alpha t)^2} dw \right) \qquad (50) \\ &= N\left(\frac{m - \alpha t}{\sqrt{t}}\right) + e^{2\alpha m} N\left(\frac{m + \alpha t}{\sqrt{t}}\right). \qquad (51) \end{aligned}$$

3.2. Geometric Brownian motion versus constant default barrier

Replacing the standard Brownian motion with drift α with our cash flow X_t, following a gBm, and substituting the lower boundary m for a default barrier, satisfying the definition of $db^{(\cdot)}$ but still being a constant, yields:

$$\begin{aligned} cd_{t,\mathbb{Q}} &= \mathbb{Q}\left\{X_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma M_t} < db^{(\cdot)}\right\} \qquad (52) \\ &= \mathbb{Q}\left\{\frac{1}{\sigma}\left(\mu - \frac{\sigma^2}{2}\right)t + M_t < ln\left(\frac{db^{(\cdot)}}{X_0}\right)\frac{1}{\sigma}\right\}. \qquad (53) \end{aligned}$$

Transforming Equation (52) into (53) reveals a structure equivalent to that in Equation (47). The term $\frac{1}{\sigma}(\mu - \frac{\sigma^2}{2})$ in Equation (53) is equivalent to α in Equation (47). Furthermore, the lower boundary m from Equations (49) to (50) has been adjusted to $ln(\frac{db^{(\cdot)}}{X_0}) \cdot \frac{1}{\sigma}$ for the gBm process used in our model:

$$cd_{t,\mathbb{Q}} = \mathbb{Q}\left\{\alpha t + M_t = \hat{M}_t < m\right\}, \tag{54}$$

with

$$\alpha = \frac{1}{\sigma}\left(\mu - \frac{\sigma^2}{2}\right), \tag{55}$$

$$m = \frac{1}{\sigma} ln\left(\frac{db^{(\cdot)}}{X_0}\right). \tag{56}$$

To conclude, pasting α and m from Equations (55) and (56) into Equations (49) and (51) yields formulae for the joint density function of $\hat{M}_t$ and $\hat{W}_t$ under the risk-neutral probability measure $\mathbb{Q}$ and for the default probability $cd_{t,\mathbb{Q}}$, if the process follows a gBm.

3.3. Geometric Brownian motion versus piecewise linear default barriers

In this section, we generalize Equations (49) and (50) for a default boundary that is a polygonal function over the course of the holding period. We extend the approach of Wang and Pötzelberger (1997) for standard Wiener processes without drift towards a gBm with drift.

To provide a general solution, we proceed under the assumption that the holding period, $0 \leq t \leq T$, can be divided into n-intervals, with $0 = t_0 < t_1 < ... < t_n = T$, and set the lower boundary m_t constant on each of the intervals $[t_{j-1}, t_j]$, $j = 1, 2, ..., n$ and $m_0 < 0$. For our specific problem of LBO valuation, it is important to note that $t_0 = 0, t_1 = 1, ..., t_n = T$ and t are the parameters describing the points in time within the holding period.

The probability that the modified Wiener Process $\hat{W}_t$ does not cross m_t on the interval $[0, T]$ can be divided into n conditional events: $\hat{W}_t$ does not cross m_t on the interval $[t_j, t_{j+1}]$ given that $\hat{W}(t)$ has not crossed $m(t)$ on the interval $[t_{j-1}, t_j]$. Equation (50) provides the probability for the complementary event on a single interval. Changing the integral area from $[-\infty, m]$ to $[m, \infty]$ yields the conditional probability that $m(t)$ has not been crossed for each of the intervals. To connect the single intervals, we restate Equation (50) but with the adjusted integral area as described and in a form with merely one integral:

$$\begin{aligned} \mathbb{Q}\left\{\hat{M}_t \geq m\right\} &= \frac{1}{\sqrt{2\pi t}}\left(\int_m^\infty e^{-\frac{1}{2t}(m-\alpha t)^2} dw - e^{2\alpha m}\int_m^\infty e^{-\frac{1}{2t}(m-2m-\alpha t)^2} dw\right) \\ &= \int_{m-\alpha t}^{\infty} \left(1 - e^{-\frac{2m(m-\alpha t-x)}{T}}\right) f(x)dx, \end{aligned} \tag{57}$$

with

$$f(x) = \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}. \tag{58}$$

Next, we apply and adjust Theorem 1 of Wang and Pötzelberger (1997) (Equation 4 on p. 56) to derive the crossing probability for a piecewise linear boundary m_t and a Brownian motion with drift α. For easier expression, we define $t_j - t_{j-1} = \Delta t_j$.

$$cd_{t,\mathbb{Q}} = \mathbb{Q}\left\{\hat{M}_t < m_t, t \leq T\right\} = 1 - \mathbb{E}_Q \left\{g\left(W_{t_1}, ..., W_{t_n}, m_{t_1}, ..., m_{t_n}\right)\right\}, \tag{59}$$

with

$$\begin{aligned} & g\left(x_1, ..., x_n, m_1, ..., m_n\right) \\ &= \prod_{j=1}^{n} \mathbb{I}_{(x_j + \alpha \Delta t_j \geq m_j)} \left(1 - e^{-\frac{2(m_{j-1} - \alpha \Delta t_{j-1} - x_{j-1})(m_j - \alpha \Delta t_j - x_j)}{\Delta t_j}}\right). \end{aligned} \tag{60}$$

By applying Equation (57) to all time steps, we can transform Equation (59) into an integral function of the form:

$$\begin{aligned} cd_{t,\mathbb{Q}} &= \mathbb{Q}\left\{\hat{M}_t < m_t, t \leq T\right\} \\ &= 1 - \int_{m-\alpha t}^{\infty} \left[\prod_{j=1}^{n} \left(1 - e^{-\frac{2(m_{j-1} - \alpha \Delta t_{j-1} - x_{j-1})(m_j - \alpha \Delta t_j - x_j)}{\Delta t_j}}\right) \right. \\ & \left. \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi \Delta t_j}} e^{-\frac{(x_j - x_{j-1})^2}{2\Delta t_j}} \right] dx \\ &= 1 - \int_{m-\alpha t}^{\infty} \left[h(m,x)k(x)\right] dx, \end{aligned} \tag{61}$$

with

$$h(m,x) = \prod_{j=1}^{n} \left(1 - e^{-\frac{2(m_{j-1} - \alpha \Delta t_{j-1} - x_{j-1})(m_j - \alpha \Delta t_j - x_j)}{\Delta t_j}}\right), \tag{62}$$

$$k(x) = \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi \Delta t_j}} e^{-\frac{(x_j - x_{j-1})^2}{2\Delta t_j}}. \tag{63}$$

Plugging in gBm-adjusted values for α and m_t allows us to arrive at an explicit formula for the default probability $cd_{t,\mathbb{Q}}$ of a gBm versus piecewise linear default barriers, thus, reflecting the dynamics of the redemption policies of LBO investments. α is defined as in Equation (55), and m_t is similar to Equation (56) but piecewise linear:

$$\alpha = \frac{1}{\sigma}\left(\mu - \frac{\sigma^2}{2}\right), \tag{64}$$

$$m_t = \frac{1}{\sigma} ln \left(\frac{db_t^{(\cdot)}}{X_0}\right). \tag{65}$$

For $n = 1$, Equation (59) collapses to the explicit analytic form of Equation (51). Otherwise, numerical integration, e.g., via adaptive strategies in MATLAB or MATHEMATICA, is required. This approach, however, is highly efficient and precise compared to an extensive Monte Carlo simulation. We provide numerical examples and comparative statics in Section 5.

4. Explicit-form solution

With Equation (59) from the previous section, we can solve all value components for equityholders (Equations (27) to (36)) and debtholders (Equations (42) to (46)) irrespective of the prevailing redemption policy. For cash sweep redemption, our model yields stochastic default boundaries in any case. Under fixed redemption, the boundaries are deterministic if we do not include the accumulation of excess cash ($\theta = 1$) and become stochastic otherwise. The structure of the nested integrals in our model, however, enables us to find solutions for any of these problems.

In general, we use the common relationship for continuous random variables:

$$\mathbb{E}(Z) = \int_{-\infty}^{\infty} Z f(z) dz, \tag{66}$$

where $f(z)$ is the density function of the random variable Z.

The value components of our model are equivalent to random variables with density function $h(m_t, x) \cdot k(x)$ as defined in Equations (62) and (63). Note that x_t, as defined in Equation (60), is equivalent to the standard normally distributed random variable W_t of our EBIT or cash flow process. Some of the conditions concerning the value components, captured in indicator functions, restrict the areas of the integrals. Table 2 summarizes value components and their conditions.

The first three conditions (going concern, default, non-negative debt) result

Table 2: Value conditions. Six conditions regarding the different LBO payoffs for equityholders and debtholders have been derived in Section 2. This table summarizes those conditions and their relationship with the NPV and IRR value components. For the present value of the holding period, we need to separate going concern from default payoffs as well as distinguish the components within the going concern part to ensure a correct mapping of conditions. In detail, the six conditions are: (1) gc - going concern condition ($X_t \geq db_t^{(\cdot)}$), (2) def - default condition ($X_t < db_t^{(\cdot)}$), (3) noneg - non-negative condition of debt ($D_t \geq 0$), (4) defEq - condition for liquidation value of equity ($Liq_d > 0$), (5) defD1 - condition 1 for liquidation value of debt ($(1-\rho)V_t^U < C_t + D_{t-1}$), and (6) defD2 - condition 2 for liquidation value of debt ($(1-\rho)V_t^U \geq C_t + D_{t-1}$). The term "both" indicates that the condition exists irrespective of the type of redemption, while "sweep" indicates conditions relevant only for cash sweep redemption. Thus, entries denoted "fixed" are exclusive to fixed redemption.

Value components		**Value conditions**					
		(1) gc	**(2) def**	**(3) noneg**	**(4) defEq**	**(5) defD1**	**(6) defD2**
NPV^{Eq}							
PV_{HP}	$\theta(X_t - NC_t)$	both					
	$\theta R_t^{(\cdot)}$	both		sweep			
	Liq_d		both		both		
PV_{Exit}		both					
NPV^{Dh}							
PV_{HP}^{Dh}	C_t	both					
	$R_t^{(\cdot)}$	both		sweep			
	$(1-\rho)V_d^U$		both			both	
	$C_d + D_{d-1}$		both				both
PV_{Exit}^{Dh}		both					

in adjustments of the integral areas because the value components affected by them are stochastic. To transform these conditions into lower and upper limits of the integral areas, they have to be rearranged for x_t. While we provide the transformations step by step in Appendix B, Equations (67) to (69) present the results.

$$1.\ \text{gc}: \qquad x_t \geq \frac{1}{\sigma} \ln\left(\frac{db_t^{(\cdot)}}{X_0}\right) - \alpha t = m_t^{(\cdot)}, \tag{67}$$

$$2.\ \text{def}: \qquad x_t < \frac{1}{\sigma} \ln\left(\frac{db_t^{(\cdot)}}{X_0}\right) - \alpha t = m_t^{(\cdot)}, \tag{68}$$

$$3.\ \text{noneg}: \qquad x_t \leq \frac{1}{\sigma} \ln\left(\frac{D_{t-1}(1+\gamma y_D(1-\tau_c))}{\gamma X_0}\right) - \alpha t = n_t^{sweep}. \tag{69}$$

Value conditions (4) to (6) are relevant in the case of default. All default payoffs are path-dependent up to the last going concern period but deterministic in the default period itself. This is because the cash flow level in the case of default equals the default boundary, $X_t = db_t$ for $t = d$, which is determined at the beginning of the period. The conditions lead to deterministic max and min relationships for the default payoffs. These payoffs are kept within the integrals up to the last period prior to default and multiplied by the incremental probability of the default period, $pd_{t,\mathbb{Q}} = \mathbb{Q}\left\{X_t < db_t^{(\cdot)}\right\} - \mathbb{Q}\left\{X_{t-1} < db_{t-1}^{\cdot}\right\}$ with $t-1 < d \leq t$. We define $pd_{t,\mathbb{Q}}$ by:

$$\begin{aligned}
pd_{t,\mathbb{Q}} &= \mathbb{Q}\left\{X_t < db_t^{(\cdot)}\right\} - \mathbb{Q}\left\{X_{t-1} < db_{t-1}^{(\cdot)}\right\} \\
&= \left(1 - \mathbb{Q}\left\{X_t \geq db_t^{(\cdot)}\right\}\right) - \left(1 - \mathbb{Q}\left\{X_{t-1} \geq db_{t-1}^{(\cdot)}\right\}\right) \\
&= \mathbb{Q}\left\{X_{t-1} \geq db_{t-1}^{(\cdot)}\right\} - \mathbb{Q}\left\{X_t \geq db_t^{(\cdot)}\right\} \\
&= cd_{t-1,\mathbb{Q}} - cd_{t,\mathbb{Q}}. \qquad (70)
\end{aligned}$$

To conclude, the underlying algebra is simple once the value conditions have been generated: We apply Equation (66) with $h(m_t, x) \cdot k(x)$ as our density function to all value components, limit the integral areas by our transformed conditions and restrict our default payoffs by deterministic min and max relationships. Thus, we are able to restate the NPV equations in explicit integral form. We begin with the equityholders, where the NPV^{Eq} is now determined by:

$$NPV^{Eq} = -I_0 + PV_{HP}^{(\cdot)} + PV_{Exit}, \qquad (71)$$

with

$$\begin{aligned}
PV_{HP}^{fixed} = &\sum_{t=1}^{T} e^{-rt} \mathbb{E}_Q \left(\left(\theta X_t - \theta NC_t - \theta R_t^{fixed} \right) \mathbb{I}_{\{gc, 0<t\leq d\}} \right) \\
&+ e^{-rd} \mathbb{E}_Q \left(Liq_d^{def} \mathbb{I}_{\{def, 0<d\leq T\}} \mathbb{I}_{\{liqEq, 0<d\leq T\}} \right) \\
= &\sum_{t=1}^{T} \left[e^{-rt} \left(\int_{m_t^{fixed}}^{\infty} \theta \left(X_t - NC_t - R_t^{fixed} \right) h(m^{fixed}, x) k(x) dx \right.\right. \\
&\left.\left. + \int_{\substack{m_s^{fixed} \\ \text{for } s<t}}^{\infty} pd_{t,Q} \max \left(Liq_d^{def}, 0 \right) h(m^{fixed}, x) k(x) dx \right) \right], \qquad (72)
\end{aligned}$$

$$
\begin{aligned}
PV_{HP}^{sweep} =& \sum_{t=1}^{T} e^{-rt}\mathbb{E}_Q\left(\left(\theta X_t - \theta NC_t - \theta R_t^{sweep}\mathbb{I}_{\{noneg,0<t\leq d\}}\right)\mathbb{I}_{\{gc,0<t\leq d\}}\right) \\
&+ e^{-rd}\mathbb{E}_Q\left(Liq_d^{def}\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqEq,0<d\leq T\}}\right) \\
=& \sum_{t=1}^{T}\Bigg[e^{-rt}\Bigg(\int_{m_t^{sweep}}^{\infty}\theta\left(X_t - NC_t\right)h(m^{sweep},x)k(x)dx \\
&- \int_{m_t^{sweep}}^{n_t^{sweep}}\theta R_t^{sweep}h(m^{sweep},x)k(x)dx \\
&+ \int_{\substack{m_s^{sweep}\\ \text{for } s<t}}^{\infty} pd_{t,Q}\max\left(Liq_d^{def},0\right)h(m^{sweep},x)k(x)dx\Bigg)\Bigg], \qquad (73) \\
PV_{Exit} =& e^{-rT}\mathbb{E}_Q\left((V_T^U + V_T^{TS} + V_T^{XC} - D_T)\mathbb{I}_{\{gc,0<t\leq T\}}\right) \\
=& e^{-rT}\int_{m_t^{(.)}}^{\infty}(V_T^U + V_T^{TS} + V_T^{XC} - D_T)h(m^{(.)},x)k(x)dx. \qquad (74)
\end{aligned}
$$

For the debtholders, we equivalently derive a definition of NPV^{Dh} by:

$$
NPV^{Dh} = -D_0 + PV_{HP}^{Dh,(.)} + PV_{Exit}^{Dh}, \qquad (75)
$$

with

$$
\begin{aligned}
PV_{HP}^{Dh,fixed} =& \sum_{t=1}^{T} e^{-rt}\mathbb{E}_Q\left((C_t + R_t^{fixed})\mathbb{I}_{\{gc,0<t\leq d\}}\right) \\
&+ e^{-rd}\mathbb{E}_Q\left((1-\rho)V_d^U\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqD1,0<d\leq T\}}\right) \\
&+ e^{-rd}\mathbb{E}_Q\left((C_d + D_{d-1})\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqD2,0<d\leq T\}}\right) \\
=& \sum_{t=1}^{T}\Bigg[e^{-rt}\Bigg(\int_{m_t^{fixed}}^{\infty}(C_t + R_t^{fixed})h(m^{fixed},x)k(x)dx \\
&+ \int_{\substack{m_s^{fixed}\\ \text{for } s<t}}^{\infty} pd_{t,Q}\min\left((1-\rho)V_d^U, C_d + D_{d-1}\right)h(m^{fixed},x)k(x)dx\Bigg)\Bigg], \\
& \qquad (76)
\end{aligned}
$$

$$
\begin{aligned}
PV_{HP}^{Dh,sweep} =& \sum_{t=1}^{T} e^{-rt}\mathbb{E}_Q\left((C_t + R_t^{sweep}\mathbb{I}_{\{noneg,0<t\leq d\}})\mathbb{I}_{\{gc,0<t\leq d\}}\right) \\
&+ e^{-rd}\mathbb{E}_Q\left((1-\rho)V_d^U\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqD1,0<d\leq T\}}\right) \\
&+ e^{-rd}\mathbb{E}_Q\left((C_d + D_{d-1})\mathbb{I}_{\{def,0<d\leq T\}}\mathbb{I}_{\{liqD2,0<d\leq T\}}\right), \\
=& \sum_{t=1}^{T}\left[e^{-rt}\left(\int_{m_t^{sweep}}^{\infty} C_t h(m^{sweep},x)k(x)dx\right.\right. \\
&+ \int_{m_t^{sweep}}^{n_t^{sweep}} R_t^{sweep} h(m^{sweep},x)k(x)dx \\
&\left.\left. + \int_{\substack{m_s^{sweep} \\ \text{for } s<t}}^{\infty} pd_{t,Q}\min\left((1-\rho)V_d^U, C_d + D_{d-1}\right)h(m^{sweep},x)k(x)dx\right)\right],
\end{aligned} \tag{77}
$$

$$
\begin{aligned}
PV_{Exit}^{Dh} =& e^{-rT}\mathbb{E}_Q\left(D_T\mathbb{I}_{\{gc,0<t\leq T\}}\mathbb{I}_{\{noneg,0<t\leq T\}}\right) \\
=& e^{-rT}\int_{m_t^{(\cdot)}}^{n_t^{(\cdot)}} D_T h(m^{(\cdot)},x)k(x)dx.
\end{aligned} \tag{78}
$$

Thus, we have established a model consisting of explicit valuation equations for all NPV components, allowing us to evaluate any leveraged buyout from the equityholder and debtholder perspectives. We are able to capture any financing structure (fixed redemption and cash sweep redemption with different initial debt levels, redemption parameters and sequences), retention policy (dividend ratios from zero to one) and performance development (drift rate, standard deviation). While the nested integrals presented in this section deliver solutions to all of these complex cases, it should be noted that for simpler cases the nested integrals collapse to explicit analytic solutions equivalent to ordinary down-and-out barriers. An example would be the case of no redemption ($\gamma = 0$ or $f_t = 0$) and full payout ($\theta = 1$), where the default boundary is constant throughout the holding period. Appendix C provides the barrier option formulae of this special case, demonstrating the connection of our general solution and the well-known barrier option framework established by Merton (1973) and subsequently extended by Rubinstein and Reiner (1991).

The next section provides a numerical example of LBO evaluation and comparative statics, allowing us to draw conclusions concerning optimal financing structures in LBOs.

5. LBO evaluation and optimal financing

In this section, we demonstrate the capabilities of our model through a numerical application. Moreover, we provide economic insights in the field of leveraged buyouts by examining the impact of flexible choices in setting up and executing the LBO, by analyzing the influence of optimizing the financing structure for either of the two main investment criteria, NPV and IRR, and by accessing the sensitivity of the results to the model parameters.

5.1. Numerical application and comparison of cash sweep and fixed debt redemption

We assume an exemplary LBO setting in which the buyer intends to keep the target company for three years ($T = 3$). Debt redemption, interest payments and dividends occur annually. The EBIT process follows a gBm with initial level of $EBIT_0 = 166.67$. The expected drift rate of the EBIT process in the absence of an LBO is $\mu_{\mathbb{P},Pre} = 2\%$. The buyer expects to increase the drift rate to $\mu_{\mathbb{P}} = 5\%$ over the holding period and to establish a post-LBO drift of $\mu_{\mathbb{P},Post} = 3\%$. The volatility σ is expected to be constant at 20%. The corporate tax rate τ_c is fixed at 40%, and the risk-free rate of return r is deterministic and constant throughout the holding period at 3%.

The debt level prior to the LBO is $D_{Pre} = 300$. D_{Post}, the debt level after exit, is state-dependent and determined as a multiple, $l^* = 3$, of the unlevered after-tax cash flow at exit (X_T). To finance the deal, the buyer targets an initial debt level of $D_0 = 650$. Under cash sweep debt redemption, the buyer will use $\gamma = 0.6$ of the net cash flows to pay down debt, while all remaining cash will be paid out to her ($\theta = 1$). To construct a comparable fixed debt redemption case, we determine the expected unconditional redemption payments under the described cash sweep regime and set them as fixed redemptions f_t:

$$f_t = \gamma \left(X_t - y_D D_{t-1}(1 - \tau_c).\right) \tag{79}$$

For both types of redemption, a debt-to-EBIT covenant of $\beta = 6.5$ exists that triggers default as soon as it is exceeded. In the event of default, the firm faces bankruptcy costs of $\rho = 25\%$ of the then-prevailing market value of assets (V_d^U). Note that all input parameters are the same for the two redemption policies. Only the redemption ratio γ is replaced by fixed cash obligations f_t, which are not path-dependent. Table 3 summarizes the basic set of parameters.

Table 3: Base case parameters of numerical application. This table shows the basic set of parameters used for the numerical application of the model.

Variable	**Description**	**Value**
T	holding period in years	3
$EBIT_0$	earnings before interests and taxes in t=0	166.67
τ_c	corporate tax rate	40%
X_0	unlevered after-tax cash flow in t=0	100
$\mu_{\mathbb{P},Pre}$	drift rate of EBIT prior to LBO	2%
$\mu_{\mathbb{P}}$	drift rate of EBIT during LBO	5%
$\mu_{\mathbb{P},Post}$	drift rate of EBIT post LBO	3%
σ	volatility of EBIT	20%
r	risk-free rate	3%
r_A	asset rate	10%
μ_{Pre}	risk-neutral drift rate of EBIT prior to LBO	-5%
μ	risk-neutral drift rate of EBIT during LBO	-2%
μ_{Post}	risk-neutral drift rate of EBIT post LBO	-4%
D_{Pre}	target company's debt level prior to LBO	300
l^*	industry average multiple for debt level after exit	3
D_0	start debt level in LBO	650
γ	cash sweep redemption ratio	0.6
θ	dividend ratio	1
β	debt-to-EBIT covenant	6.5
ρ	bankruptcy cost ratio	25%

With the information at hand, we determine the promised yield of debt according to the approach described in Section 2.6. Under cash sweep redemption, we arrive at a promised yield of $y_D^{sweep} = 3.72\%$, while fixed redemption results in $y_D^{fixed} = 3.81\%$. Again, those yields ensure an NPV of zero for the debtholders under the risk of default. The corresponding cumulative risk-neutral probabilities of default for the total holding period are $cd_{T,\mathbb{Q}}^{sweep} = 13.41\%$ and $cd_{T,\mathbb{Q}}^{fixed} = 26.78\%$.

Having determined the pricing of debt, we complete the first comparative analysis concerning cash sweep versus fixed debt redemption. Table 4 provides a comparison of the financing parameters and model outputs for both cases.

Table 4: Comparison of numerical results for cash sweep redemption and fixed redemption. This table compares the results obtained for both types of redemption using the model from Section 4 concerning promised yield of debt (y_D) and the cumulative probability of default ($cd_{t,\mathbb{Q}}$), NPV and IRR. We use the variable definitions as in Table 1 and the parameters as in Table 3. The financing parameters illustrate how the default boundaries (db_t) are derived for each period of time. The values of f_t, int_t, co_t, cov_t and db_t are simple expected values without reflecting the risk of default. Note that in case of cash sweep redemption, the financing parameters become path-dependent beginning in period 2. The formulae for $\mathbb{Q}_1$ to $\mathbb{Q}_3$ (Equation (61)), I_0 (Equation (33)),PV_1 to PV_3 (Equations (72) and (73)), PV_{Exit} (Equation (74)), NPV (Equation (27)) and IRR (Equation (28)) have been applied as derived in Sections 2 to 4.

Financing Parameters

Variable	Description	Cash Sweep	Fixed
D_0	start debt level	650	650
γ	redemption ratio	0.6	0.6
β	debt-to-EBIT covenant	6.5	6.5
$\boldsymbol{y_D}$	**promised yield of debt**	**3.72%**	**3.81%**
R_1	debt redemption in t=1	50.1	50.1
NC_1	after-tax interest payment in t=1	14.5	14.9
co_1	cash obligation in t=1	14.5	65.0
cov_1	covenant condition in t=1	60.0	60.0
$\boldsymbol{db_1}$	**default boundary in t=1**	**60.0**	**65.0**
R_2	debt redemption in t=2	*49.6*	49.6
NC_2	after-tax interest payment in t=2	*13.4*	13.7
co_2	cash obligation in t=2	*13.4*	63.4
cov_2	covenant condition in t=2	*55.4*	55.4
$\boldsymbol{db_2}$	**default boundary in t=2**	***55.4***	**63.4**
R_3	debt redemption in t=3	*49.1*	49.1
NC_3	after-tax interest payment in t=3	*12.3*	12.6
co_3	cash obligation in t=3	*12.3*	61.7
cov_3	covenant condition in t=3	*50.8*	50.8
$\boldsymbol{db_3}$	**default boundary in t=3**	***50.8***	**61.7**

Model Output

Variable	Description	Cash Sweep	Fixed
$cd_{1,\mathbb{Q}}$	cum. default probability up to t=1	1.75%	4.73%
$cd_{2,\mathbb{Q}}$	cum. default probability up to t=2	7.78%	16.70%
$cd_{1,\mathbb{Q}}$	cum. default probability up to t=3	13.41%	26.78%
I_0	initial equity investment	-597.56	-597.56
PV_1^{gc}	PV of dividend in t=1	32.12	32.07
PV_2^{gc}	PV of dividend in t=2	29.89	30.88
PV_3^{gc}	PV of dividend in t=3	27.92	29.71
PV_1^{def}	PV of potential default in t=1	0.00	0.00
PV_2^{def}	PV of potential default in t=2	0.00	3.49
PV_3^{def}	PV of potential default in t=3	0.00	5.17
PV_{Exit}	PV of LBO exit	746.57	680.70
$\boldsymbol{NPV}$	**net present value**	**238.94**	**184.45**
$\boldsymbol{IRR}$	**internal rate of return**	**15.83%**	**13.19%**

Note that fixed redemption f_t, after-tax interest payments NC_t, cash obligations co_t, covenant conditions cov_t and default boundaries db_t are unconditional expected values, i.e., expected values without reflecting default risk. Under fixed debt redemption, the cash obligations co_t define the default boundaries db_t throughout the full holding period. Thus, the probability of default is consistently higher than under cash sweep redemption. The impact on NPV and IRR is straightforward: Given the specification selected here, cash sweep redemption is superior under both investment criteria ($NPV^{sweep} = 238.94 > 184.45 = NPV^{fixed}$, $IRR^{sweep} = 15.83 > 13.19 = IRR^{fixed}$).

The results may create the impression that cash sweep debt redemption is dominating fixed debt redemption. However, analyzing the dynamics of this relationship reveals contrary insights.

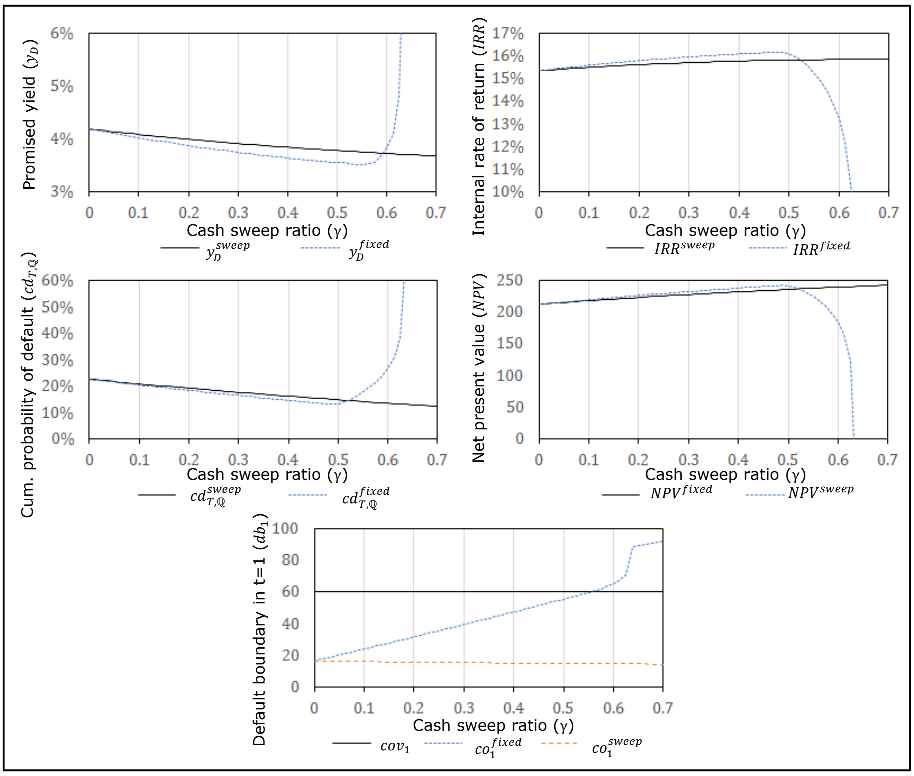

Figure 3: Cash sweep redemption versus fixed redemption for different cash sweep ratios γ and the basic set of parameters as in Table 3. This figure depicts the promised yield (y_D), the cumulative risk-neutral probability ($cd_{T,\mathbb{Q}}$), the internal rate of return (IRR), the net present value (NPV) and the default boundary of period 1 over the interval of $\gamma \in [0, 0.7]$ for cash sweep redemption (black, solid line) and fixed redemption (blue, dotted line). The fixed debt redemption payments are matched to the cash sweep redemption ratio γ as defined by Equation (79).

Figure 3 shows that for moderate levels of the cash sweep ratio ($0 < \gamma \leq 0.59$), translating to moderate fixed cash obligations, the promised yield y_D is lower under fixed debt redemption. This triggers a lower cumulative default probability ($cd_{T,\mathbb{Q}}$), a higher IRR and an increased NPV over a γ interval of $0 < \gamma \leq 0.52$. The bottom graph in Figure 3 provides the first part of the explanation: For moderate γ values, both redemption policies face a default boundary determined by the covenant condition rather than the cash obligation. The fixed cash obligation increases substantially with increasing values of γ, resulting in a switch in the default trigger at $\gamma = 0.56$. The rationale from a debtholders' perspective is straightforward: As long as the default boundary is identical for both types of redemption, fixed redemption delivers higher payoffs in adverse (going concern) states. In other words, the flexibility of the cash sweep redemption requires a premium. As soon as the default boundary for the fixed case exceeds the one of the cash sweep case, the higher probability of default for the fixed case makes the cash sweep case more favorable, generating a trade-off that quickly results in an increasing promised yield. The second part of the explanation is less trivial. While the two curves for default boundary and promised yield cross at $\gamma = 0.59$ and $\gamma = 0.56$, respectively, the curves for probability of default, IRR and NPV intersect earlier at $\gamma = 0.52$. This is caused by the path-dependency of the default boundary under cash sweep debt redemption. Note that the graph only depicts the first period's default boundary because the subsequent ones are path-dependent and, thus, not comparable on an aggregated level. A high cash flow in the first period reduces the covenant trigger of the subsequent periods, resulting in a lower probability of default followed by higher expected payoffs. This effect of the reduction of the default boundary causes the IRR and NPV shift as illustrated.

Based on this analysis, we can clearly state that the flexibility of cash sweep debt redemption adds significant value for investors in LBOs and reduces the risk of default for target companies as long as the corresponding fixed cash obligations in any period exceed the covenant restriction. However, if fixed cash obligations are moderate, i.e., remain well below the covenant condition, the pricing of debt will be lower because cash flows to the debtholders are fixed reducing the credit risk. In such cases, fixed debt redemption is more beneficial for the equityholders because the lower promised yield translates into higher payoffs, lower default probability (due to higher payoffs) and, thus, to increases in NPV and IRR.

To conclude, our results provide an economic rationale for the widespread phenomenon of mixed (cash sweep and fixed) financing structures in LBOs. Fixed debt is available at lower prices as long as the cash obligations are not critical to the firm's going concern. Buyers in LBOs exploit this fact by filling senior tranches with fixed debt and adding junior cash sweep debt to exploit the benefits of debt without material increases in the probability of default.

5.2. Optimal financing for NPV and IRR maximization

Investors in LBOs typically apply the IRR, both for investment and financing decisions and for performance measurement (see, e.g., Gompers et al., 2015), although academics have postulated for more than 50 years that there are serious difficulties and pitfalls associated with this decision criterion (see, e.g., Lorie and Savage, 1949, and Hirshleifer, 1958). Brealey, Myers and Allen provided the classic critique in the first edition of "Principles of Corporate Finance" and continue to do so (Brealey et al., 2013). We skip the discussion of these well-known pitfalls, Footnote 2 of the introduction section contains a brief summary. Our focus is on the impact of the investment criterion choice in a dynamic setting with stochastic cash flows and explicit risk of default.

For the analysis, we consider the cash sweep policy introduced in the previous section but vary the initial debt level to identify the optimal leverage that maximizes NPV or IRR. Figure 4 summarizes the results graphically.

Choosing the initial debt level based on a maximized IRR implies $D_0 = 650$ and results in $IRR = 15.83\%$. The corresponding NPV of this structure is 238.94. In turn, when we optimize the initial debt level for maximizing the NPV criterion, we arrive at $D_0 = 460$ with $NPV = 267.55$. Thus, optimizing the initial debt level by the IRR criterion led to an NPV reduction of 10.69%. Moreover, the IRR criterion fostered risk taking: The cumulative default probability over the holding period is $cd_{T,\mathbb{Q}} = 13.41\%$, while for the NPV-optimal debt level ($D_0 = 460$) $cd_{T,\mathbb{Q}}$ only amounts to 0.99%.

The results provide insights into the IRR vs. NPV discussion beyond the classical arguments based on static frameworks. Our model allows for a precise comparison concerning value creation and risk taking. The numerical results indicate that the IRR criterion encourages investors to choose higher leverage, which in turn increases default probability and reduces NPV creation.

A formal mathematical proof of this finding is beyond the scope of this paper and remains for further research. However, we present the basic intuition: The IRR approach discounts future expected cash flows stronger than the NPV method, because it uses the risk-neutral IRR as a discount rate instead of the risk-free rate r. Therefore, low initial equity investments, which are driven by high initial debt levels, receive a higher weight. In contrast, reductions of the expected cash flows over the holding period (particularly towards the end of the holding period), driven by an increased risk of default, have a lower impact because of the higher discount rate. Thus, the optimal trade-off between benefits and costs of debt are different for the two investment criteria.

The sensitivity analyses in the next section add further robustness to our findings. Note that the results under fixed debt redemption are equivalent. We provide them in Appendix D.1.

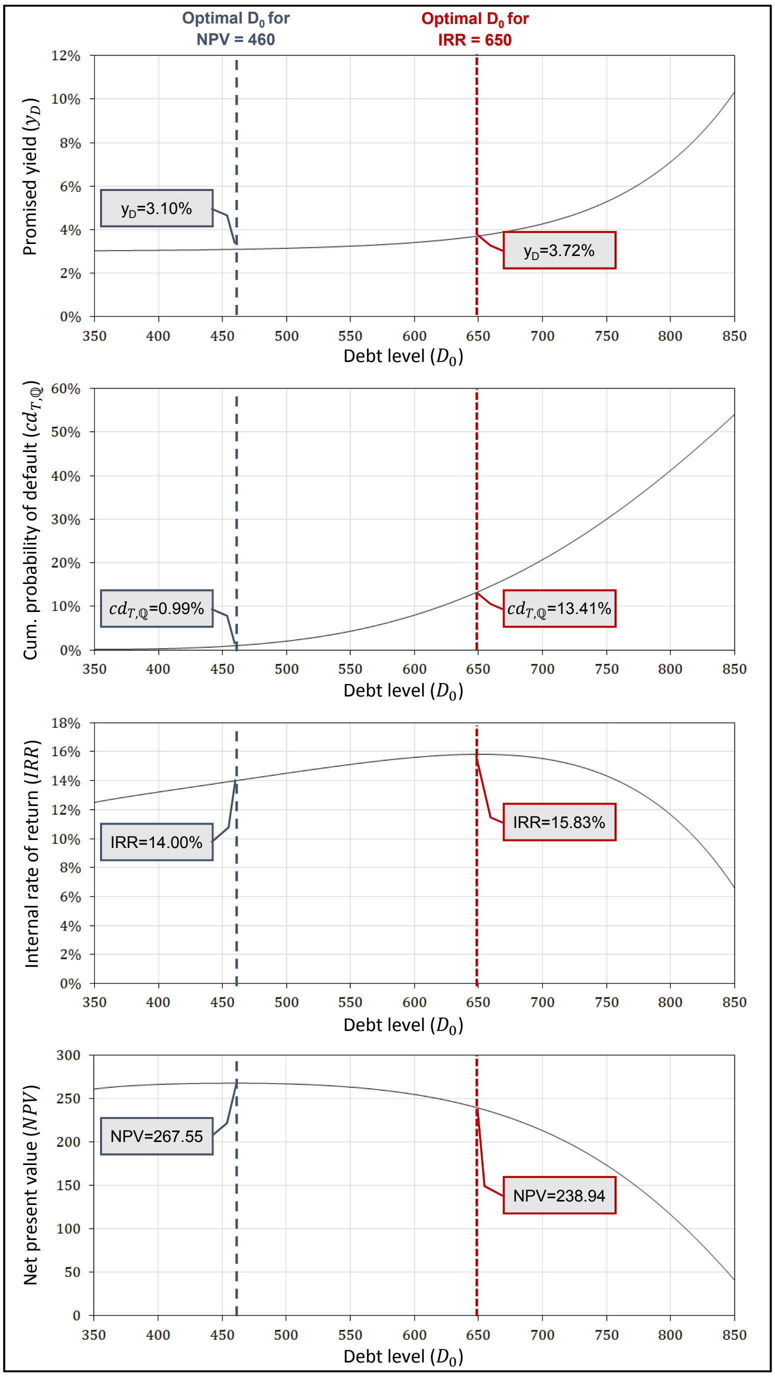

Figure 4: Optimal initial debt level for maximizing NPV or IRR. This figure depicts the promised yield y_D (first graph), the cumulative risk-neutral probability over the full holding period $cd_{T,\mathbb{Q}}$ (second graph), the IRR (third graph), and the NPV (fourth graph), each as a function of the initial debt level over the interval of $D_0 \in [350, 850]$. We use the basic set of parameters reported in Table 3 and the financing parameters for cash sweep redemption as illustrated in Table 4. The optimal initial debt level D_0 for maximizing the NPV (blue, dashed line) or the IRR (red, small dashed line) are shown in all four graphs.

5.3. Comparative Statics

Subsequently, we provide the comparative statics for the case of cash sweep redemption. Each parameter is individually adjusted while holding all other inputs equal to the base case, i.e., ceteris paribus condition. Table 5 presents the results.

Table 5: Comparative Statics. This table reports the comparative statics of the model derived in Section 4 for cash sweep redemption with respect to all model parameters. We use the basic set of parameters illustrated in Table 3 and for the financing parameters listed in Table 4. Output parameters are the promised yield (y_D), the cumulative risk-neutral probability ($cd_{T,\mathbb{Q}}$), the internal rate of return (IRR^{Eq}), the net present value (NPV^{Eq}), the IRR-maximizing debt level (D^*_{IRR}), the corresponding IRR ($IRR^{Eq,*}$), the NPV-maximizing debt level (D^*_{NPV}) and the corresponding NPV ($NPV^{Eq,*}$). Model parameters are defined as follows: μ is the risk-neutral drift rate; σ is the $EBIT$'s volatility; r is the risk-free rate; τ_c is the corporate tax rate; β is the debt-to-EBIT covenant; ρ is the bankruptcy cost ratio; θ is the dividend ratio; and γ is the redemption ratio. The basic parameter values are shown in bold.

Var	**1**	**2**	**3**	**4**	**5**	**Var**	**1**	**2**	**3**	**4**	**5**
$\boldsymbol{\mu_{\mathbb{P}}}$	1.0%	3.0%	**5.0%**	7.0%	9.0%	$\boldsymbol{\sigma}$	10.0%	15.0%	**20.0%**	25.0%	30.0%
y_D	4.34%	3.98%	3.72%	3.53%	3.39%	y_D	3.05%	3.19%	3.72%	4.81%	6.54%
$cd_{T,\mathbb{Q}}$	21.93%	17.30%	13.41%	10.22%	7.65%	$cd_{T,\mathbb{Q}}$	0.12%	3.69%	13.41%	25.64%	37.44%
IRR^{Eq}	6.64%	11.33%	15.83%	20.18%	24.42%	IRR^{Eq}	17.38%	17.04%	15.83%	14.02%	11.96%
NPV^{Eq}	62	149	239	333	432	NPV^{Eq}	277	267	239	200	159
D^*_{IRR}	526	595	651	700	742	D^*_{IRR}	830	734	651	579	532
$IRR^{Eq,*}$	7.81%	11.60%	15.83%	20.46%	25.47%	$IRR^{Eq,*}$	21.09%	17.81%	15.83%	14.49%	13.35%
D^*_{NPV}	420	440	460	478	499	D^*_{NPV}	690	558	460	444	435
$NPV^{Eq,*}$	107	185	268	355	448	$NPV^{Eq,*}$	277	272	268	259	236
$\boldsymbol{r}$	1.0%	2.0%	**3.0%**	4.0%	5.0%	$\boldsymbol{\tau_c}$	0.0%	20.0%	30.0%	**40.0%**	50.0%
y_D	1.75%	2.73%	3.72%	4.72%	5.73%	y_D	3.29%	3.33%	3.34%	3.72%	4.60%
$cd_{T,\mathbb{Q}}$	16.18%	14.75%	13.41%	12.17%	11.01%	$cd_{T,\mathbb{Q}}$	9.18%	11.03%	12.09%	13.41%	15.00%
IRR^{Eq}	12.57%	14.20%	15.83%	17.45%	19.09%	IRR^{Eq}	12.45%	13.44%	14.30%	15.83%	19.22%
NPV^{Eq}	218	229	239	249	258	NPV^{Eq}	387	312	274	239	210
D^*_{IRR}	620	636	651	666	680	D^*_{IRR}	617	618	627	651	717
$IRR^{Eq,*}$	12.65%	14.22%	15.83%	17.48%	19.19%	$IRR^{Eq,*}$	12.51%	13.51%	14.34%	15.83%	19.83%
D^*_{NPV}	417	438	460	480	500	D^*_{NPV}	0	441	448	460	483
$NPV^{Eq,*}$	259	263	268	272	277	$NPV^{Eq,*}$	430	344	307	268	228
$\boldsymbol{\beta}$	5.5	6	**6.5**	7	7.5	$\boldsymbol{\rho}$	15.0%	20.0%	**25.0%**	30.0%	35.0%
y_D	4.35%	3.67%	3.72%	3.65%	3.54%	y_D	3.36%	3.43%	3.72%	4.01%	4.32%
$cd_{T,\mathbb{Q}}$	30.76%	20.18%	13.41%	8.88%	5.87%	$cd_{T,\mathbb{Q}}$	13.26%	13.29%	13.41%	13.54%	13.67%
IRR^{Eq}	11.93%	14.43%	15.83%	16.55%	16.94%	IRR^{Eq}	16.19%	15.98%	15.83%	15.67%	15.51%
NPV^{Eq}	158	209	239	255	264	NPV^{Eq}	246	242	239	236	233
D^*_{IRR}	532	587	651	719	793	D^*_{IRR}	678	662	651	641	631
$IRR^{Eq,*}$	14.24%	14.93%	15.83%	16.96%	18.41%	$IRR^{Eq,*}$	16.27%	15.99%	15.83%	15.68%	15.55%
D^*_{NPV}	407	431	460	492	529	D^*_{NPV}	469	463	460	455	453
$NPV^{Eq,*}$	265	266	268	269	270	$NPV^{Eq,*}$	268	268	268	267	267
$\boldsymbol{\theta}$	0.0%	40.0%	60.0%	80.0%	**100.0%**	$\boldsymbol{\gamma}$	0.0%	40.0%	**60.0%**	80.0%	100.0%
y_D	3.67%	3.70%	3.71%	3.72%	3.72%	y_D	4.18%	3.84%	3.72%	3.62%	3.54%
$cd_{T,\mathbb{Q}}$	9.33%	10.80%	11.61%	12.48%	13.41%	$cd_{T,\mathbb{Q}}$	22.49%	16.05%	13.41%	11.17%	9.30%
IRR^{Eq}	15.73%	15.79%	15.81%	15.82%	15.83%	IRR^{Eq}	15.33%	15.76%	15.83%	15.82%	15.71%
NPV^{Eq}	249	245	243	241	239	NPV^{Eq}	212	232	239	245	249
D^*_{IRR}	673	665	660	656	651	D^*_{IRR}	597	635	651	665	680
$IRR^{Eq,*}$	15.78%	15.81%	15.82%	15.82%	15.83%	$IRR^{Eq,*}$	15.62%	15.78%	15.83%	15.84%	15.81%
D^*_{NPV}	485	474	469	464	460	D^*_{NPV}	395	433	460	512	572
$NPV^{Eq,*}$	269	268	268	268	268	$NPV^{Eq,*}$	268	268	268	266	259

Our findings are in line with the general intuition concerning the model and, thus, provide a sanity check. The drift rate $\mu_{\mathbb{P}}$ has a strong positive impact on IRR and NPV and a negative impact on debt pricing and the default probability. A higher drift rate, i.e., increased EBIT performance over the holding period, allows for higher optimal debt capacities irrespective of the applied decision criterion.

For the cash flow's volatility σ, the effects are reversed: Greater uncertainty requires a higher promised yield for the debtholders, raises the risk of default and reduces the results of both decision criteria. Consistently, the optimal debt levels for maximizing IRR and NPV decrease.

The impact of the risk-free rate r is twofold. On the one hand, an increased risk-free rate means higher discount factors, leading to a higher promised yield y_D and a lower NPV. Subsequently, an increase in y_D yields higher interest payments and, thus, forces down the IRR and entails a higher default probability. On the other hand, ceteris paribus, a rise in r yields a higher risk-neutral drift due to the relationship $\mu = \mu_{\mathbb{P}} - (r_A - r)$. As analyzed above, increasing drift rates have exactly the opposite effects on IRR, NPV and default probability. Because the second effect exceeds the first, overall, we find a positive relationship between r and the decision criteria, as well as a negative relationship with respect to the cumulative default probability.

The corporate tax rate τ_c has a positive impact on the promised yield y_D and cumulative default probability $cd_{T,\mathbb{Q}}$, i.e., a higher τ_c drives increases in y_D and $cd_{T,\mathbb{Q}}$ because less cash is available to serve the debtholders during the holding period and the liquidation value in event of default is also reduced. Hence, a negative relationship with respect to the NPV is a logical consequence. For the IRR, we find the opposite effect, which at first glance may appear surprising. However, the nature of the IRR determination perfectly explains the phenomenon. An increased corporate tax rate reduces future cash flows but also reduces the initial investment, which has a much higher weight under the IRR criterion in contrast to the NPV criterion. This property of the IRR also causes the optimal initial debt level for maximizing the IRR (D^*_{IRR}) to move contrary to D^*_{NPV}.

For the covenant ratio β, we find the same straightforward relationships as for the drift rate $\mu_{\mathbb{P}}$. A less tight covenant, i.e., β moves upwards, leads to decreasing default probabilities $cd_{T,\mathbb{Q}}$ and a lower promised yield y_D and has a positive impact on the investment criteria IRR and NPV. Consistently, the optimal debt levels are higher.

A rise in the bankruptcy cost ratio ρ triggers the exact opposite results, which are clearly due to the reduced liquidation value in the event of default.

The impact of the dividend ratio θ on the model parameters is of very small magnitude relative to the other parameters. Moreover, the direction of the effects is as expected: Lower dividends foster excess cash holdings, creating a cushion against default. Thus, the dividend ratio and probability of default $cd_{T,\mathbb{Q}}$ have a positive relationship, which triggers a negative impact on the promised yield y_D and the NPV. We find the opposite regarding the IRR, again caused by the fact that earlier cash flows (dividends received over the holding period) are valued higher than later cash flows (excess cash paid out at exit).

The effects generated by a change in the cash sweep ratio γ depend on a trade-off. A high γ reduces the debt burden more quickly, relaxes the covenant condition, and subsequently generates lower costs of debt y_D, lower default probabilities $cd_{T,\mathbb{Q}}$ and thus increased expected future payoffs. By contrast, a more rapid reduction of debt gives away tax savings and shifts cash flows from the beginning of the holding period towards the end (relevant for the IRR criterion). Which of the two effects dominates depends on the initial debt level. Figure 5 presents in detail the trade-off we described.

Thus, we find different effects concerning NPV and IRR for our base case debt level of $D_0 = 650$. The NPV-curves for the different γ-values have already crossed such that a higher γ generates an increased NPV. The IRR-curves, however, have not fully switched their positions. The first time the IRR is consistently increasing with γ is at $D_0 = 684$. At our base case debt level, $\gamma = 0.6$ generates the highest IRR.

Overall, the comparative statics demonstrate the robustness of the model. All model outputs created by changes in the input parameters conform to economic intuition. A complete comparative statics report regarding fixed debt redemption has also been drafted and is presented in Appendix D.2.

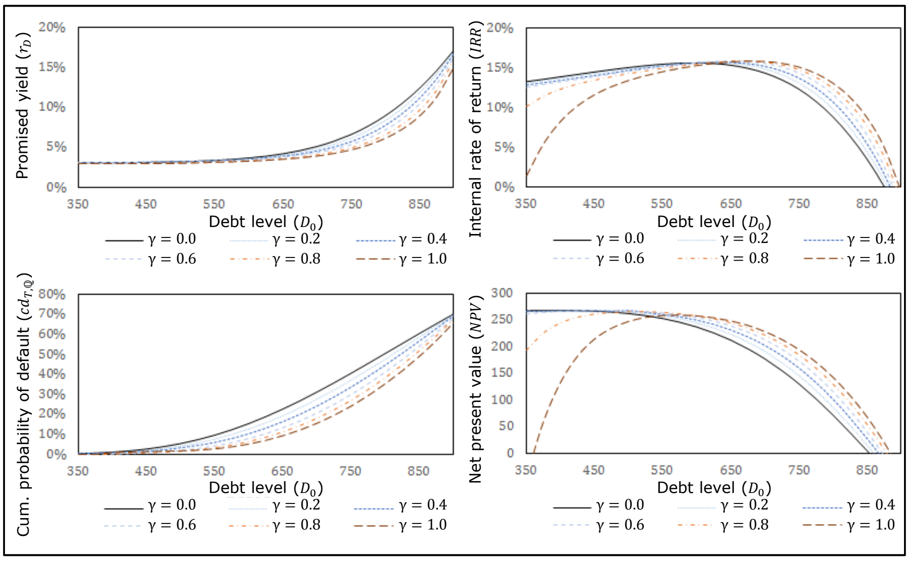

Figure 5: Detailed comparative analysis for cash sweep ratio γ. The figure illustrates the promised yield y_D (upper-left graph), the cumulative risk-neutral probability over the full holding period $cd_{T,\mathbb{Q}}$ (lower-left graph), the IRR (up-right graph), and the NPV (lower-right graph), each as a function of the initial debt level over the interval of $D_0 \in [350, 875]$ for $\gamma \in [0, 1]$. All other input parameters are as defined in the basic set reported in Table 3.

6. Conclusion

Target firms in an LBO setting follow a different capital structure and redemption policy than their peers. Initially financed by a large portion of debt, leverage is reduced stepwise over the holding period. In this article, we implement these dynamics in a model based on a boundary-crossing approach that allows for a non-differentiable functional form of the barrier to evaluate its financial effects.

Methodologically, our contribution can be summarized as follows. The method employed in this paper allows to capture both types of debt redemption found in LBO settings: the fixed, predetermined one and the dynamic, path-dependent one known as "cash sweep". Based upon these stepwise redemptions, we implement a discontinuous, non-differentiable lower boundary for mapping the default trigger. We allow for flexibility, as these boundaries can be either derived from cash obligations (redemption plus interest payments) or covenants (e.g., debt-to-EBIT ratio). By extending the approach of Wang and Pötzelberger (1997), wea are able to determine default probabilities using nested integrals that can be solved numerically. This enables us to provide explicit-form solutions for the evaluation of LBO investments using the NPV and IRR criteria. Additionally, the model captures the pricing of debt endogenously, which allows to also evaluate LBO structures from a debtholder perspective.

Our model makes some significant contributions to economic theory. We show that firms using a cash sweep redemption can create significant additional value for investors by reducing the risk of default due to the path-dependency of the redemption payments. However, we also find that for moderate fixed cash obligations, i.e., cash obligations are not critical as a default trigger, the promised yield of debt decreases as cash flows to debtholders are less exposed to credit risk. Equityholders are better off in such setups irrespective of the decision criterion employed because interest payments are reduced. Thus, our model explains why buyers in LBOs apply mixed financing structures, senior moderate levels of fixed debt and junior cash sweep debt. Furthermore, when stepwise optimizing the capital structure for either NPV or IRR, we find novel insights beyond the classic critics of the IRR. The entry debt level maximizing the IRR is strictly higher than the debt level implied by maximizing the NPV. In turn, this is reflected by a higher default probability for the IRR-maximizing case. Comparing the two decision criteria with respect to value creation, we find that the IRR criterion causes a significant NPV reduction.

Nonetheless, some limitations of our approach have to be addressed. We do not address the positive and negative wealth effects of corporate debt beyond tax savings and default. Debt provides several advantages for an LBO investor by reducing overinvestment and agency problems and, thus, the cost associated with them (Jensen and Meckling, 1976). On the opposite side, debt overhang and underinvestment are particular debt-related disadvantages not covered by our model. Additionally, as our model optimizes IRR and NPV at the firm level, we do not differentiate between general and limited partners usually engaged in PE investments. Therefore, including their typical compensation schemes might be a potential area of further research.

Finally, because the nested integral approach allows for several conditions, the model is easily extendable, e.g., to rules limiting the tax deductibility of interest payments.

A. Appendix: Stochastic calculus for Brownian motion with drift and constant lower boundary

A.1. Density function of Brownian motion with drift and its minimum

In this section, we derive the joint density function of a Brownian motion with drift $\hat{W}_t$ and its minimum $\hat{M}_t$.

First, we begin with a Brownian motion without drift W_t and adjust it in a way that a Brownian motion with drift $\hat{W}_t$ is generated:

$$\hat{W}_t = \alpha t + W_t. \tag{80}$$

Next, we define the minimum $\hat{M}_t$ of such a process under the prerequisites $\hat{M}_t \leq 0$ and $\hat{W}_t \geq \hat{M}_t$:

$$\hat{M}_t = \min_{0 \leq t \leq T} \hat{W}_t. \tag{81}$$

According to the Girsanov Theorem, we define a new probability measure $\hat{\mathbb{Q}}$ under which $\hat{W}_t$ has zero drift:

$$\hat{Z}_t = e^{-\alpha W_t - \frac{1}{2}\alpha^2 \cdot t} = e^{-\alpha \hat{W}_t + \frac{1}{2}\alpha^2 t} \tag{82}$$

$$\hat{Q}(A) = \int_A \hat{Z}_T d\mathbb{Q} \tag{83}$$

For a process without drift, we know the joint density function with its minimum from the Reflection Principle (for detailed derivation, see, for example, Shreve, 2004):

$$\hat{f}_{\hat{M}_t, \hat{W}_t}(m, w) = \frac{2(w - 2m)}{t\sqrt{2\pi t}} e^{\frac{-(2m-w)^2}{2t}} \tag{84}$$

Finally, we can derive the density of $\hat{M}_t$ and $\hat{W}_t$ under $\mathbb{Q}$, the risk-neutral probability measure:

$$
\begin{aligned}
\mathbb{Q}\{\hat{M}_t \geq m, \hat{W}_t \geq w\} &= \mathbb{E}_Q\{\mathbb{I}_{\{\hat{M}_t \geq m, \hat{W}_t \geq w\}}\} \\
&= \mathbb{E}_Q\{\frac{1}{\hat{Z}_t}\mathbb{I}_{\{\hat{M}_t \geq m, \hat{W}_t \geq w\}}\} \\
&= \mathbb{E}_Q\{e^{\alpha \hat{W}_t - \frac{1}{2}\alpha^2 t}\mathbb{I}_{\{\hat{M}_t \geq m, \hat{W}_t \geq w\}}\} \\
&= \int_m^\infty \int_w^\infty e^{\alpha Y - \frac{1}{2}\alpha^2 T} \hat{f}_{\hat{M}_t, \hat{W}_t}(x, y) dx dy \\
\frac{\delta^2 \mathbb{Q}\{\hat{M}_t \geq m, \hat{W}_t \geq w\}}{\delta m \delta w} &= e^{\alpha w - \frac{1}{2}\alpha^2 t} \hat{f}_{\hat{M}_t, \hat{W}_t}(m, w) \\
&= \frac{2(w - 2m)}{t\sqrt{2\pi t}} e^{\alpha w - \frac{1}{2}\alpha^2 t - \frac{(2m-w)^2}{2t}}. \qquad (85)
\end{aligned}
$$

A.2. Default and going concern probability of Brownian motion with drift versus a constant default barrier

Based on the density function of $\hat{M}_t$ and $\hat{W}_t$ under $\mathbb{Q}$ (Equation (85)), we derive the default and going concern probabilities of a Brownian motion with drift versus a constant default barrier. We begin with the going concern probability. Thus, the relevant set of values m and w is $\{(m, w); w \geq m, m \leq 0\}$. We integrate the density function over this region to determine:

$$
\begin{aligned}
\mathbb{Q}\{\hat{M}_T \geq m\} = &\int_m^0 \int_m^w \frac{2(v - 2m)}{T\sqrt{2\pi T}} e^{\alpha w - \frac{1}{2}\alpha^2 T - \frac{(2v-w)^2}{2T}} dv dw \\
&+ \int_0^\infty \int_m^0 \frac{2(v - 2m)}{T\sqrt{2\pi T}} e^{\alpha w - \frac{1}{2}\alpha^2 T - \frac{(2v-w)^2}{2T}} dv dw \qquad (86) \\
= &\int_m^0 \left[\frac{1}{\sqrt{2\pi T}} e^{\alpha v - \frac{\alpha^2 T}{2} - \frac{(2v-w)^2}{2T}}\right]_m^v dw \\
&+ \int_0^\infty \left[\frac{1}{\sqrt{2\pi T}} e^{\alpha v - \frac{\alpha^2 T}{2} - \frac{(2v-w)^2}{2T}}\right]_0^m dw \qquad (87) \\
= &\int_m^0 \frac{1}{\sqrt{2\pi T}} e^{\alpha w - \frac{\alpha^2 T}{2} - \frac{w^2}{2T}} dw - \int_m^0 \frac{1}{\sqrt{2\pi T}} e^{\alpha w - \frac{\alpha^2 T}{2} - \frac{(2m-w)^2}{2T}} dw \\
&+ \int_0^\infty \frac{1}{\sqrt{2\pi T}} e^{\alpha w - \frac{\alpha^2 T}{2} - \frac{w^2}{2T}} dw - \int_0^\infty \frac{1}{\sqrt{2\pi T}} e^{\alpha w - \frac{\alpha^2 T}{2} - \frac{(2m-w)^2}{2T}} dw. \qquad (88)
\end{aligned}
$$

Next, we follow the approach of Shreve (2004) and complete the squares:

$$\alpha w - \frac{\alpha^2 T}{2} - \frac{(2m-w)^2}{2T} = -\frac{1}{2T}(w - 2m - \alpha T)^2 + 2\alpha m, \tag{89}$$

$$\alpha w - \frac{\alpha^2 T}{2} - \frac{w^2}{2T} = -\frac{1}{2T}(w - \alpha T)^2, \tag{90}$$

Thus, we can transform Equation (88) to:

$$\mathbb{Q}\{\hat{M}_T \geq m\} = \frac{1}{\sqrt{2\pi T}} \left(\int_m^\infty e^{-\frac{1}{2T}(w-\alpha T)^2} dw - e^{2\alpha m} \int_m^\infty e^{-\frac{1}{2T}(w-2m-\alpha T)^2} dw \right) \tag{91}$$

By substituting $y_1 = \frac{w-\alpha T}{\sqrt{T}}$ in the first integral and $y_2 = \frac{w-2m-\alpha T}{\sqrt{T}}$ in the second integral, we obtain:

$$\begin{aligned}
\mathbb{Q}\{\hat{M}_T \geq m\} &= \frac{1}{\sqrt{2\pi T}} \left(\int_{\frac{m-\alpha T}{\sqrt{T}}}^\infty e^{-\frac{1}{2}y_1^2} dy_1 - e^{2\alpha m} \int_{\frac{-m-\alpha T}{\sqrt{T}}}^\infty e^{-\frac{1}{2}y_2^2} dy_2 \right) \\
&= N\left(-\frac{m-\alpha T}{\sqrt{T}}\right) - e^{2\alpha m} N\left(\frac{m+\alpha T}{\sqrt{T}}\right).
\end{aligned} \tag{92}$$

Equation (92) represents the going concern probability. The default probability is simply the probability of the counter event, hence we arrive at:

$$\begin{aligned}
\mathbb{Q}\{\hat{M}_T < m\} &= 1 - \left[N\left(-\frac{m-\alpha T}{\sqrt{T}}\right) - e^{2\alpha m} N\left(\frac{m+\alpha T}{\sqrt{T}}\right) \right] \\
&= N\left(\frac{m-\alpha T}{\sqrt{T}}\right) + e^{2\alpha m} N\left(\frac{m+\alpha T}{\sqrt{T}}\right).
\end{aligned} \tag{93}$$

Equation (93) is equal to Equation (51) from Section 3.

B. Appendix: Transformation of value conditions

In Section 2 of this paper, we define conditions concerning the payoffs in our model. During the derivation of our explicit integral-form solutions in Section 4, we present the first three of these conditions rearranged for the standard normally distributed random variable x_t, thereby allowing us to restrict the limits of the integrals (Equations (67) to (69)). Subsequently, we show the transformation in detail and examine the three deterministic conditions for liquidation value of equity and debt.

B.1. Going concern and default condition

We begin with the going concern condition, which is fulfilled if the after-tax cash flow X_t does not cross the default boundary $db_t^{(\cdot)}$. Rearranging for the standard normally distributed random variable x_t is independent of the underlying debt redemption case. Thus, we have:

$$\begin{aligned} X_t &\geq db_t^{(\cdot)} \\ X_0 e^{\alpha\sigma t + \sigma x_t} &\geq db_t^{(\cdot)} \\ x_t &\geq \frac{1}{\sigma} \ln\left(\frac{db_t^{(\cdot)}}{X_0}\right) - \alpha t = m_t^{(\cdot)}. \end{aligned} \tag{94}$$

Note that the default condition is equivalent with opposite sign ("<").

B.2. Non-negative condition of debt

The non-negative condition of debt is relevant for cash sweep debt redemption. It prevents redemption payments from exceeding the current level of debt. The transformed condition stated in Equation (69) is derived as follows:

$$\begin{aligned} D_t &\geq 0 \\ D_{t-1} - \gamma\left(X_t - D_{t-1} y_D (1-\tau_c)\right) &\geq 0 \\ X_t &\leq \frac{D_{t-1}\left(1 + \gamma y_D (1-\tau_c)\right)}{\gamma} \\ X_0 e^{\alpha\sigma t + \sigma x_t} &\leq \frac{D_{t-1}\left(1 + \gamma y_D (1-\tau_c)\right)}{\gamma} \\ x_t &\leq \frac{1}{\sigma} \ln\left(\frac{D_{t-1}\left(1 + \gamma y_D (1-\tau_c)\right)}{\gamma X_0}\right) - \alpha t = n_t^{sweep}. \end{aligned} \tag{95}$$

B.3. Condition for liquidation value of equity

The condition for the liquidation value of equity is deterministic in the default period itself but path-dependent in the periods before. This is because the cash flow X_t equals the default boundary db_t in the event of default. The default boundary is determined at the beginning of the period, and thus, the value of the cash flow is known up front in the event that a default is triggered. However, we show how to rearrange for db_t.

$$X_t + XC_t + (1-\rho)V_t^U - D_{t-1} - D_{t-1}y_D(1-\tau_c) > 0$$
$$X_t\left(1 + (1-\rho)\frac{e^\mu}{e^r - e^\mu}\right) > D_{t-1} + D_{t-1}y_D(1-\tau_c) - XC_t$$

Substituting $X_t = db_t^{(\cdot)}$ yields:

$$db_t^{(\cdot)} > \frac{D_{t-1} + D_{t-1}y_D(1-\tau_c) - XC_t}{1 + (1-\rho)\frac{e^\mu}{e^r - e^\mu}}. \tag{96}$$

B.4. Condition for liquidation value of debt

The condition for the liquidation value of debt is also deterministic in the default period itself but path-dependent in the periods before. This is because the cash flow X_t equals the default boundary db_t in the event of default. The default boundary is determined at the beginning of the period, and thus, the value of the cash flow is known up front in the event that a default is triggered. However, we show how to rearrange condition 1 for db_t.

$$(1-\rho)V_t^U > D_{t-1}y_D + D_{t-1}$$
$$X_t\frac{e^\mu}{e^r - e^\mu} > \frac{D_{t-1}(1+y_D)}{1-\rho}$$

Substituting $X_t = db_t^{(\cdot)}$ yields:

$$db_t^{(\cdot)} > \frac{D_{t-1}(1+y_D)}{(1-\rho)\frac{e^\mu}{e^r - e^\mu}}. \tag{97}$$

Note that condition 2 for the liquidation value of debt is equivalent with opposite sign ("$\leq$").

C. Appendix: Explicit analytic-form solution of value components for $\theta = 1$ and $\gamma = 0$

While our approach works for any LBO financing structure, delivering solution formulae in explicit integral form, there exists a special case that collapses to an explicit analytic form. This case requires three special assumptions: (1) no excess cash at the beginning of the holding period ($XC_0 = 0$), (2) no debt redemption over the holding period ($\gamma = 0$), and (3) full payout of free equity cash flows over the holding period ($\theta = 1$).

Under these prerequisites, the default boundary is constant over the course of the LBO, and there exists no difference between redemption cases ($db = db_t^{sweep} = db_t^{fixed}$). The default barrier db is defined as the maximum of covenant and cash obligation, which can be obtained deterministically from the outset:

$$db = \max\left(D_0 y_D(1-\tau_c), \frac{D_0}{\beta}(1-\tau_c)\right). \tag{98}$$

The formula for the cumulative default probability is equivalent to Equation (51):

$$cd_{t,\mathbb{Q}} = \mathbb{Q}\left\{\hat{M}_t < m\right\} = N\left(\frac{m-\alpha t}{\sqrt{t}}\right) + e^{2\alpha m} N\left(\frac{m+\alpha t}{\sqrt{t}}\right), \tag{99}$$

with

$$\alpha = \frac{1}{\sigma}\left(\mu - \frac{\sigma^2}{2}\right), \tag{100}$$

$$m = \frac{1}{\sigma} ln\left(\frac{db}{X_0}\right). \tag{101}$$

Based on Equation (99), we obtain the incremental default probability of a single period as:

$$pd_{t,\mathbb{Q}} = cd_{t,\mathbb{Q}} - cd_{t-1,\mathbb{Q}}. \tag{102}$$

The present values for the equity cash flows over the course of the holding period are equivalent to barrier down-and-out call options with rebate values. The formula for such an option translated to our problem is as follows:

$$\begin{aligned} PV_t =& C(X_t, db) - \left(\frac{db}{X_0}\right)^{2\frac{\alpha}{\sigma}} C(\frac{db^2}{X_t}, db) \\ &+ (db - co)e^{-rt}\left(N(d_2(X_t, db)) - \left(\frac{db}{X_0}\right)^{2\frac{\alpha}{\sigma}} N(d_2(\frac{db^2}{X_t}, db))\right) \\ &+ pd_t(1-\rho)db\frac{e^{\mu}}{e^r - e^{\mu}}, \end{aligned} \tag{103}$$

with

$$C(X_t, db) = X_0 e^{(\mu - r_A)t} N(d_1(X_t, db)) - db e^{-rt} N(d_2(X_t, db)), \tag{104}$$

$$C(\frac{db^2}{X_t}, db) = \frac{db^2}{X_t} e^{(\mu - r_A)t} N(d_1(\frac{db^2}{X_t}, db)) - db e^{-rt} N(d_2(\frac{db^2}{X_t}, db)), \tag{105}$$

$$d_1(X_t, db) = \frac{\ln\left(\frac{X_0}{db}\right) + (\mu + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}, \tag{106}$$

$$d_2(X_t, db) = d_1(X_t, db) - \sigma\sqrt{t}, \tag{107}$$

$$d_1(\frac{db^2}{X_t}, db) = \frac{\ln\left(\frac{db}{X_0}\right) + (\mu + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}, \tag{108}$$

$$d_2(\frac{db^2}{X_t}, db) = d_1(\frac{db^2}{X_t}, db) - \sigma\sqrt{t}. \tag{109}$$

$$\tag{110}$$

A complete derivation can be found in Zhang (1998).

Table 6 proves that the model outputs of analytic explicit-form solution and integral explicit-form solution are equal. Furthermore, it is shown why a model being able to reflect dynamic debt redemption is important. If one would have applied a simplified model with constant default boundary to our base case scenario, the NPV would have been underestimated by 11.3%.

Table 6: Financing parameters and model output. The first two columns of the table describe the model parameters. The results for the special case of $\theta = 1$ and $\gamma = 0$ are depicted in columns three (analytic explicit-form solution) and four (integral explicit-form solution). Clearly, both solution approaches yield the same results. Additionally, we present the cash sweep base case with $\gamma = 0.6$ in column five to illustrate the impact of flexible redemption mappable by our model.

Variable	Description	Expl. Anal. Form	Expl. Int. Form	Expl. Int. Form
D_0	start debt level	650	650	650
γ	redemption ratio	0	0	0.6
β	debt-to-EBIT covenant	6.5	6.5	6.5
$\boldsymbol{y_D}$	**promised cost of debt**	**4.18%**	**4.18%**	**3.72%**
$cd_{1,\mathbb{Q}}$	cum. default probability up to t=1	1.75%	1.75%	1.75%
$cd_{2,\mathbb{Q}}$	cum. default probability up to t=2	11.49%	11.49%	7.78%
$\boldsymbol{cd_{3,\mathbb{Q}}}$	**cum. default probability up to t=3**	**22.49%**	**22.49%**	**13.41%**
I_0	initial equity investment	-597.56	-597.56	-597.56
PV_1^{gc}	PV of dividend in t=1	78.56	78.56	32.12
PV_2^{gc}	PV of dividend in t=2	70.47	70.47	29.89
PV_3^{gc}	PV of dividend in t=3	62.44	62.44	27.92
PV_1^{def}	PV of potential default in t=1	0.00	0.00	0.00
PV_2^{def}	PV of potential default in t=2	0.00	0.00	0.00
PV_3^{def}	PV of potential default in t=3	0.00	0.00	0.00
PV_{Exit}	PV of LBO exit	598.13	598.13	746.57
$\boldsymbol{NPV}$	**net present value**	**212.03**	**212.03**	**238.94**
$\boldsymbol{IRR}$	**internal rate of return**	**15.33%**	**15.33%**	**15.83%**

The impact of a dynamic debt redemption is best illustrated by examining the cumulative default probability over time. We present these cumulative default probabilities for the case of no redemption ($\gamma = 0.0$), full redemption ($\gamma = 1.0$) and our base case ($\gamma = 0.6$) in Figure 6. It is clearly shown how the cumulative default probability is lowered by dynamic debt redemption after the first period due to a decreased default boundary.

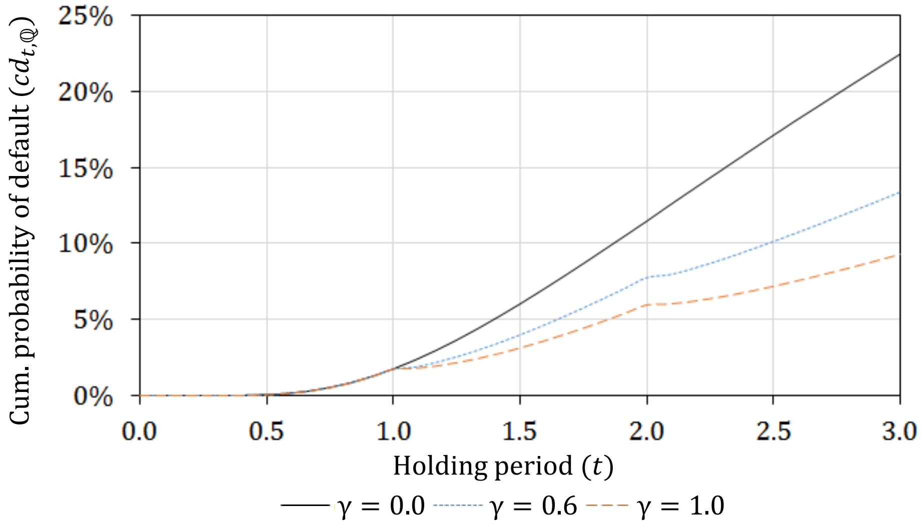

Figure 6: Cumulative default probability (cd_t until point in time T for different cash sweep ratios γ. The figure depicts cd_t over the holding period for $\gamma = 0.0$ (black, solid line), $\gamma = 0.6$ (blue, dotted line), and $\gamma = 1.0$ (orange, dashed line). All other input parameters are as defined in the basic set reported in Table 3.

D. Appendix: Detailed numerical application for fixed debt redemption

D.1. Optimal financing for NPV and IRR maximization

While Section 5.2 of the paper was devoted to optimal financing under cash sweep debt redemption, we execute the same analysis for fixed debt redemption in this section. We use the base case scenario specified in Table 3. As depicted in Figure 7 the results are equivalent to the cash sweep redemption case: The IRR-optimizing debt level ($D_0 = 715$) is significantly higher than the one optimizing the NPV ($D_0 = 254$). Again, we find an increased promised yield, a higher risk of default and an NPV reduction of 9.60%.

By constructing a fixed debt redemption case based on the cash sweep ratio $\gamma = 0.6$ (as described in Section 5.1), we examine a scenario where path dependent switches between the two default triggers (cash obligation vs. covenant) are likely. For debt levels above $D_0 = 713$ the default trigger is set by the cash obligation right from the beginning of the holding period. The observed double-peak in the NPV-curve at $D_0 = 254$ (our global maximum) and at $D_0 = 680$ is a direct consequence of these path-dependent switches.

Please note that if one chooses a lower cash sweep ratio (e.g.,$\gamma = 0.5$), the graphs look similar to the ones under cash sweep debt redemption (see e.g. Figure 4) as the default trigger switches to the debt-to-EBIT covenant condition for all debt levels.

D.2. Comparative statics

We provide a sensitivity analysis for fixed debt redemption equivalent to the comparative statics regarding cash sweep debt redemption (Table 5. To ensure a valid comparison between the redemption cases, we match the fixed redemption payments again to the expected cash sweep redemption payments as described in Equation (79). This matching triggers a specific feature we outline at the beginning: Whenever a change in parameters lead to changes in the expected $EBIT$, we face an adjustment of the fixed redemption payments.

Thus, some effects depicted in Table 7 are slightly different than in Section 5.3. Exemplarily, an increase in $\mu_{\mathbb{P}}$ causes higher expected $EBIT$ figures and, consequently, higher fixed cash obligations. The cumulative risk of default ($cd_{T,\mathbb{Q}}$) increases because the effect of higher cash obligations exceed the effect of the improved $EBIT$ process. Beyond this peculiarity, the results generated by changes in $\mu_{\mathbb{P}}$ are consistent with the cash sweep redemption case.

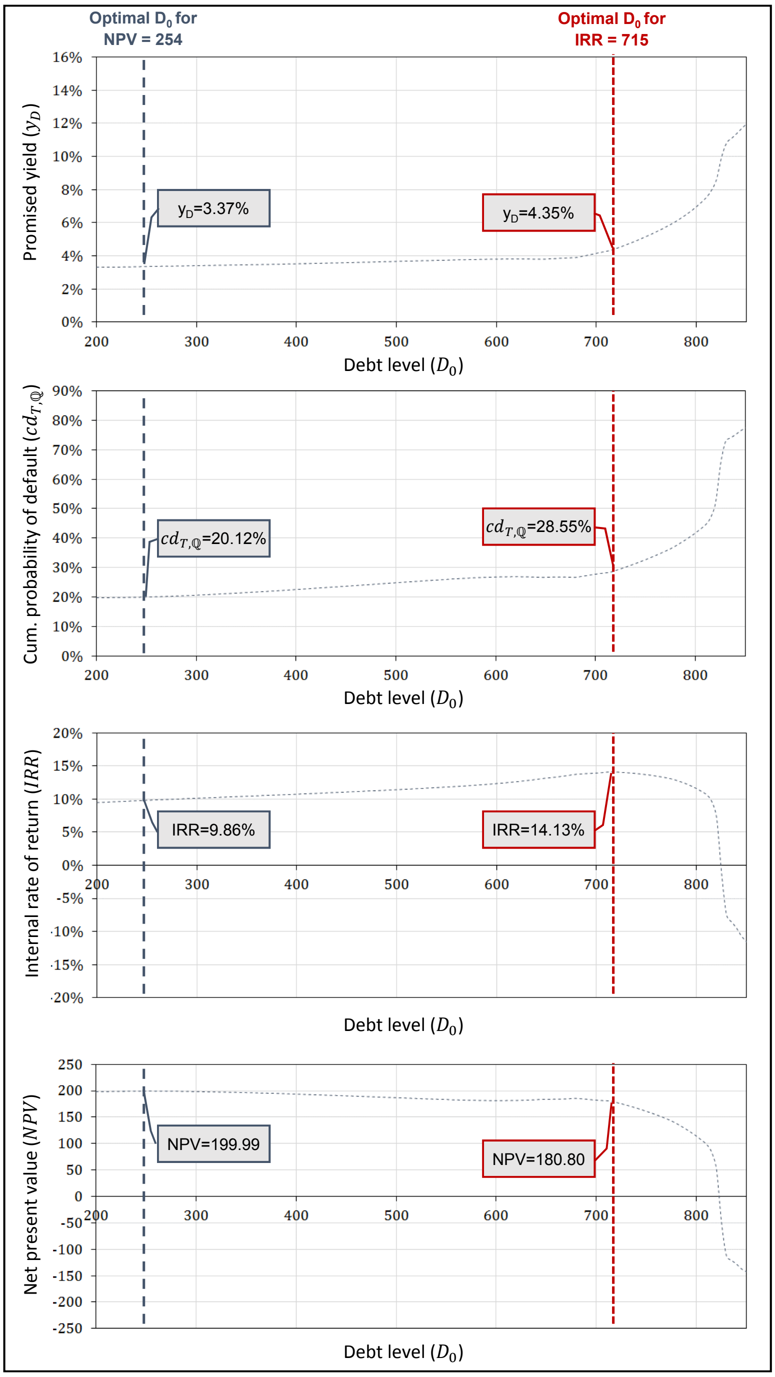

Figure 7: Optimal initial debt level for maximizing NPV or IRR. This figure depicts the promised yield y_D (first graph), the cumulative risk-neutral probability over the full holding period $cd_{T,\mathbb{Q}}$ (second graph), the IRR (third graph), and the NPV (fourth graph), each as a function of the initial debt level over the interval of $D_0 \in [200, 850]$. We use the basic set of parameters reported in Table 3 and the financing parameters for fixed redemption as illustrated in Table 4. The optimal initial debt level D_0 for maximizing the NPV (blue, dashed line) or the IRR (red, small dashed line) are shown in all four graphs.

Table 7: Comparative Statics for fixed debt redemption. This table reports the comparative statics of the model derived in Section 4 for fixed debt redemption with respect to all model parameters. We use the basic set of parameters illustrated in Table 3 and for the financing parameters those listed in Table 4. Output parameters are the promised yield (y_D), the cumulative risk-neutral probability ($cd_{T,\mathbb{Q}}$), the internal rate of return (IRR^{Eq}), the net present value (NPV^{Eq}), the IRR-maximizing debt level (D^*_{IRR}), the corresponding IRR ($IRR^{Eq,*}$), the NPV-maximizing debt level (D^*_{NPV}) and the corresponding NPV ($NPV^{Eq,*}$). Model parameters are defined as follows: μ is the risk-neutral drift rate; σ is the $EBIT$'s volatility; r is the risk-free rate; τ_c is the corporate tax rate; β is the debt-to-EBIT covenant; ρ is the bankruptcy cost ratio; θ is the dividend ratio; and γ is the redemption ratio. The basic parameter values are shown in bold.

Var	**1**	**2**	**3**	**4**	**5**	**Var**	**1**	**2**	**3**	**4**	**5**
$\boldsymbol{\mu_{\mathbb{P}}}$	1.0%	3.0%	**5.0%**	7.0%	9.0%	$\boldsymbol{\sigma}$	10.0%	15.0%	**20.0%**	25.0%	30.0%
y_D	4.04%	3.77%	3.81%	3.92%	4.04%	y_D	3.05%	3.14%	3.81%	9.13%	10.04%
$cd_{T,\mathbb{Q}}$	25.59%	25.26%	26.78%	28.45%	30.11%	$cd_{T,\mathbb{Q}}$	1.24%	10.64%	26.78%	72.81%	77.95%
IRR^{Eq}	5.89%	9.75%	13.19%	16.54%	19.81%	IRR^{Eq}	17.22%	15.98%	13.19%	-4.12%	-6.04%
NPV^{Eq}	48	118	184	252	322	NPV^{Eq}	273	244	184	-101	-124
D^*_{IRR}	625	675	715	750	760	D^*_{IRR}	835	775	715	303	120
$IRR^{Eq,*}$	5.94%	9.89%	14.13%	18.22%	22.37%	$IRR^{Eq,*}$	21.46%	17.97%	14.13%	8.09%	6.61%
D^*_{NPV}	245	249	254	259	263	D^*_{NPV}	725	675	254	121	0
$NPV^{Eq,*}$	55	125	200	278	359	$NPV^{Eq,*}$	275	244	200	159	109
$\boldsymbol{r}$	1.0%	2.0%	**3.0%**	4.0%	5.0%	$\boldsymbol{\tau_c}$	0.0%	20.0%	30.0%	**40.0%**	50.0%
y_D	1.49%	2.57%	3.81%	5.44%	9.98%	y_D	3.82%	3.90%	3.95%	3.81%	6.41%
$cd_{T,\mathbb{Q}}$	19.40%	22.43%	26.78%	34.03%	66.99%	$cd_{T,\mathbb{Q}}$	27.38%	28.34%	28.93%	26.78%	42.71%
IRR^{Eq}	12.09%	12.83%	13.19%	12.58%	0.00%	IRR^{Eq}	10.18%	10.84%	11.44%	13.19%	10.56%
NPV^{Eq}	207	200	184	150	-73	NPV^{Eq}	282	225	197	184	91
D^*_{IRR}	675	700	715	700	450	D^*_{IRR}	715	725	750	715	825
$IRR^{Eq,*}$	12.39%	13.43%	14.13%	13.29%	12.52%	$IRR^{Eq,*}$	10.38%	11.36%	12.34%	14.13%	21.48%
D^*_{NPV}	285	270	254	241	225	D^*_{NPV}	0	318	298	254	213
$NPV^{Eq,*}$	206	203	200	197	193	$NPV^{Eq,*}$	430	262	231	200	168
$\boldsymbol{\beta}$	5.5	6	**6.5**	7	7.5	$\boldsymbol{\rho}$	15.0%	20.0%	**25.0%**	30.0%	35.0%
y_D	4.14%	3.83%	3.81%	3.84%	3.89%	y_D	4.00%	3.96%	3.81%	4.06%	4.88%
$cd_{T,\mathbb{Q}}$	30.30%	27.06%	26.78%	27.19%	28.00%	$cd_{T,\mathbb{Q}}$	29.51%	28.89%	26.78%	27.23%	31.69%
IRR^{Eq}	12.20%	13.19%	13.19%	13.03%	12.77%	IRR^{Eq}	13.73%	13.25%	13.19%	12.61%	10.81%
NPV^{Eq}	162	184	184	182	177	NPV^{Eq}	193	185	184	173	139
D^*_{IRR}	627	674	715	698	671	D^*_{IRR}	782	750	715	675	600
$IRR^{Eq,*}$	12.74%	13.38%	14.13%	13.57%	13.14%	$IRR^{Eq,*}$	15.58%	14.87%	14.13%	12.65%	11.50%
D^*_{NPV}	265	260	254	240	212	D^*_{NPV}	270	266	254	239	212
$NPV^{Eq,*}$	201	200	200	200	200	$NPV^{Eq,*}$	214	207	200	193	187
$\boldsymbol{\theta}$	0.0%	40.0%	60.0%	80.0%	**100.0%**	$\boldsymbol{\gamma}$	0.0%	20.0%	40.0%	50.0%	**60.0%**
y_D	3.83%	3.82%	3.82%	3.81%	3.81%	y_D	4.18%	3.87%	3.63%	3.54%	3.81%
$cd_{T,\mathbb{Q}}$	27.05%	26.88%	26.82%	26.76%	26.78%	$cd_{T,\mathbb{Q}}$	22.49%	18.29%	14.52%	13.15%	26.78%
IRR^{Eq}	12.21%	12.54%	12.75%	12.97%	13.19%	IRR^{Eq}	15.33%	15.78%	16.07%	16.12%	13.19%
NPV^{Eq}	174	177	179	182	184	NPV^{Eq}	212	226	237	241	184
D^*_{IRR}	712	714	714	715	715	D^*_{IRR}	597	620	642	655	715
$IRR^{Eq,*}$	12.87%	13.34%	13.58%	13.83%	14.13%	$IRR^{Eq,*}$	15.62%	15.87%	16.08%	16.16%	14.13%
D^*_{NPV}	296	286	275	264	254	D^*_{NPV}	395	440	525	588	254
$NPV^{Eq,*}$	195	196	197	199	200	$NPV^{Eq,*}$	268	268	268	247	200

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