Adiabatic Preparation of Spin-Spin-Hamiltonians for Long-Distance Entanglement

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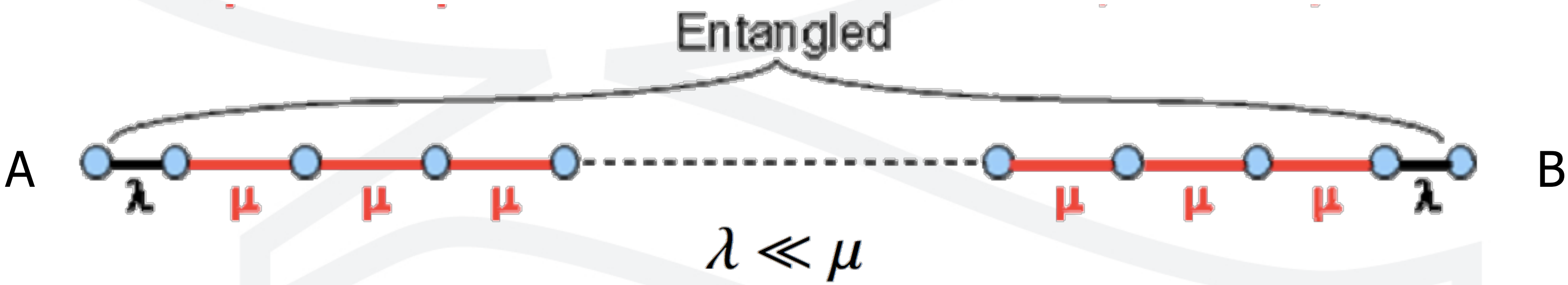
iQSim, Brighton, 16 December 2013

cost
European Science Foundation
Bundesministerium für Bildung und Forschung
Deutsche Forschungsgemeinschaft DFG
PICC
iQIT
Universität Siegen

Long distance entanglement (LDE)

- Ground-state, indirect, end-to-end entanglement in spin-chains, useful for quantum bus
- Weakly coupled end/messenger ions



L. Campos Venuti et al., Phys. Rev. Lett., 96, 247206 (2006);
L. Campos Venuti et al., Phys. Rev. A 76, 052328 (2007);
S. M. Giampaolo et al., New J. of Phys. 12, 025019 (2010).

LDE

- Examples: models with competing interactions along orthogonal axes as XY, XYZ, Heisenberg...

$$H_{XY}=\sum_{j,k}\left(J_x\sigma_j^x\sigma_k^x+J_y\sigma_j^y\sigma_k^y\right)$$

$$H_{XYZ}=\sum_{j,k}\left(J_x\sigma_j^x\sigma_k^x+J_y\sigma_j^y\sigma_k^y+J_z\sigma_j^z\sigma_k^z\right)$$

$$H_{Heisenberg}=J\sum_{j,k}\left(\sigma_j^x\sigma_k^x+\sigma_j^y\sigma_k^y+\sigma_j^z\sigma_k^z\right)$$

LDE Road Map

1. Spin-spin interaction: Magnetic gradient induced coupling (MAGIC)
   - Experimental implementation and full characterization
2. Tailoring spin-spin couplings:
   - Weakly coupled end-spins

$$\sum_{i<j}^{N} \sigma_{z,i}\sigma_{z,j} J_{ij}$$

3. Engineering of Hamiltonian:
   - Interaction along z and along another axis

$$\sum_{i<j}^{N} \sigma_i^z \sigma_j^z J_{ij}^z + \sigma_i^x \sigma_j^x J_{ij}^x$$

4. Preparation of the ground state:
   - Adiabatic variation of Hamiltonian

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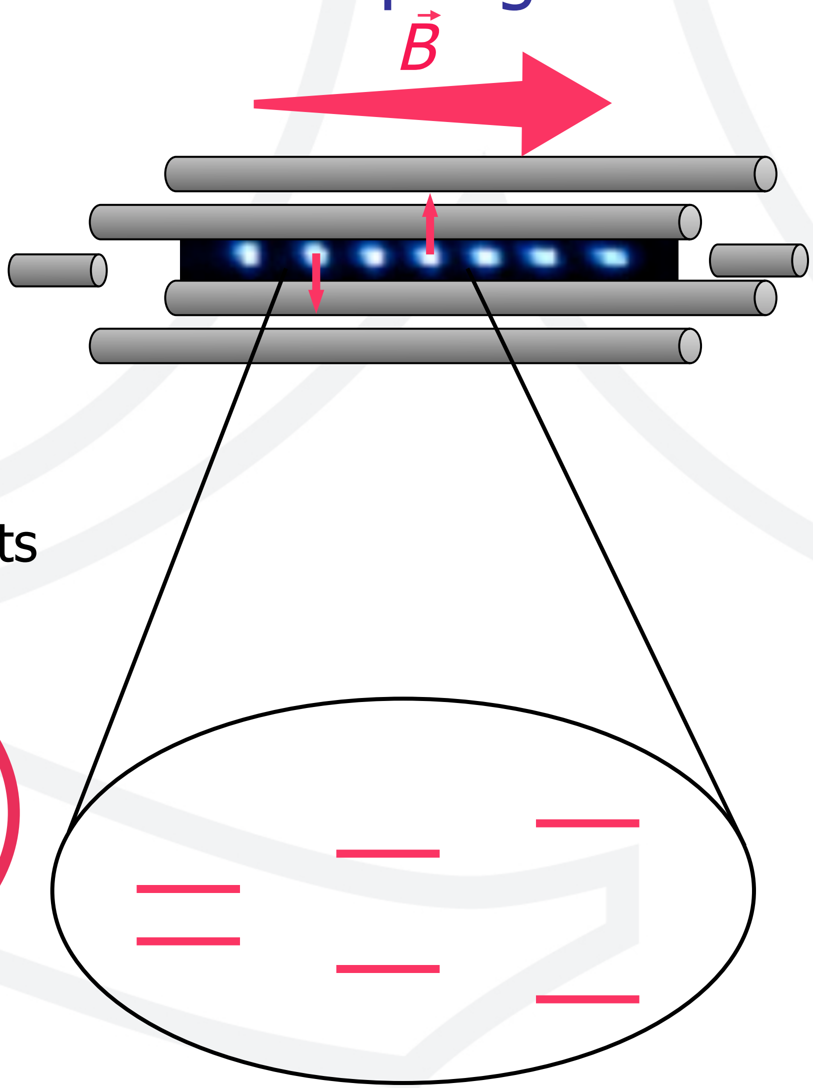
Magnetic Gradient Induced Coupling: MAGIC

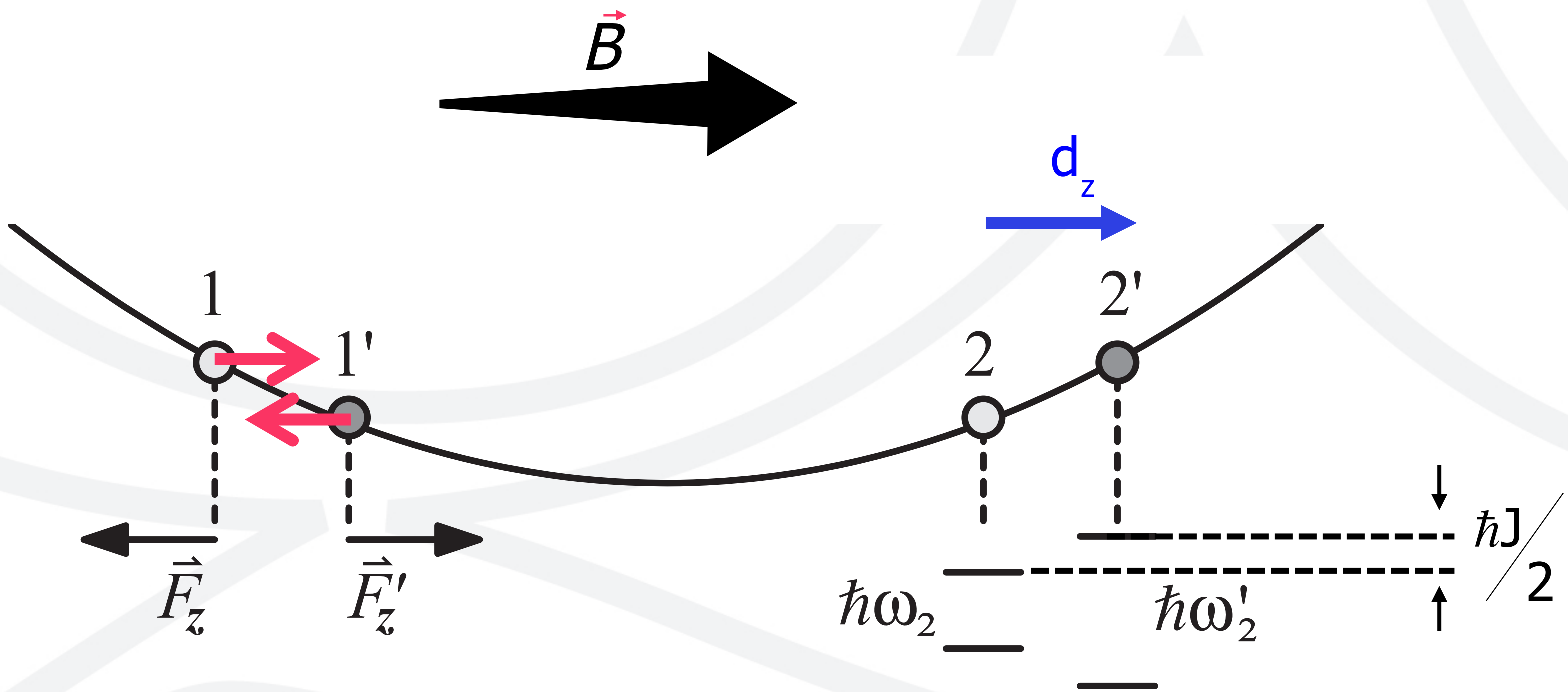


1. Qubit resonances shifted individually
2. Spin-Spin coupling between individual qubits

$$-\frac{\hbar}{2}\sum_{i<j}^{N}\sigma_{z,i}\sigma_{z,j}J_{ij}$$

F. Mintert, CW, PRL (2001)
CW in *Laser Physics at the Limit*, Springer, 2002. quant-ph/0111158.
CW, Ch. Balzer, Adv.At.Mol.Opt.Phys. (2003). quant-ph/0305129

MAGIC Example: Two Ions

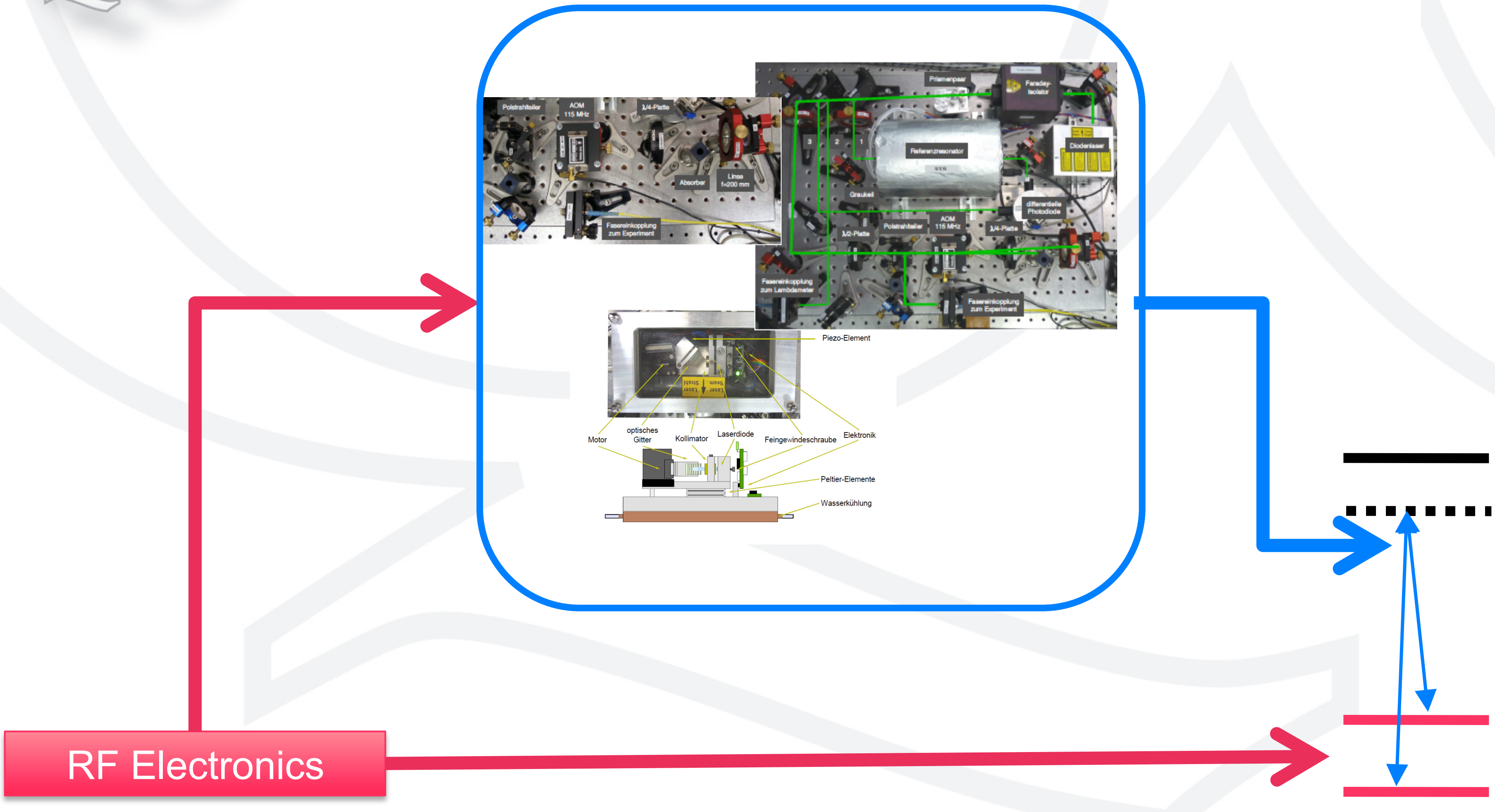


Spin flip $\Rightarrow d_z = \mp F_z / (m\nu^2)$

$$\hbar J_{12} = -F_z d_z = -F_z^2 / (m\nu^2) \propto (b / \nu)^2.$$

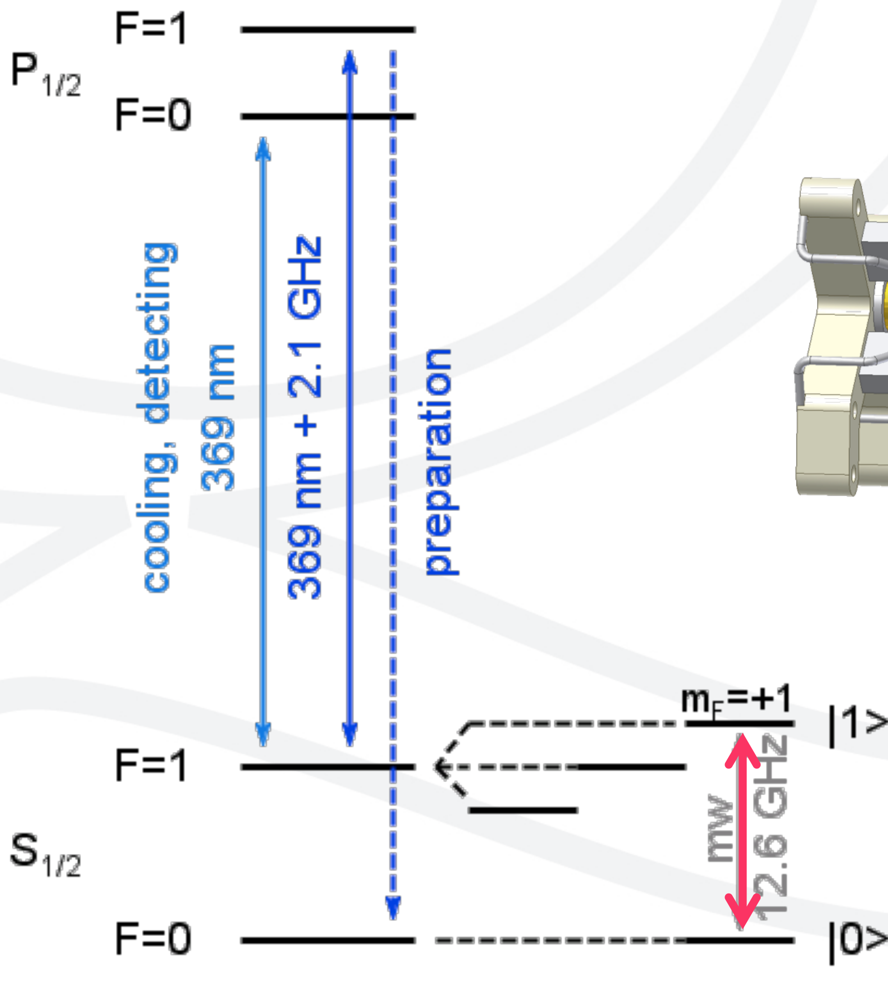
MAGIC

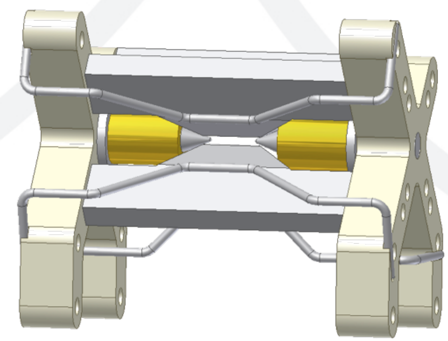
Coupling and Addressing Qubits using **RF-waves**



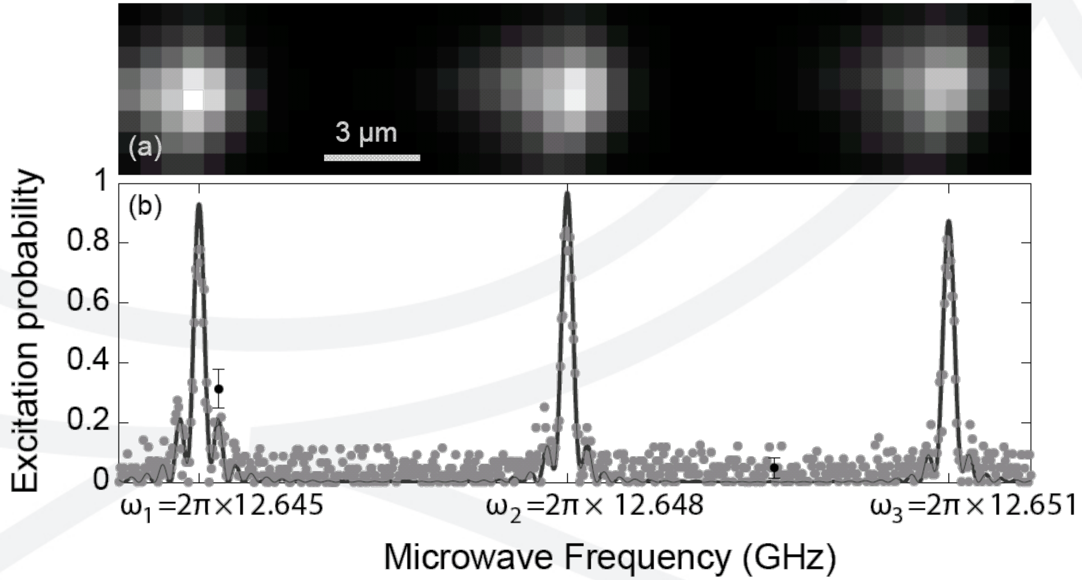
See also Boulder, Hannover, Oxford, Sussex, ...

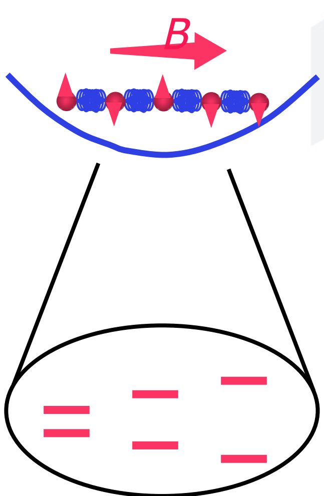
$^{171}Yb^+$


F=1
$P_{1/2}$
F=0
cooling, detecting
369 nm
369 nm + 2.1 GHz
preparation
m_F=+1
|1>
F=1
$S_{1/2}$
mw
12.6 GHz
F=0
|0>



Individual Addressing of Spins

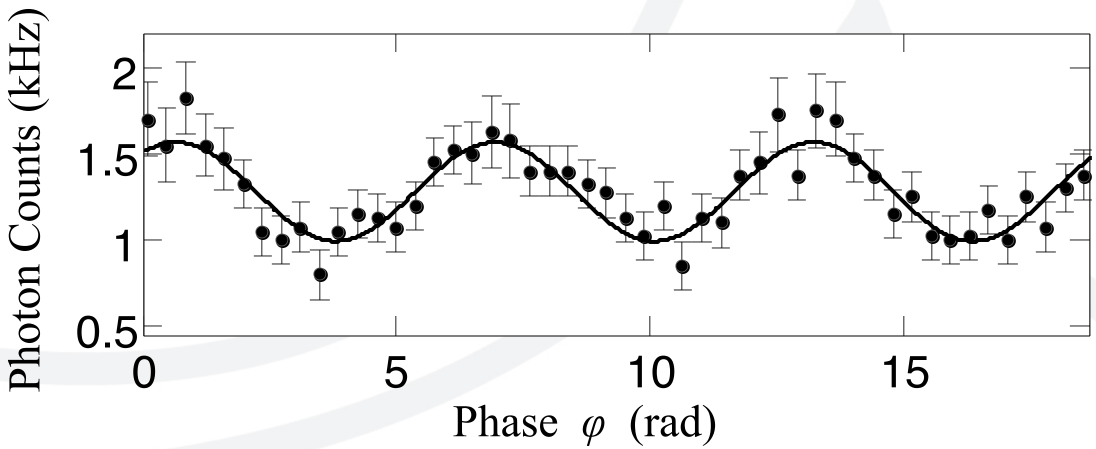


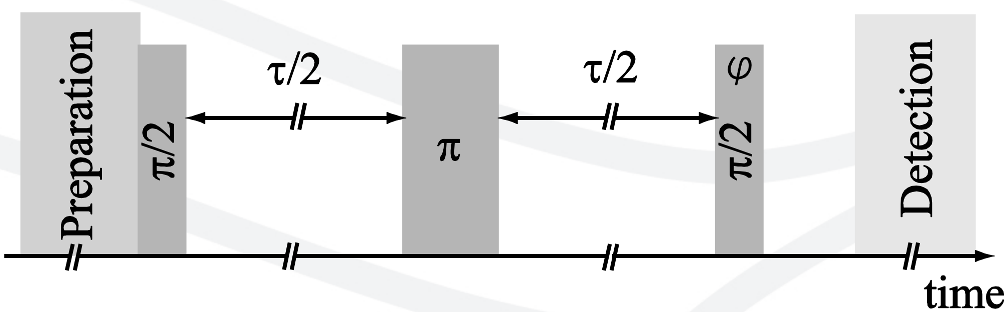


- Microwave-optical double resonance
- Cross-talk (max. spurious exc. prob.) < 4×10^{-4} (meanwhile < 10^{-5})

Phys. Rev. Lett. **108**, 220502 (2012)

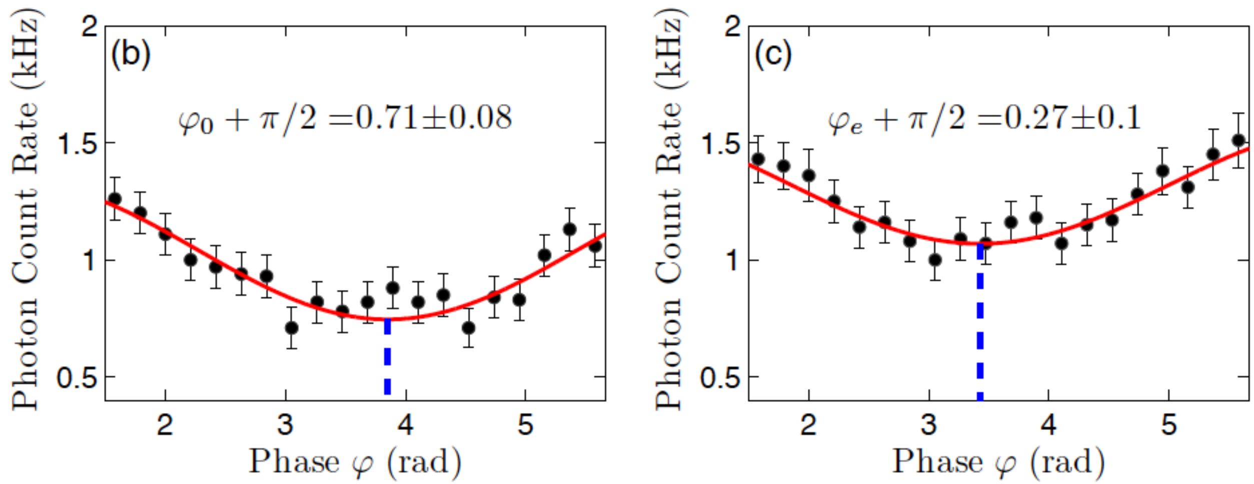
MAGIC - Measuring *J*
Ramsey-type experiment

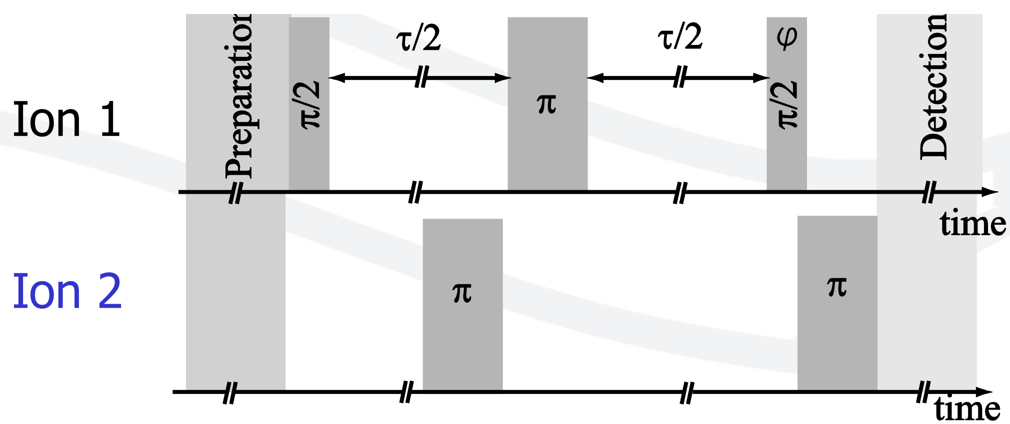




MAGIC - Measuring *J*

$$J_{12} = \frac{\phi_2 - \phi_1}{\tau}$$

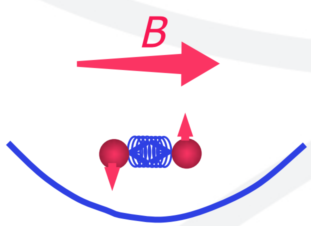


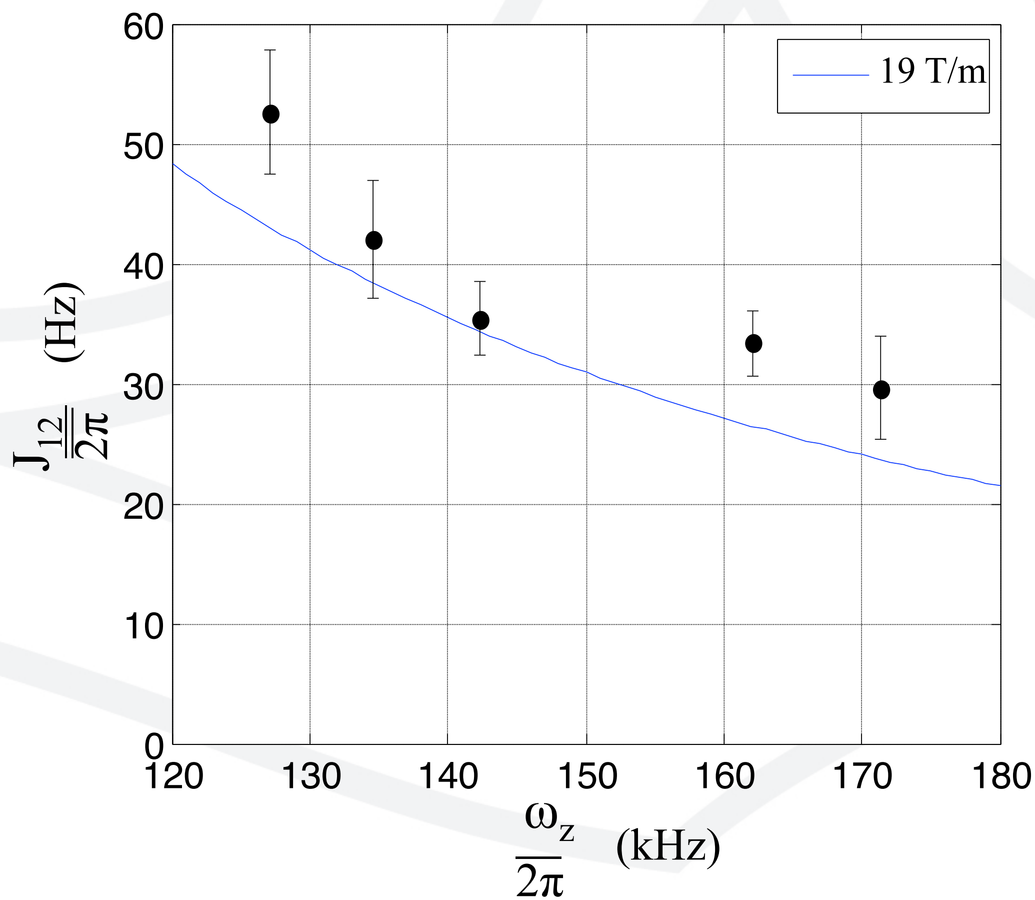


MAGIC - Measuring *J*

Varying the trapping potential

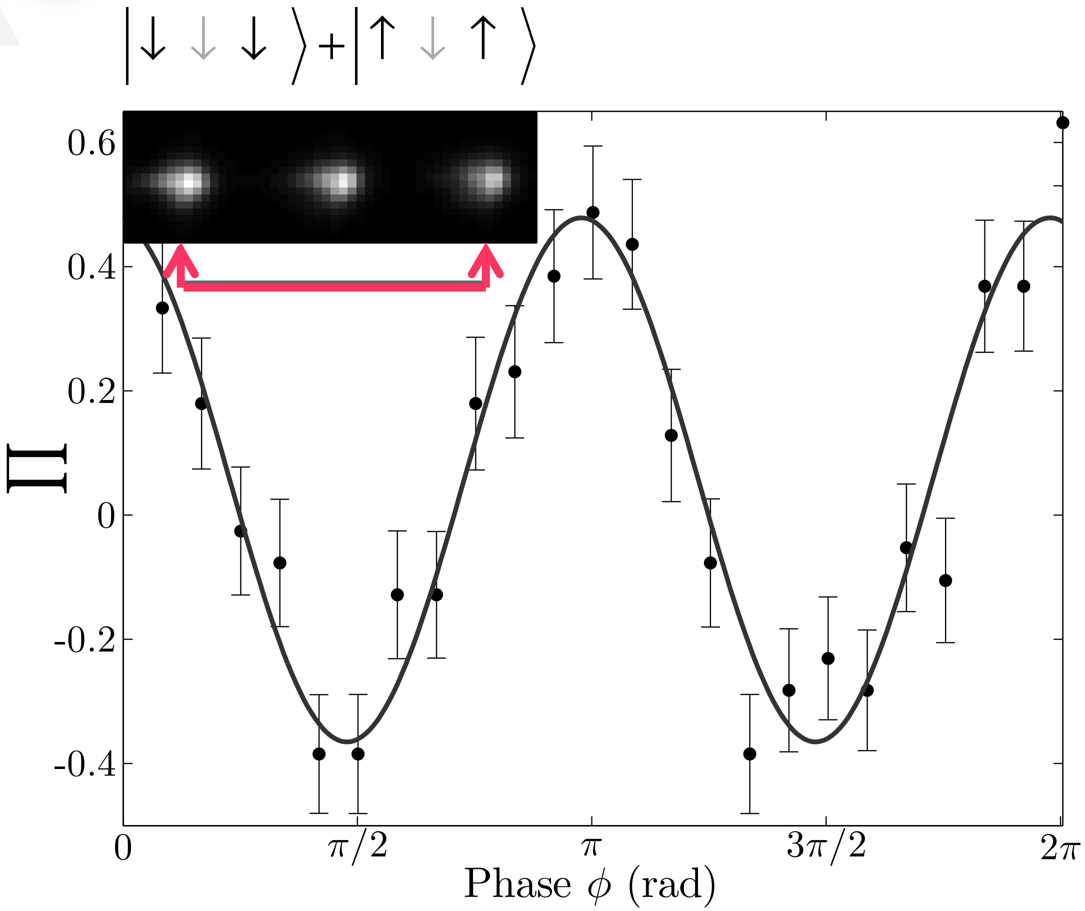
$$J \propto \left(\frac{\partial_z B}{\nu}\right)^2$$





A Khromova et al., PRL. **108**, 220502 (2012)

MAGIC – Entanglement

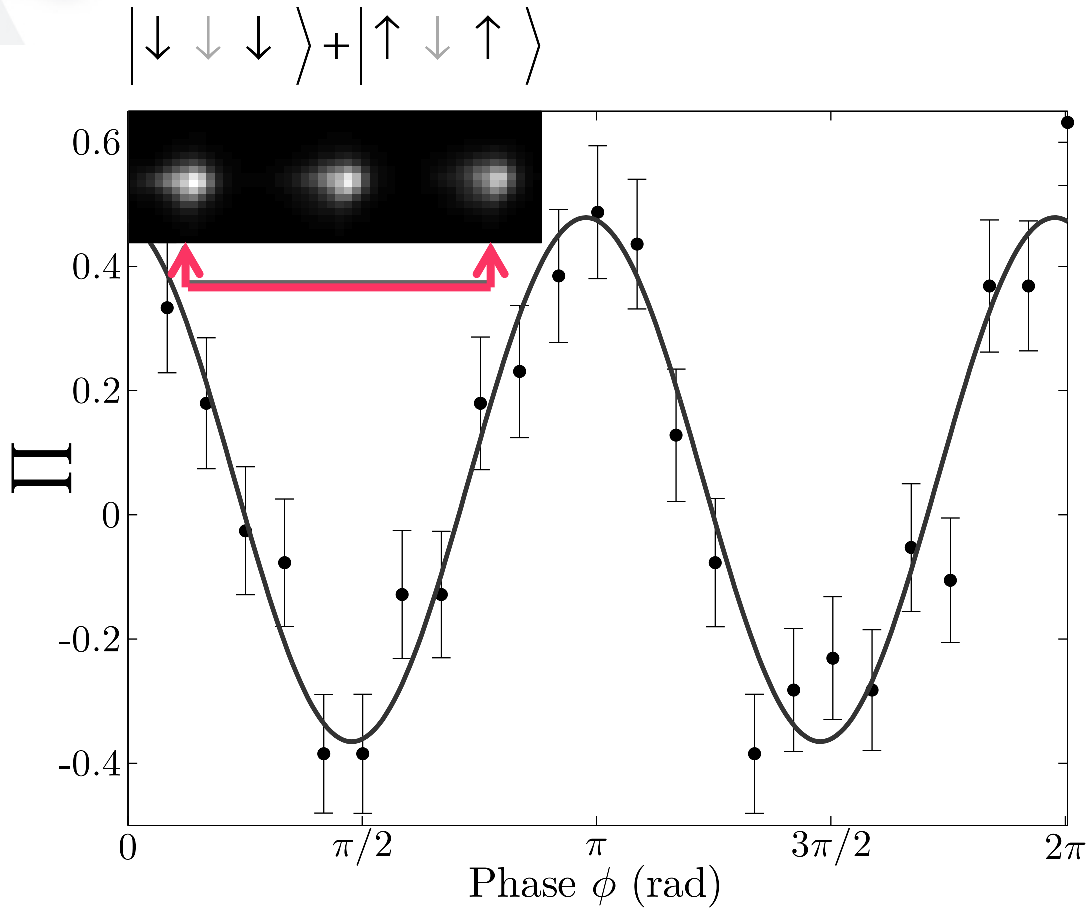


$$\Pi = P_{00} + P_{11} - (P_{10} + P_{01})$$

$$F = \frac{\Pi_{zz} + 1}{4} + \frac{A}{2} > 0.5$$

Phys. Rev. Lett. **108**, 220502 (2012)

MAGIC – Entanglement

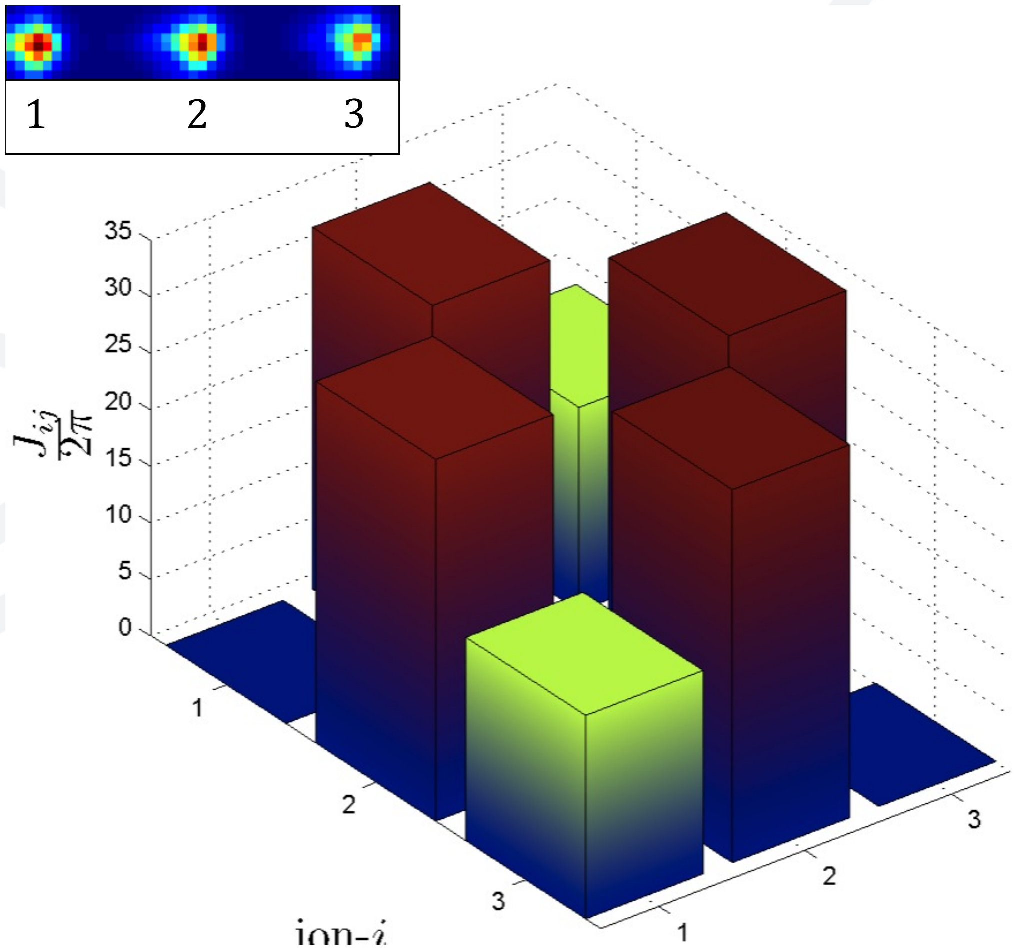


Thermal excitation:

$\langle n_1 \rangle = 23(7)$

Phys. Rev. Lett. **108**, 220502 (2012)

MAGIC: Measured J-type couplings



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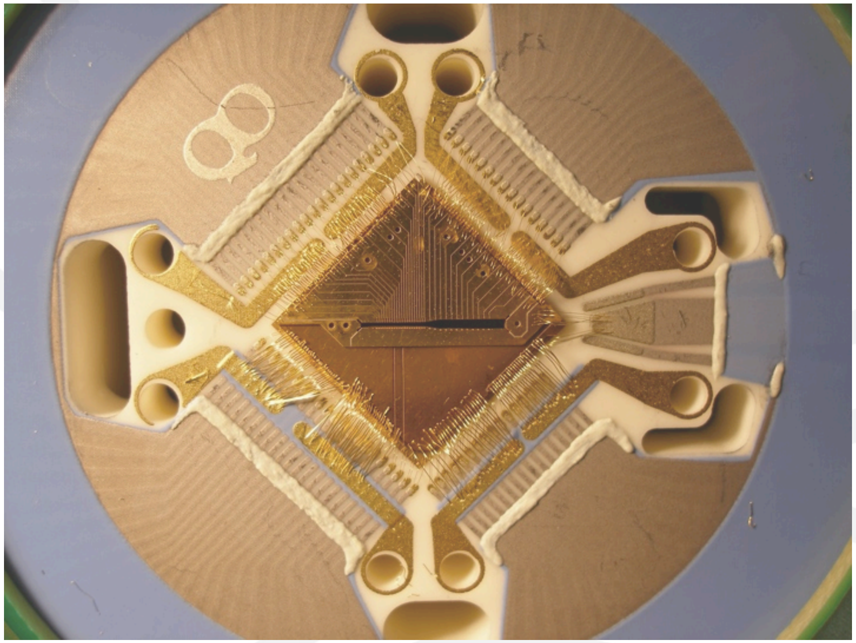
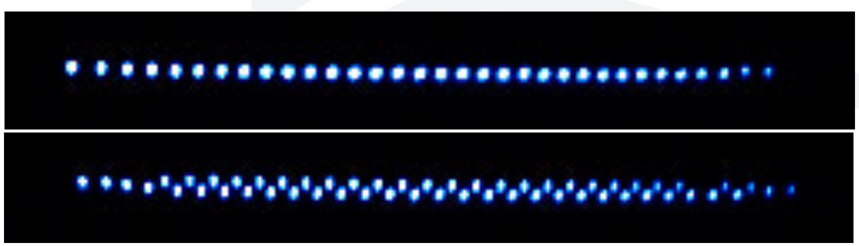
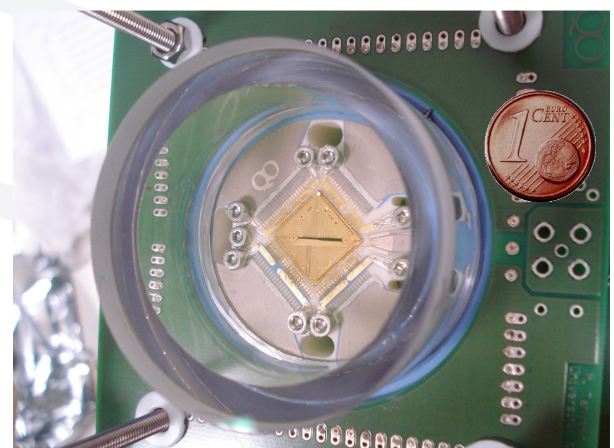
Tailoring Couplings: Segmented µtrap



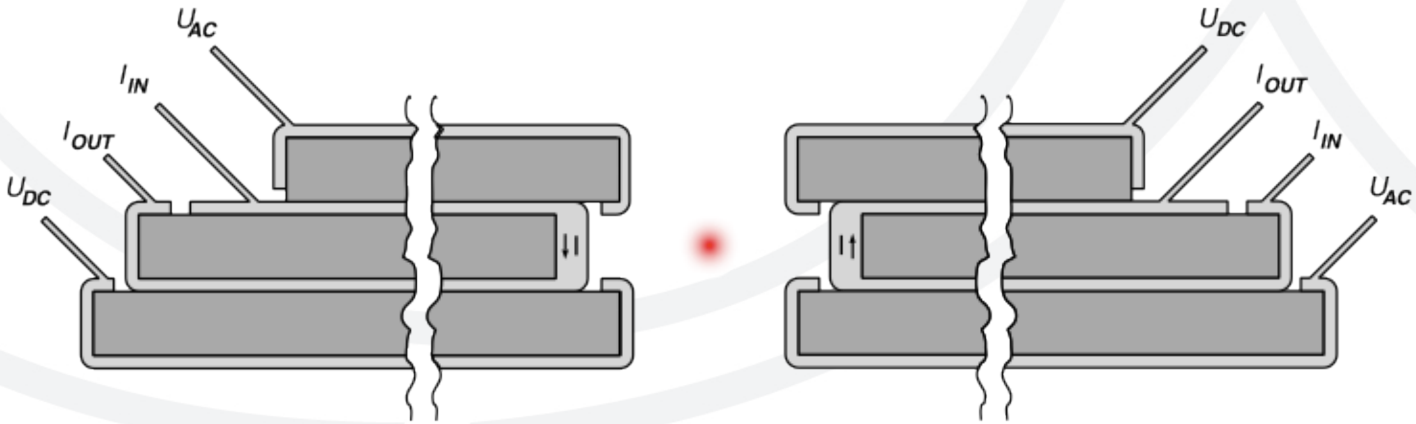


- linear Paul trap,
  three layer structure
  (collab. w/ F. Schmidt-Kaler)
- segments allow flexible axial trapping potentials and multiple trapping zones
- carrier acts as vacuum interface



Appl. Phys. B **107**, 935 (2012)

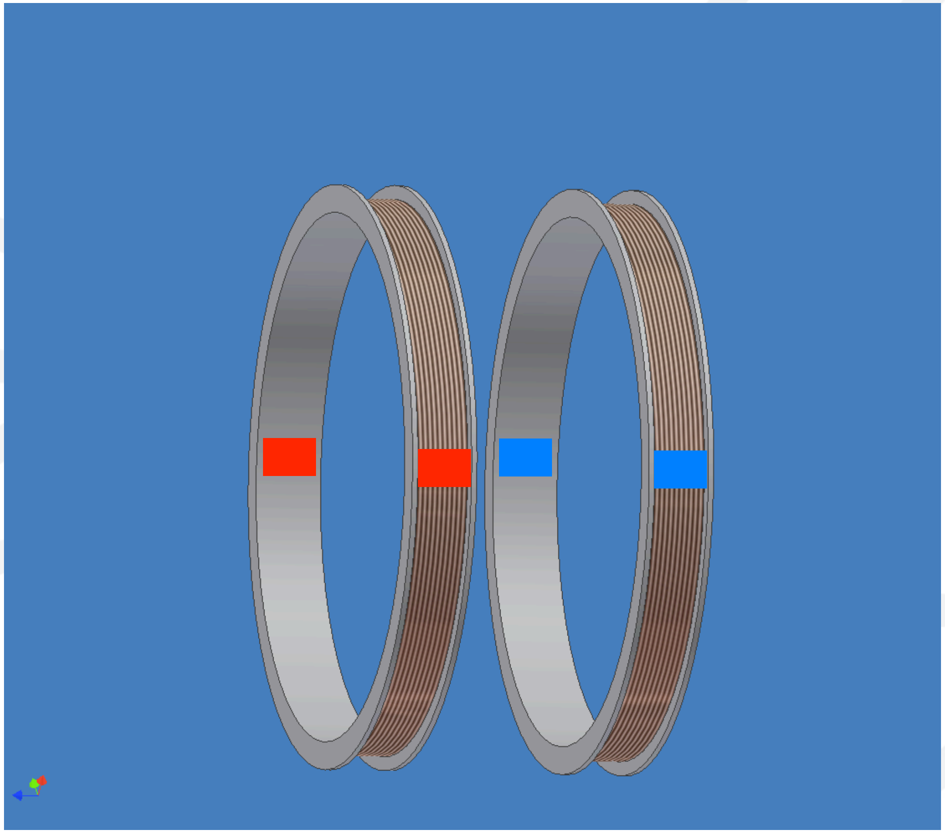
Tailoring Couplings: Segmented µtrap



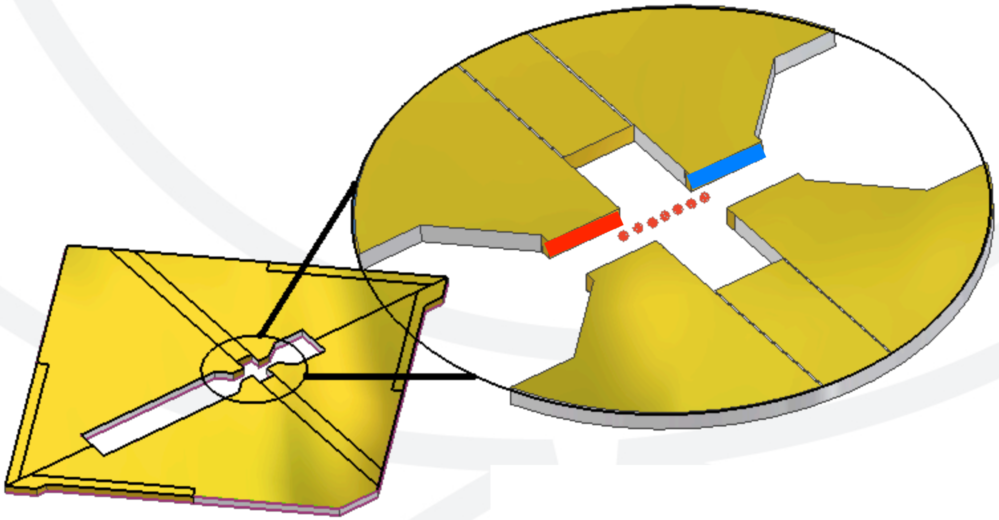
Middle Layer: gradient coil

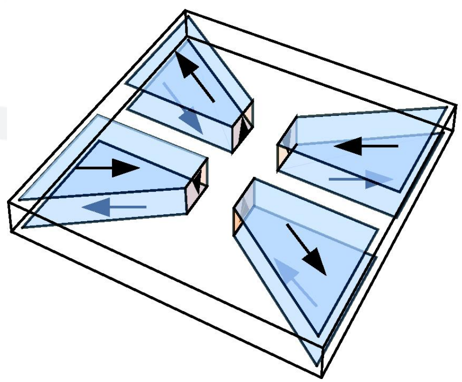
Micro-structured trap

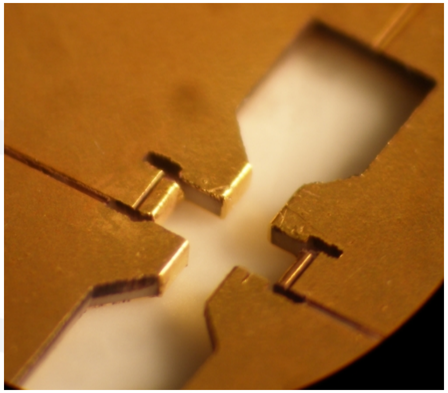
Anti-Helmholtz coils



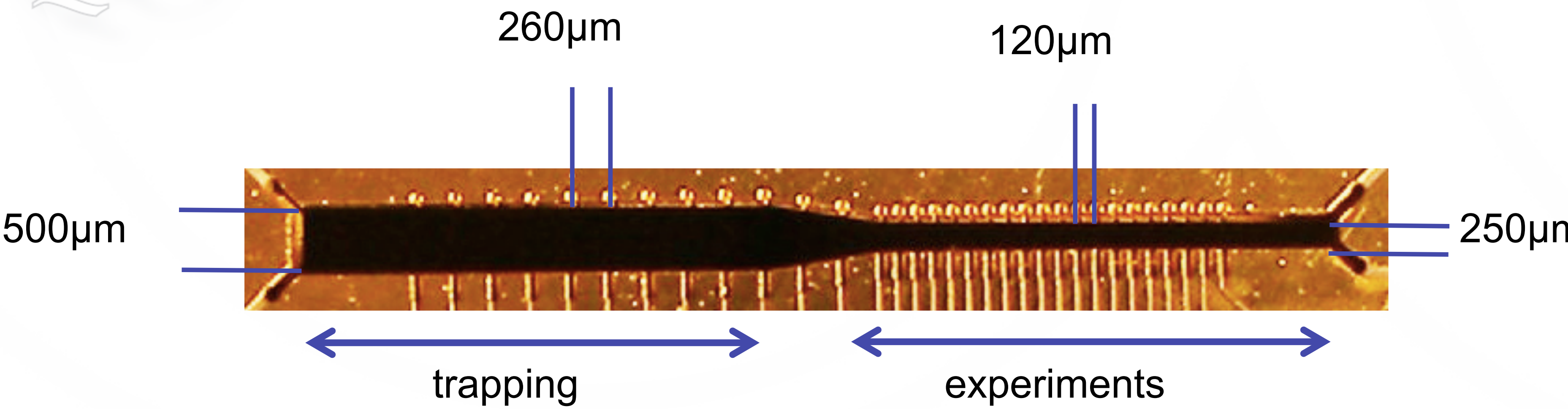
Anti-Helmholtz coils







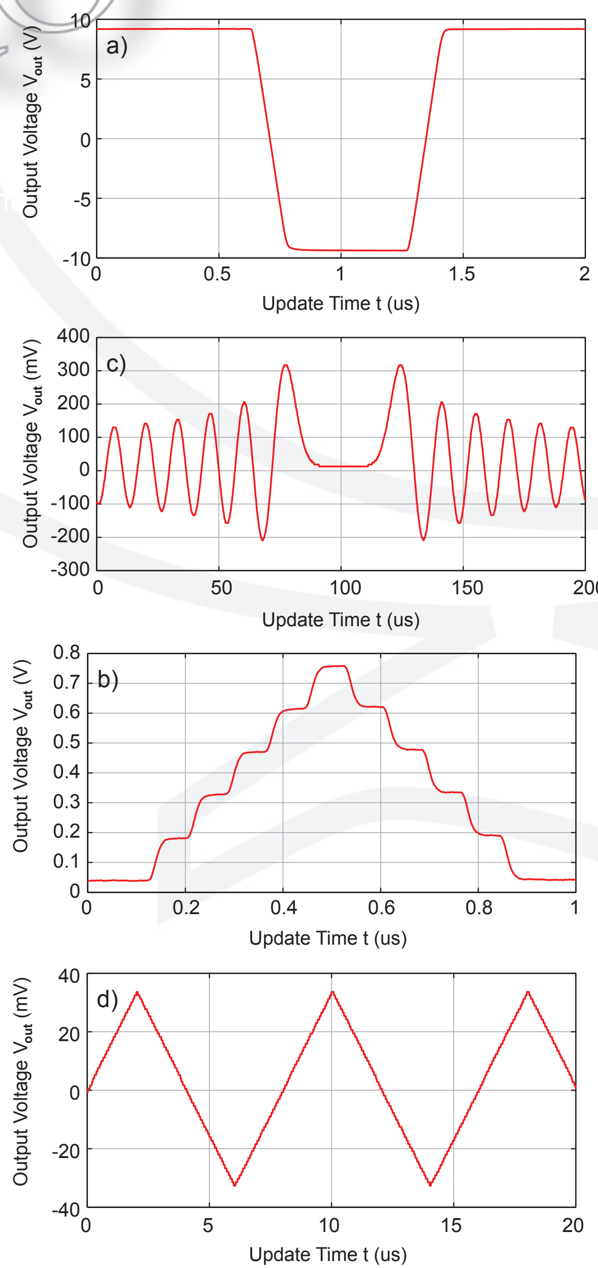
Tailoring Couplings: Segmented μtrap



- Wide trapping zone
- Narrow region for large gradient

Tailoring Couplings: Segmented µtrap

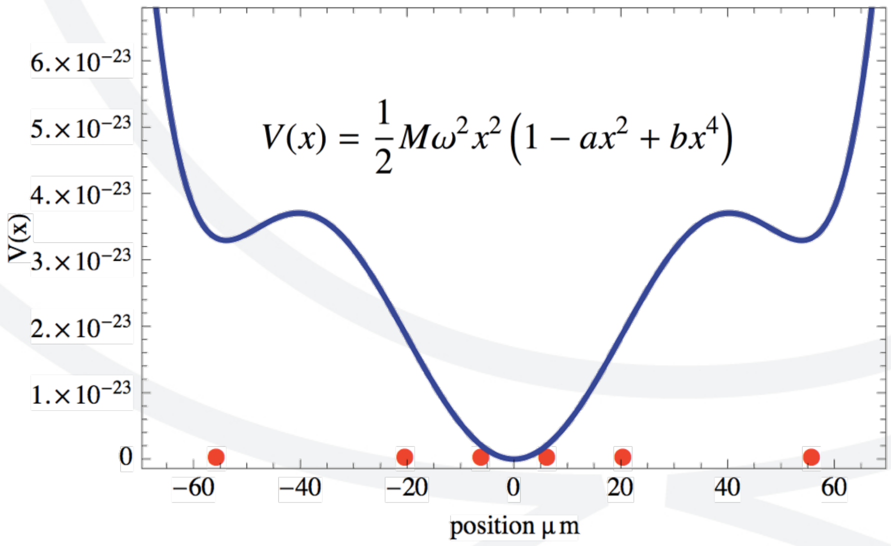
Multichannel-Arbitrary Waveform Generator

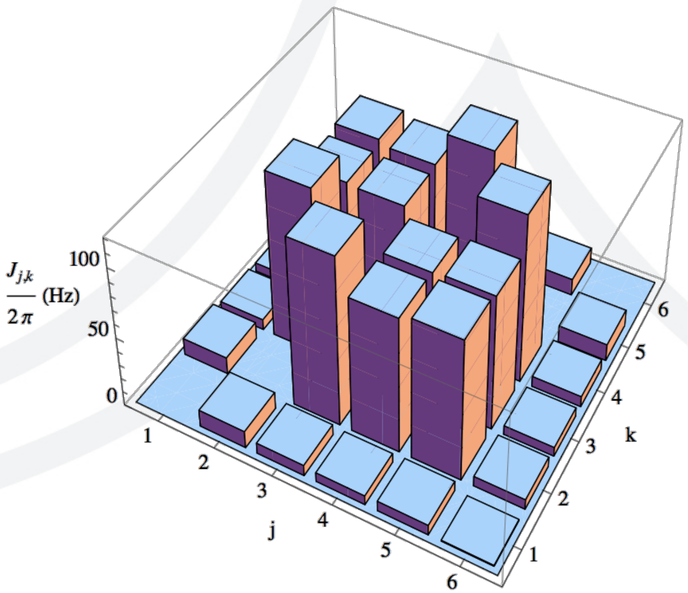


- Presently 24 independent channels
- Synchronous, arbitrary sequences
- Amplitude ±10 V
- Update rate 20 MHz
- 16 bit resolution
- Low noise
- Lossfree transmission up to 2 m
- Freely programmable via USB

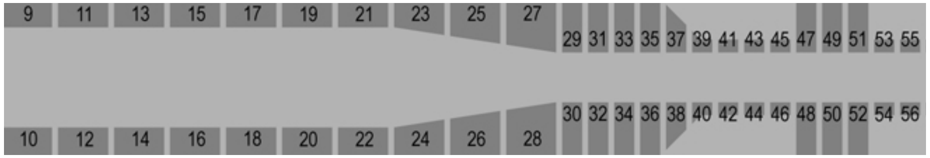
Rev. Sci. Instr. **84** (2013)
Patent DE 10 2011 001 399 A1 (2012).

Tailoring Coupling Constants, Example





- Harmonic trap: couplings of end-spins are the largest, but
- Couplings of end-spins can be controlled by tailoring the trapping potential



S. Zippilli et al., arXiv:1304.0261

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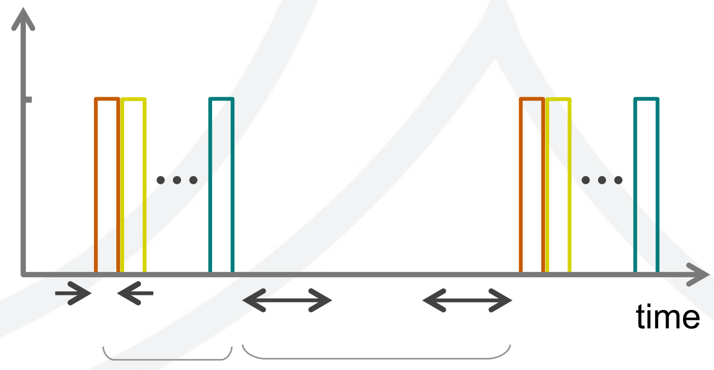
$$\sum_{i<j}^{N} \sigma_i^z \sigma_j^z J_{ij}^z + \sigma_i^x \sigma_j^x J_{ij}^x$$

4. Preparation of the ground state:
   - Adiabatic variation of Hamiltonian

Engineering other Hamiltonians

- Simulation of the dynamics induced by an Ising model, with interactions along x



Note $$e^{-i\pi\sigma_j^y/4}\sigma_j^z e^{i\pi\sigma_j^y/4} = \sigma_j^x$$

sequence of driving pulses: each pulse is quasi-resonant with a single spin

free evolution:

and $$e^{-i\frac{\pi}{4}\sigma_N^y} \cdots e^{-i\frac{\pi}{4}\sigma_1^y} e^{-iH_{\text{Ising}}^z t} e^{i\frac{\pi}{4}\sigma_1^y/4} \cdots e^{i\frac{\pi}{4}\sigma_N^y/4} = e^{-iH_{\text{Ising}}^x t}$$

Engineering other Hamiltonians

- Simulation of the dynamics induced by an Ising model, with interactions along *x*

- Simultaneous Interaction along *x* and *z* by Trotterization:

$$e^{-i\left[H^x_{\text{Ising}}+H^z_{\text{Ising}}\right]\tau} \simeq \left[e^{-i\left[H^x_{\text{Ising}}\right]\tau/n} e^{-i\left[H^z_{\text{Ising}}\right]\tau/n}\right]^n$$

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Adiabatic Preparation, Example

- Initial Hamiltonian in rotating frame:

$$H_I^{zz} = h/2\sum_j \sigma_j^z - 1/2\sum_{j,k} J_{j,k}\sigma_j^z\sigma_k^z , \quad h = \omega_j - \omega_d$$

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- During evolution:

$$H_{eff}\left(h,\alpha\right) = H_I^{zz}\left(h\right) + \alpha H_I^{xx}\left(h\right)$$

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- During evolution:

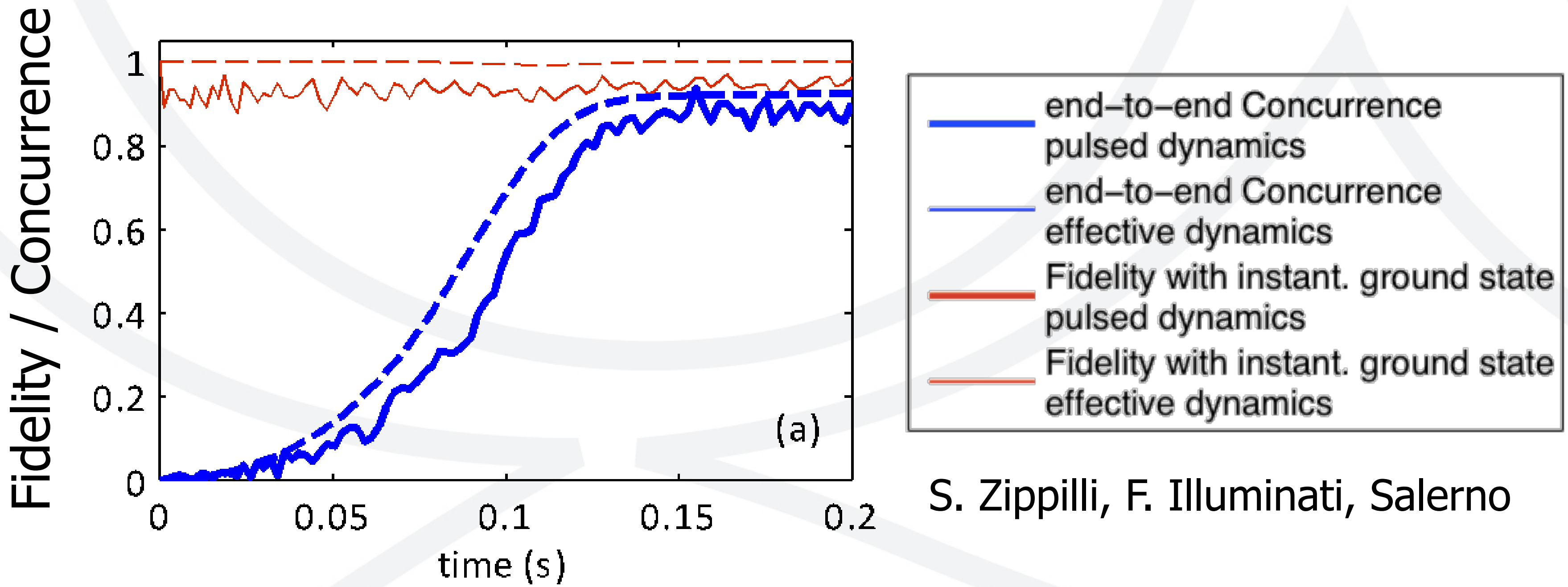
$$H_{eff}\left(h,\alpha\right) = H_I^{zz}\left(h\right) + \alpha H_I^{xx}\left(h\right)$$

- Adiabatic preparation:

- $\begin{cases} h(t_0) = h_0 \\ \alpha(t_0) = 0 \end{cases}$ → $\begin{cases} h(t_f) = 0 \\ \alpha(t_f) = 1 \end{cases}$
- $H(h_0, 0) = H_I^{(z)}(h_0)$ → $H(0,1) = \sum_{j,k} J_{j,k}\left(\sigma_j^x\sigma_k^x + \sigma_j^z\sigma_k^z\right)$
- $|\uparrow,\uparrow,\cdots\uparrow,\uparrow\rangle$ → Long distance entanglement

S. Zippilli et al., arXiv:1304.0261

Simulation: Adiabatic Transformation

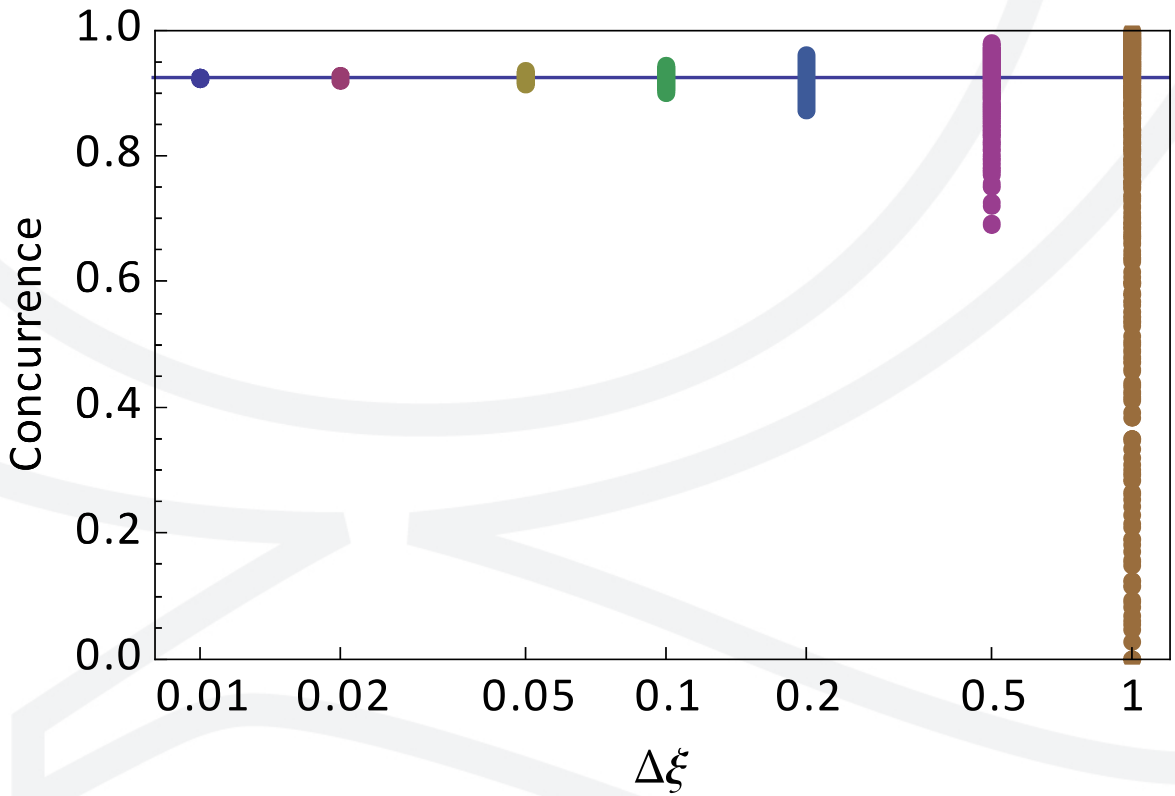


S. Zippilli, F. Illuminati, Salerno

- good agreement with effective Hamiltonian
- quantum simulation
- ground state entanglement

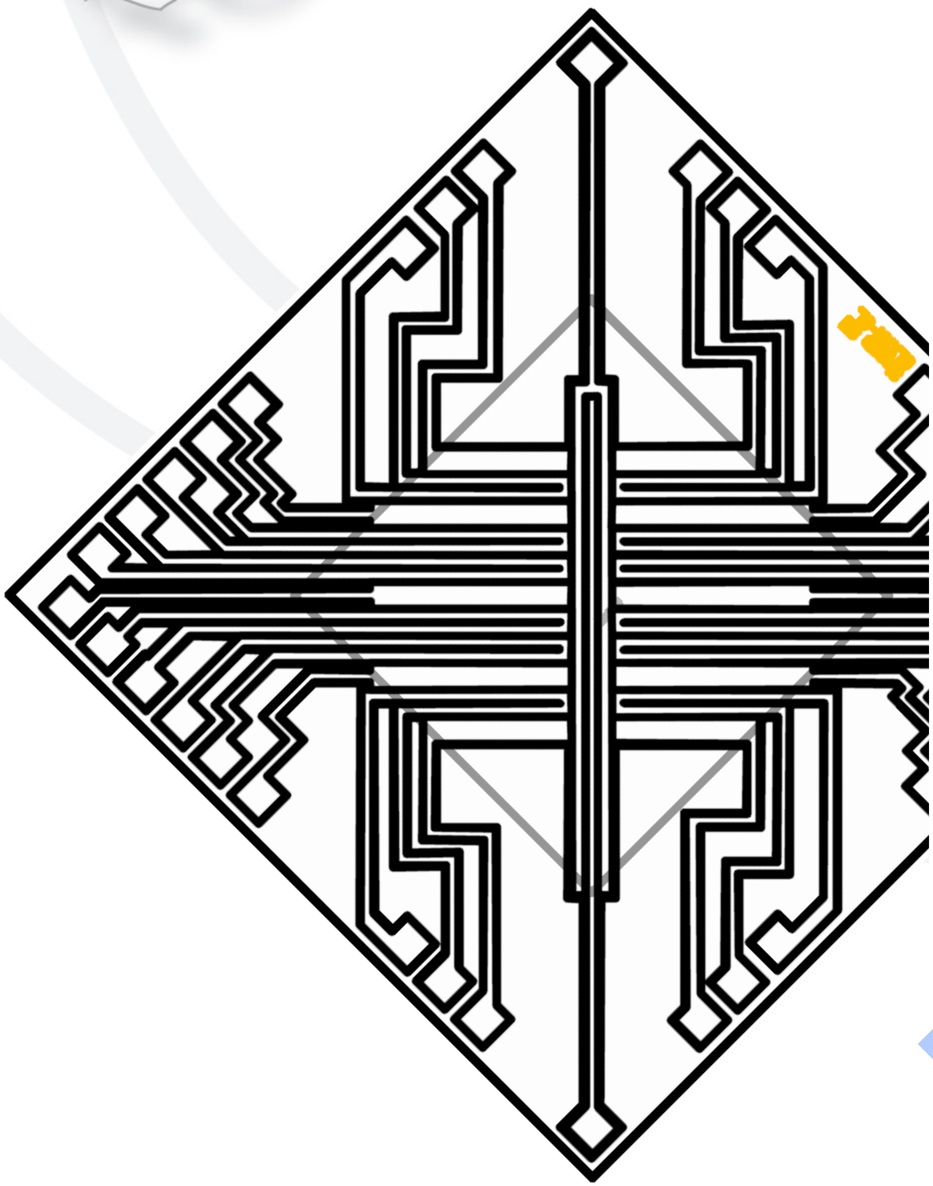
S. Zippilli et al., arXiv:1304.0261

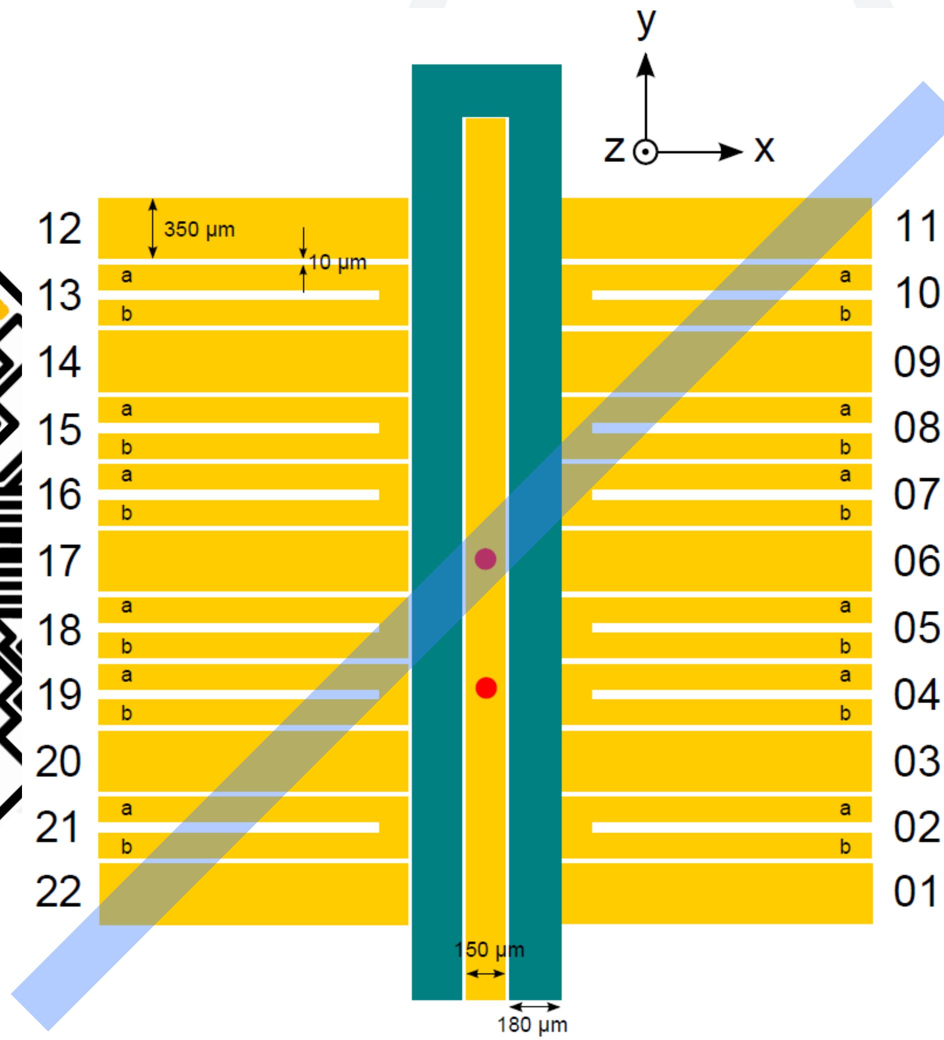
Simulation: Fluctuating Couplings



- concurrence remains high even for relative fluctuations of the couplings of 20 %

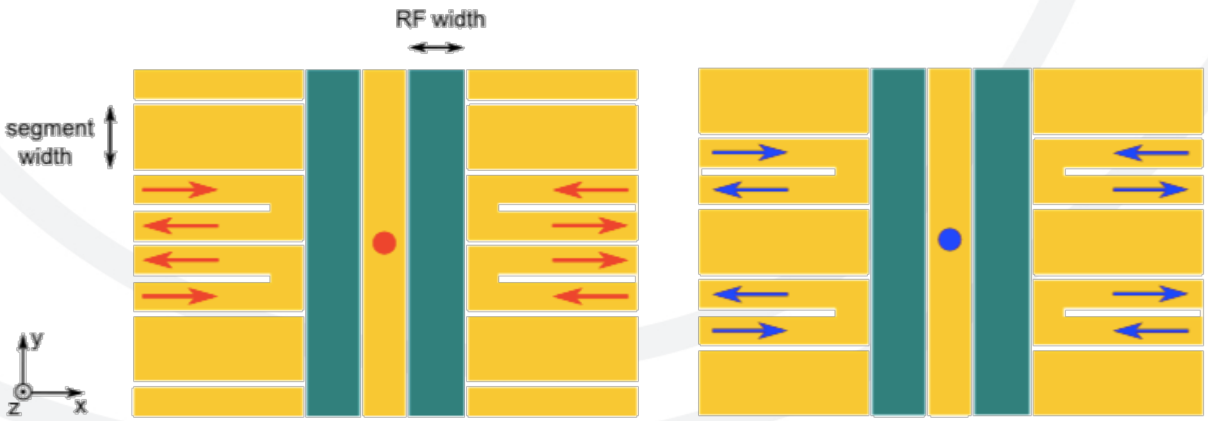
Surface ion trap for MAGIC

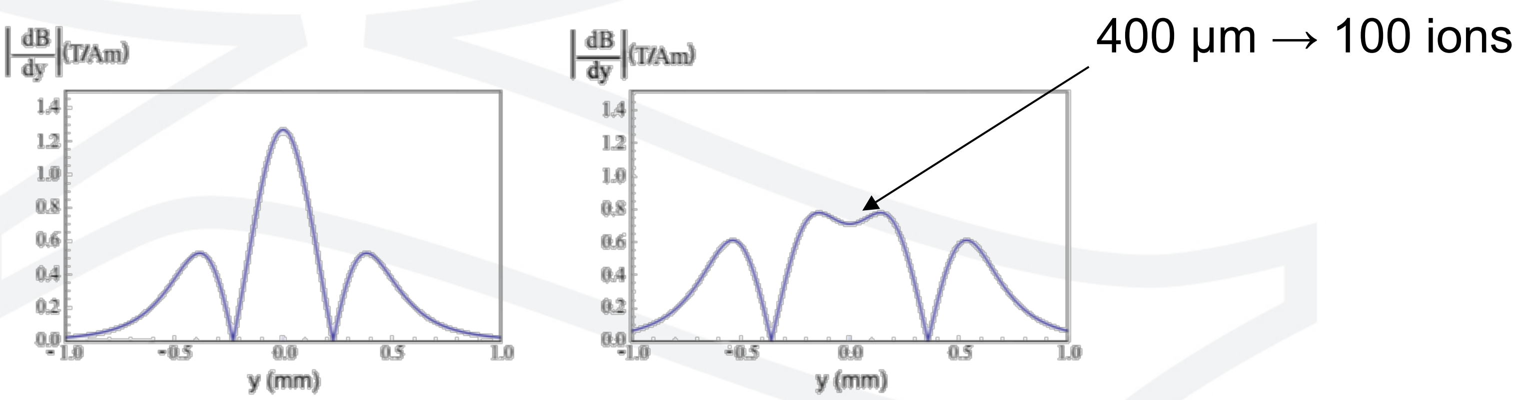




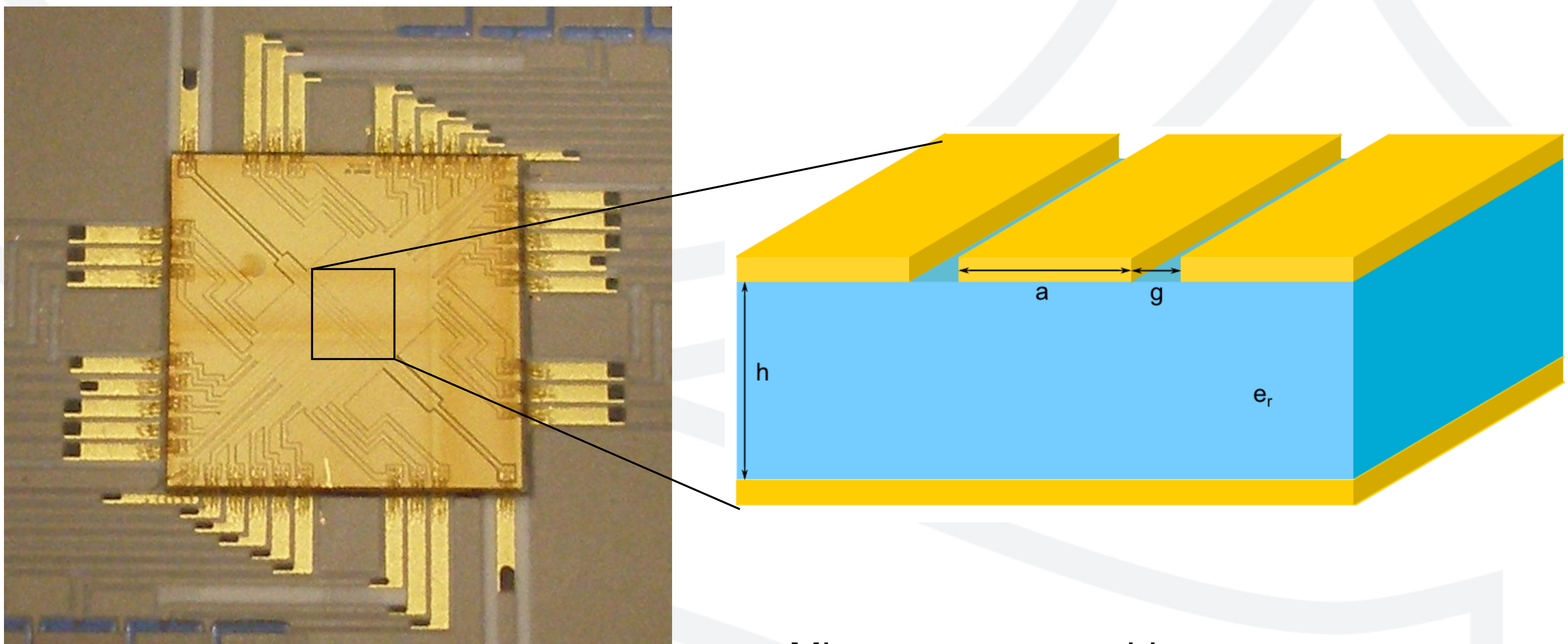
P. Kunert et. al. arXiv:1307.0949

Surface trap: versatile gradient





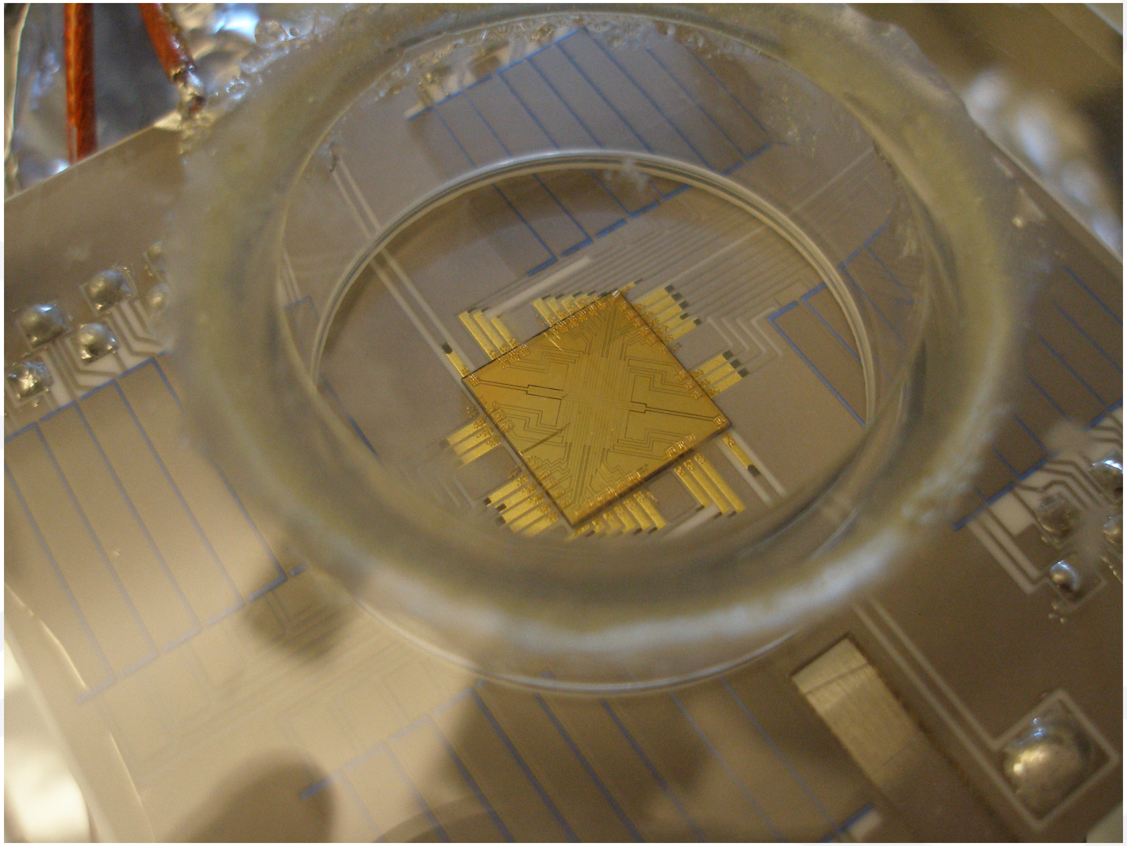
Microwave on-chip



Microwave waveguide

12,64 GHz for 171Yb+

Surface trap for MAGIC



Surface trap characteristics

• trapping 172-Yb-ions in several regions

• trapping height: (160 ± 10) µm

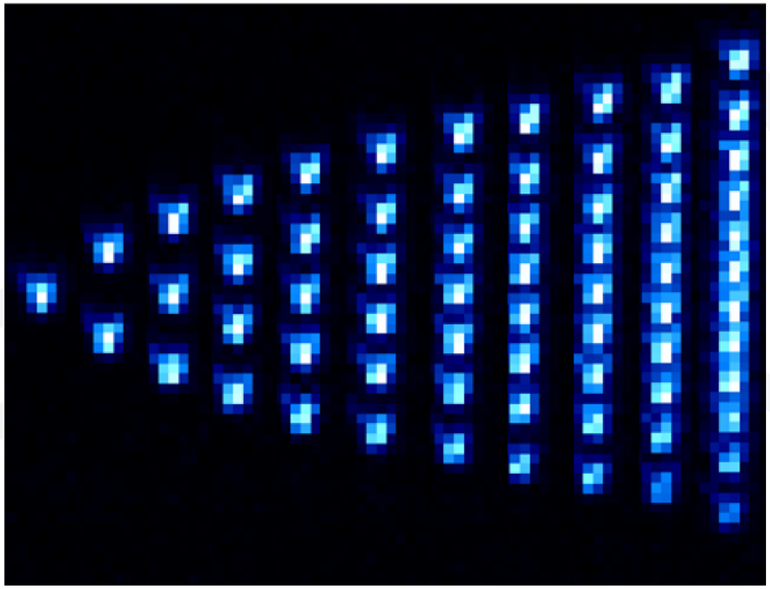
• storage times ~ hours

• pressure < 3 * 10^-11 mbar

• signal-to-background ratio: ~ 200:1
(1.5 Mio counts/sec)

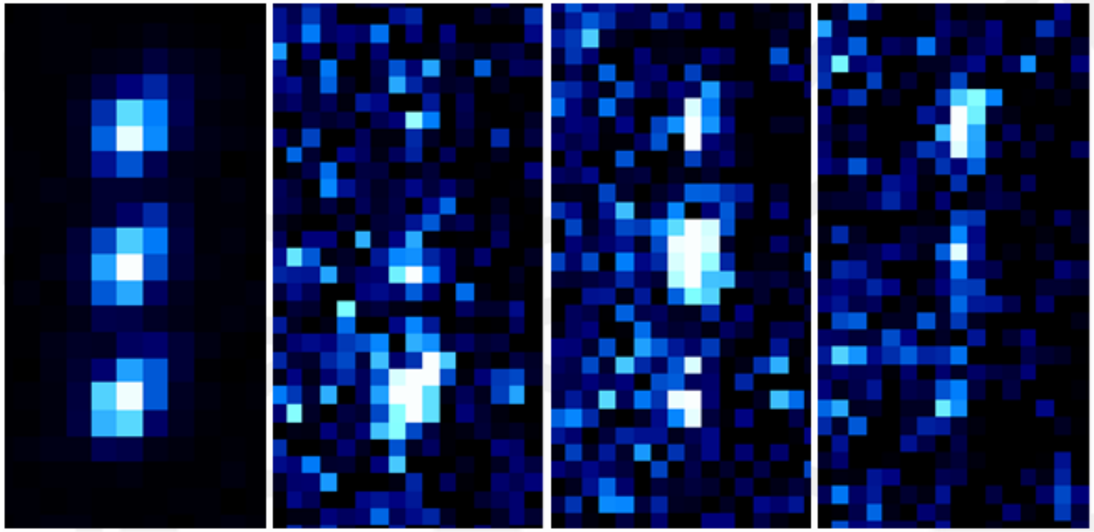
• trapping between RF amplitude: 150 – 400 V_pp

@ 250 V_pp and 14,71 MHz:
q = 0.23; trap depth = 73.4 meV



P.Kunert et.al. arXiv:1307.0949

Addressing



≈15 mT/m

P.Kunert et.al. arXiv:1307.0949

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