Mathematical Concepts (G6012)

Introduction

Thomas Nowotny

Chichester I, Room CI-105

Office hours: Tuesdays 15:00-16:45

T.Nowotny@sussex.ac.uk

Smile …

- I will record all lectures and make recordings available on Study Direct
- Don’t worry – only I will be visible in the recordings
- We do record attendance – please fill in the attendance sheet

Notes

- Taking notes? – Yes, but don’t copy the slides: Slides will always be provided
- Note taking is good to
  - Note down questions
  - Note down something you understood that is not on the slides
  - Note down if you need to think about something later
  - Try something while I am doing it on the blackboard / camera

About the Blackboard / Camera

- I will use the blackboard to develop stuff, because it is easier to follow along
- Most of the stuff I do on the blackboard is in the lecture notes.
- But I will also use the blackboard to answer questions in detail – some of that material might not be in the notes

Blackboard

- In the slides I will mark content that I have prepared for the blackboard with **BB**, and there are (sometimes hidden!) slides with the content (also, sometimes added after lecture)
- Lectures will be final & available on study direct on the evening of the day they were given.
- If you see the mark **BG** it is additional BackGround

Course structure

- 2 **Lectures**/week (1 hour each), 1 hour "Seminar" in 5 groups, 2 "Problem Sets"

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Mon												
Tue	**L1** S1	**L3** S2	**L5** S3	**L7** S4	**L9** S4	**L11** S5	**L13** S6	**L15** S7	**L17** S8	**L19** S9	**L21** S10	**L23** S11
Wed	S1	S2	S3	S4	S4	S5	S6	S7	S8	S9	S10	S11
Thu	**L2**	**L4**	**L6**	**L8**	**L10**	**L12** **P1**	**L14**	**L16**	**L18**	**L20**	**L22**	**L24** **P2**
Fri	S1	S2	S3	S4	S4	S5	S6	S7	S8	S9	S10	S11

Problem sets due on Thursdays, 16:00 in the School Office

Course Content

- What is this course going to cover?
  - Basic minimum maths skills that you will need to study computer science effectively
  - “Models of computing”: Automata theory

Topics

Math Fundamentals

Introduction
Numbers
Set theory, intervals
Proof by induction
Vectors and matrices
Norms and distances
Functions
Differentiation & Integration
Numerical integration
Probability theory
Statistics

Automata

Regular Expressions
Finite state models
Pushdown automata
Turing machines
The RAM model

Seminars

- Are there to **answer questions**
- Will cover more examples
- Will discuss coursework solutions
- Will discuss problems similar to coursework problems and similar to unseen exam

- Lab classes will be given by myself and GTAs. Attendance will be recorded.

Assessment

- **Problem sets**: 50%
  - 2 sets, due weeks 6 and 12
  - 25% weighting each.
- **Unseen exam**: 50%
  - In the January assessment period.

Coursework: Problem sets

- Problem sets for course work will be posted online on Study Direct.
- You should work on the problems **on your own** and in your own time and submit problem sets on Thursdays **before 16:00** in the School office.
- “Last retrieval date” will be the Friday (one day late) after the coursework was due
- Model answers will be available from Tuesday

Matlab

- Matlab can be very useful
- But: You will ***not be required*** to use matlab in this course
- You are welcome to use it to cross-check your workings ***but expected to do them on your own first***
- I might occasionally provide an example you can run in matlab for demonstration or additional exercises that may be fun

How to use Matlab …

- Matlab is installed on all Informatics computers
- Start -> All programs -> Matlab R2013a*
- I have put a small tutorial on study direct
- Built-in Matlab help is excellent

*search for “matlab” in “Start” if you can’t find it

Textbook(s)

- Everyone learns differently
- In principle, you can do this module without textbook. I will provide online resources and full lecture notes on Study Direct.
- If you do like to work with a textbook: I have reserved copies of three books in the library:

Textbook(s)

- *David Makinson: Sets, Logic and Maths for Computing (Undergraduate Topics in Computer Science), Springer 2nd ed. (2012)*
  (for the first parts and some of the Calculus)
- *John A. Vince: Mathematics for Computer Graphics (Undergraduate Topics in Computer Science), Springer*
  (for the linear Algebra, i.e. vectors and matrices)
- *John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman: Introduction to Automata Theory, Languages, and Computation, Addison-Wesley*
  (for the automata content)

Important

- Please check your email regularly (daily) as important messages may be distributed exclusively by email
- Please do not collude or plagiarise when working on assessed coursework*
- If you are having difficulties please let me know sooner rather than later

*If in any doubt about this, see sussex.ac.uk/academicoffice/resources/misconduct and links therein.

NUMBER SYSTEMS

What is a number?

- Leopold Kronecker: "God created the natural numbers, and all the rest is the work of man."
- There are several **number systems**:

$\mathbb{N}$ – the **natural** numbers

$\mathbb{Z}$ – the **integers**

$\mathbb{Q}$ – the **rational** numbers

$\mathbb{R}$ – the **real** numbers

$\mathbb{C}$ – the **complex** numbers

… (e.g. quaternions)

$\mathbb{N}$: The natural numbers

- $\mathbb{N}$ = {1,2,3 …}, operations "+", "-", "·", "/"
- If one adds or multiplies two natural numbers, the result is another natural number:
  "$\mathbb{N}$ is **closed** under addition and multiplication."
- $\mathbb{N}$ is **not closed** under subtraction or division: $3-5 \notin \mathbb{N}$ $1/3 \notin \mathbb{N}$

Formal definition

$\mathbb{N}$ can be generated with a successor relation:

Any natural number may be obtained from 1 by applying the 'successor relation' [S(n) = n+1] a finite number of times.

If 0 is included one (often) writes $\mathbb{N}_0$

$\mathbb{Z}$: The integers

- $\mathbb{Z}$ = { …,-2, -1, 0, 1,2, … }
- $\mathbb{Z}$ stands for 'Zahlen', German for 'numbers'.
- Unlike $\mathbb{N}$, the integers $\mathbb{Z}$ are closed under subtraction.
- But, something is lost: It is **not** true that any integer can be obtained from 1 by applying the 'successor relation' [S(n) = n+1] a finite number of times.
  Rather, both successors *and* predecessors must be considered.

$\mathbb{Q}$: The rational numbers

- $\mathbb{Q}$ = all those numbers that can be written in the form ***a* / *b***, where ***a*** and ***b*** are integers
  a = "nominator", ***b*** = "denominator"
- Unlike $\mathbb{Z}$, $\mathbb{Q}$ is closed under division (so long as ***b* ≠0**).
- Unlike $\mathbb{Z}$ and $\mathbb{N}$, $\mathbb{Q}$ has an ordering ("less than" or "greater than" relation) but no successor relation:
  Between any two rational numbers there is another. Proof: **BB**

Quick refresher: Manipulating fractions (rational numbers)

- Multiplying: $$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

- Dividing: $$\frac{a}{b} \Big/ \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

Quick refresher: Manipulating fractions (rational numbers)

- Addition & Subtraction:

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d}{b \cdot d} + \frac{c \cdot b}{d \cdot b}$$

$$= \frac{a \cdot d}{b \cdot d} + \frac{c \cdot b}{b \cdot d} = \frac{a \cdot d + c \cdot b}{b \cdot d}$$

$$= \frac{ad + cb}{bd}$$

BB Between and two rational numbers lies another

- Given two rational numbers, we can write them $\frac{a}{b}$ and $\frac{c}{d}$ and assume without loss of generality that $\frac{a}{b} < \frac{c}{d}$.
- We can rewrite these as

$$\frac{a}{b} = \frac{ad}{bd} < \frac{cb}{db} = \frac{c}{d}$$

BB Between and two rational numbers lies another

- But then we also know $ad \leq \frac{ad+cb}{2} \leq cb$
- … and therefore $\frac{ad}{bd} \leq \frac{ad+cb}{2bd} \leq \frac{cb}{bd}$
- In other words $\frac{ad+cb}{2bd}$ is between $\frac{a}{b}$ and $\frac{c}{d}$

$\mathbb{R}$: The real numbers

- There are many numbers that you know that are not rational numbers
- They have quite simple “physical (geometrical) interpretations”
- Can you think of any?

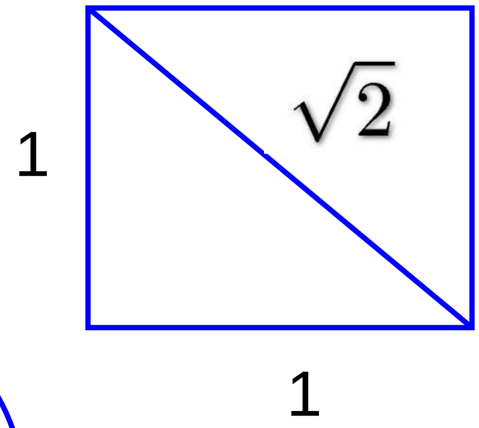
$\mathbb{R}$: The real numbers

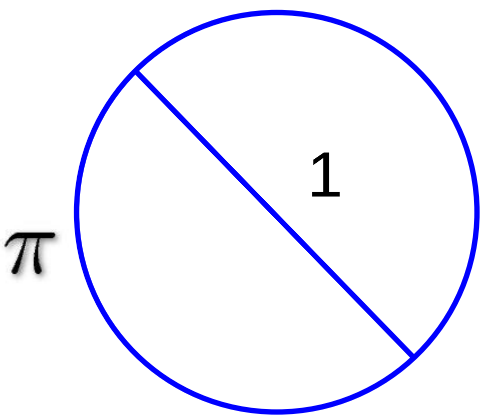
- There are many numbers that have physical interpretations but which **are not rational** numbers.

- For example:

  $\sqrt{2}$,

  π





Proof by contradiction:

- $\sqrt{2}$ **is not a rational number: BB**

BB: Lemma on even numbers

Lemma: “x is even” implies “x^2 is even”

Proof:

Assume x is odd, i.e. $x = 2n + 1$ for $n \in \mathbb{Z}$

Then,

$$x^2 = (2n+1)^2 = 4n^2 + 4n + 1 = 2(2n^2 + 2n) + 1$$

Therefore, x^2 is odd.

Inverting “x is odd” implies “x^2 is odd”, we obtain:

“ x is even” implies “ x^2 is even”.

q.e.d.

BB: Note

- We have used logic in the last part, the so-called contra-positive:
  A implies B is equivalent to (not B) implies (not A)
- In symbols:

$$(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$$

BB: $\sqrt{2} \notin \mathbb{Q}$: Proof by contradiction

- Assume $\sqrt{2}$ is rational, i.e. $\sqrt{2} = a/b$, where *a* and *b* are integers, expressed in lowest terms (in irreducible form).
- Then, $2 = a^2/b^2$, and $a^2 = 2b^2$
- Since b^2 is an integer, a^2 is even.
- Since the square of any odd number is odd, and a^2 is even, then a must be even.
- So we can write $a = 2c$, then $b^2 = 2c^2$, hence b^2 is even and, as before b is even.

BB: Proof continued …

- If both a and b are even, a/b wasn't expressed in lowest terms!
- This is a contradiction.

$\sqrt{2}$ **is not a rational number!**