HIDMS-PSO with Bio-inspired Fission-Fusion Behaviour and a Quorum Decision Mechanism

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***Abstract*—In this study, we propose a new variant of the HIDMS-PSO algorithm with a bio-inspired fission-fusion behaviour and a quorum decision mechanism (FFQ-HIDMS-PSO). In the new algorithm, units are conceptualised as self-organising fission-fusion societies that determine and adopt a suitable behaviour using unit-based quorum decisions. The incorporation of the two bio-inspired mechanisms provide "diversity aware" self-organising units that react to stagnation of particles by adopting a suitable fission-fusion behaviour, leading to a more efficient algorithm capable of maintaining significantly better population diversity throughout the search. The performance of the proposed algorithm was verified with three distinct experiments conducted using CEC17 and CEC05 test suites at 30 and 50 dimensions, comparing against 12 state-of-the-art metaheuristics and 12 state-of-the-art PSO variants. The proposed algorithm showed superior performance in these experiments by outperforming all 24 algorithms in all three experiments at 30 and 50 dimensions. The empirical evidence suggests that the proposed method also maintains significantly superior population diversity in comparison to the original HIDMS-PSO.**

***Index Terms*—particle swarm optimisation, swarm intelligence, meta-heuristics**

I. Introduction

Particle swarm optimisation, proposed in 1995 by Kennedy et al [1] is an optimisation algorithm widely used for a range of problems. Since its invention, due to its simple structure and effectiveness, PSO has attracted a lot of attention from researchers which resulted in many variants [2] and applications in a range of fields [3] [4]. The vast majority of PSO variants proposed in the literature address the problem of premature convergence to improve the performance of the algorithm. The HIDMS-PSO algorithm is a state-of-the-art PSO variant proposed in 2020 by Varna and Husbands [5]. The algorithm performs search using two fixed subpopulations, one homogeneous and one heterogeneous, and an explicit communication model to slow down the loss of population diversity. The intention of this paper is to further improve the depletion of population diversity by redesigning the unit structures in the standard HIDMS-PSO algorithm as self-organising social groups. Self-organising groups are widespread in nature, occurring in species from bacteria colonies to humans. Eusocial animals commonly exhibit self-organising behaviour to resolve various issues in order to survive. In this study, we propose a mechanism composed of a bio-inspired fission-fusion behaviour and a quorum decision mechanism to form self-organised units capable of reacting to improve their diversity. As a result, from this low-level behaviour emerges an overall higher-level population with a significantly improved diversity, which reduced premature convergence and improves search efficiency. Fission-fusion behaviour involves social groups changing their formation over time through either splitting into smaller groups (fission) or merging with other groups (fusion). It is observed in many organisms, including social insects, birds, fish and even humans, as a form of fitness beneficiary mechanism in a social group or colony, used to maximise survival or reproduction, or to minimise the chances of becoming prey. In our algorithmic model, fission-fusion behaviour is employed as a reactive mechanism through the creation of "diversity aware" units that exhibit fission-fusion behaviour when a unit's diversity exhibits a downward trend. Many colony and social group based species, such as ants and honeybees, make group decisions, and in our behavioural model units make a group decision on when to adopt fission-fusion behaviour, based on a quorum response. As a result, these two incorporated mechanisms provide units with the ability to self-organise by reaching a decision and adopting a behaviour through consensus, which in turn significantly improves population diversity and the overall performance of the algorithm.

II. Background

This section provides the necessary background information about the canonical PSO and HIDMS-PSO.

A. Canonical PSO

In canonical PSO, particles are initially randomly distributed in the search space. Throughout the search process, particles learn and retain certain information about the environment, namely its position, velocity and personal best position found. At each iteration, the position of the particle is updated by adding together its current position and the velocity. The velocity has the most significant influence on the next position of the particle, and it's calculated using two pieces of information, namely the particle's personal best-known position and the best position found within the swarm. The velocity and position calculation of the canonical PSO is as follows:

$$\vec{v}_i^{(t+1)} = \omega\vec{v}_i^{(t)} + c_1\vec{r}_1(\vec{pbest}_i - \vec{x}_i^{(t)}) + c_2\vec{r}_2(\vec{gbest} - \vec{x}_i^{(t)}) \quad (1)$$

$$\vec{x}_i^{(t+1)} = \vec{x}_i^{(t)} + \vec{v}_i^{(t)} \tag{2}$$

Where ω, $_1$ and c_2 are control parameters, namely the inertia weight and acceleration coefficients, $\vec{v}_i^{(t)}$ is the i^{th} particle's velocity, $\vec{pbest}$ is the personal best position, $\vec{gbest}$ is the globally best known solution and $\vec{x}_i^{(t)}$ is the current position of the i^{th} particle. Here, $\vec{r}_1$ and $\vec{r}_2$ are random variables with components in the range [0,1].

B. HIDMS-PSO

The HIDMS-PSO algorithm is a recent state-of-the-art PSO algorithm introduced by Varna and Husbands [6]. The algorithm introduced a new master-slave inspired dynamic topological structure with homogeneous and heterogeneous subpopulations and two movement strategies, namely, inward and outward-oriented strategies. The small subswarm entities in the HIDMS-PSO framework are called units and each unit constitutes a single master particle and 3 slave particles with distinct types. Master and slave particles retain their roles throughout the search process. The distinction in type between the slave particles allows heterogeneous behaviour, restricting information flow to avoid premature convergence and depletion of diversity. Fig. 1 shows the structure of a single unit in the HIDMS-PSO framework.

Information flow and the way particles interact with one another has an immense impact on the population diversity and particles' guidance, hence the overall search process. The HIDMS-PSO algorithm employs a communication model to control the flow of information and the interaction between particles. The communication model restricts information flow and allows particles to exchange information through master-to-master and slave-to-slave communication (see Fig. 2). The main communication is governed by the following rules:

1) Arbitrary particles of the ith unit cannot directly and freely communicate with arbitrary particles of the jth unit. Communication is established via the slave particles only.
2) Master particles can only exchange information with one of their slaves.
3) Slave particles can only communicate with the slaves of the same type; hence they cannot communicate with the other slaves within their unit.

a) Search Behaviour: In the HIDMS-PSO algorithm, the initial population is divided into two equal subpopulations, one homogeneous and one heterogeneous, and each subpopulation adopts a distinct movement strategy (Fig. 3). The homogeneous subpopulation uses the update equation of the canonical PSO algorithm, whereas the heterogeneous subpopulation is used to form N unit structures and adopts inward and outward-oriented strategies. The inward-oriented behaviour guides particles using the information obtained from members of the unit the particle belongs to. In contrast, the outward-oriented behaviour guides particles based on the information obtained from other units.

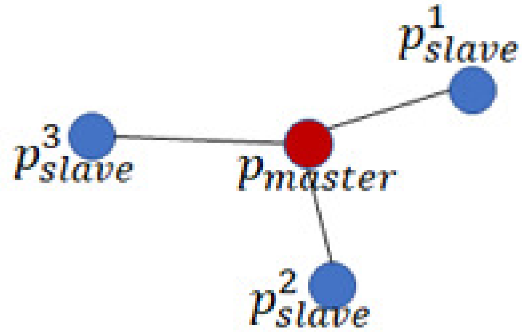

Fig. 1. Topological structure of a single unit.

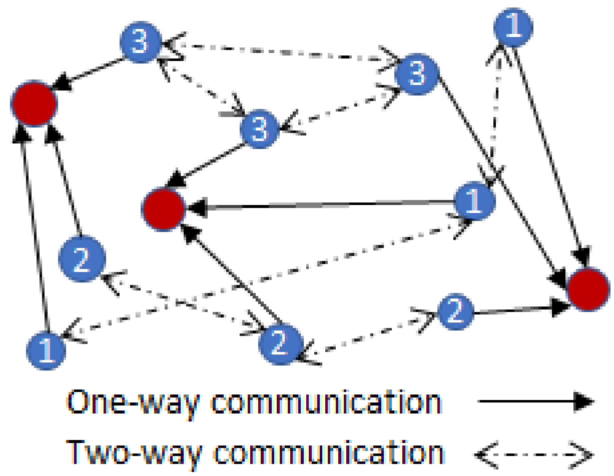

Fig. 2. The visual depiction of the communication model between 3 units.

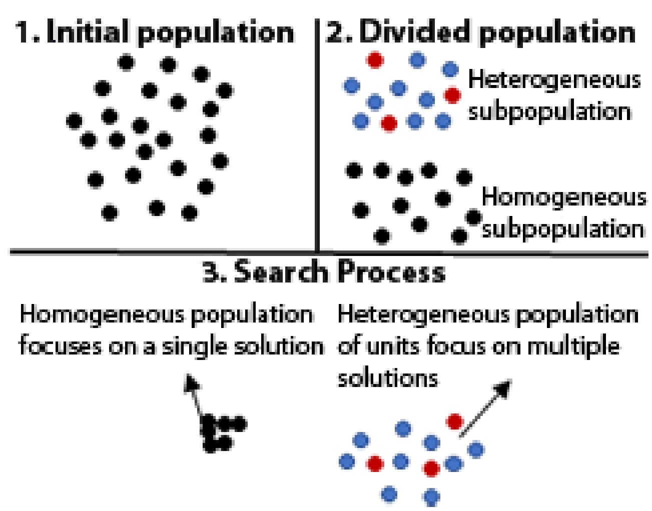

Fig. 3. Search phases of the HIDMS-PSO algorithm.

b) Inward-oriented strategy: The inward-oriented strategy uses information from members of its unit to guide its particles. For master particles of the N^{th} unit, this strategy involves particles updating their velocities by randomly selecting one of Eqs. 3-5:

$$\vec{v}_m^{(t+1)} = \omega^{(t)}\vec{v}_m^{(t)} + c_1\vec{r}_1(p\vec{best}_m - \vec{x}_m^{(t)}) + c_2\vec{r}_2(\vec{x}_s^{dis} - \vec{x}_m^{(t)}) \quad (3)$$

Where $\vec{v}_m^{(t)}$ is the velocity, $p\vec{best}_m$ is the personal best position, $\vec{x}_m^{(t)}$ is the position of the master particle at time t and, $\vec{x}_s^{dis}$ is the most dissimilar slave particle (positional dissimilarity) in the unit N. Movement towards the most dissimilar slave particle boosts the diversity of the master particle, hence the whole unit, as slave particles of a unit are highly influenced by the master particle's position.

$$\vec{v}_m^{(t+1)} = \omega^{(t)}\vec{v}_m^{(t)} + c_1\vec{r}_1(p\vec{best}_m - \vec{x}_m^{(t)}) + c_2\vec{r}_2(\vec{x}_s^{best} - \vec{x}_m^{(t)}) \quad (4)$$

Where $\vec{x}_s^{best}$ is the slave particle with the lowest cost in unit N. Local exploration is performed by guiding the master particle towards the best slave particle.

$$\vec{v}_m^{(t+1)} = \omega^{(t)}\vec{v}_m^{(t)} + c_1\vec{r}_1(p\vec{best}_m - \vec{x}_m^{(t)}) + c_2\vec{r}_2(\vec{x}_s^{avg} - \vec{x}_m^{(t)}) \quad (5)$$

Where $\vec{x}_s^{avg}$ is the average position of all slaves within the master's current unit. On the contrary, for the slave particles, the only option provided for this strategy is to move towards the unit master and personal best position of the slave particle, as shown in Eq. 6.

$$\vec{v}_s^{(t+1)} = \omega^{(t)}\vec{v}_i^{(t)} + c_1\vec{r}_1(p\vec{best}_s - \vec{x}_s^{(t)}) + c_2\vec{r}_2(\vec{x}_m - \vec{x}_s^{(t)}) \quad (6)$$

Where $\vec{v}_s^{(t)}$ is the velocity, $p\vec{best}_s$ is the personal best position, $\vec{x}_s$ is the position of the slave particle and, $\vec{x}_m$ is the position of master particle of the N^{th} unit.

c) Outward-oriented strategy: As opposed to the inward-oriented strategy, the outward-oriented movement enables particles to learn from other units while maintaining their hierarchical master-slave structure. The master particle randomly selects one of the following equations (7-9) to guide its behaviour:

$$\vec{v}_m^{(t+1)} = \omega^{(t)}\vec{v}_m^{(t)} + c_1\vec{r}_1(p\vec{best}_m - \vec{x}_m^{(t)}) + c_2\vec{r}_2(\vec{x}_{unit}^{avg} - \vec{x}_m^{(t)}) \quad (7)$$

Where $\vec{v}_m^{(t)}$ is the velocity, $p\vec{best}_m$ is the personal best position, $\vec{x}_m^{(t)}$ is the position of the master particle at time t and, $\vec{x}_{unit}^{avg}$ is the average position of the N^{th} unit's particles.

$$\vec{v}_m^{(t+1)} = \omega^{(t)}\vec{v}_m^{(t)} + c_1\vec{r}_1(p\vec{best}_m - \vec{x}_m^{(t)} + c_2\vec{r}_2(\vec{x}_{unit}^{m} - \vec{x}_m^{(t)}) \quad (8)$$

Where $\vec{x}_{unit}^{m}$ is the position of the master of a randomly selected unit.

$$\vec{v}_m^{(t+1)} = \omega^{(t)}\vec{v}_m^{(t)} + c_1\vec{r}_1(\vec{x}^{avg} - \vec{x}_m^{(t)}) + c_2\vec{r}_2(\vec{x}_{unit}^{m} - \vec{x}_m^{(t)}) \quad (9)$$

Where $\vec{x}^{avg}$ is the average position of particle's own unit members and $\vec{x}_{unit}^{m}$ is the position of the master particle of a randomly selected unit. Similar to the slave particle's movement in the inward-oriented strategy, in this case, the slave particles employ a single update equation to move towards a random slave of the same type that belongs to another unit, using:

$$\vec{v}_s^{(t+1)} = \omega^{(t)}\vec{v}_s^{(t)} + c_1\vec{r}_1(p\vec{best}_s - \vec{x}_s^{(t)}) + c_2\vec{r}_2(\vec{x}_{unit}^{rnd} - \vec{x}_s^{(t)}) \quad (10)$$

Where $\vec{v}_s^{(t)}$ is the velocity, $p\vec{best}_s$ is the personal best position, $\vec{x}_s$ is the position of the slave particle and, $\vec{x}_{unit}^{rnd}$ is the position of a random slave of the same type that belongs to another unit.

The combination of homogenous and heterogeneous populations in the HIDMS-PSO framework maintains the balance of exploration and exploitation while inward and outward-oriented learning strategies allow particles to initiate single-time behavioural fluctuations that enhance individual unit's diversity and help escape from local minima [5].

III. The Proposed Algorithm

As oppose to the standard HIDMS-PSO, in the proposed algorithm (FFQ-HIDMS-PSO) the search process initiates with a single population of n units of the type shown in Fig 1. Each unit's diversity (δ_n) is calculated at specific intervals defined by $\frac{FF_{period}}{10}$, where FF_{period} is the period of time particles are allowed to adopt the fission-fusion behaviour. FF_{period} may range from 1-10% of the maximum number of iterations. After this period, fissioned/fused units are randomly reformed as units while retaining their current particle positions. The δ_n is only calculated for units that have not undergone fission-fusion behaviour and is used as a threshold to initiate the fission-fusion behaviour. The δ_n is calculated as

$$\delta_n = \sum_{j=1}^{N} MSE(x_m, x_j) \quad (11)$$

Where MSE is the mean square error, x_m is the position of the master particle of the n^{th} unit, x_j is the jth slave particle of the n^{th} unit and N is the number of slave particles. if the n^{th} unit's δ is less than the average δ , the unit qualifies for fission-fusion behaviour. The type of behaviour (fission or fusion) is randomly selected and the behaviour is only adopted after a group decision. The group decision is based on the quorum response of each member of the unit and it is calculated as

$$QR_j = \frac{MSE(M, S_j)}{1 + (\frac{\alpha_j}{\beta})^{\gamma/10}} \quad (12)$$

Where QR_j is the quorum response of the jth member, α is the number of fissioned/fused conspecific particle at time t, β is the total number of particles with fission/fusion behaviour

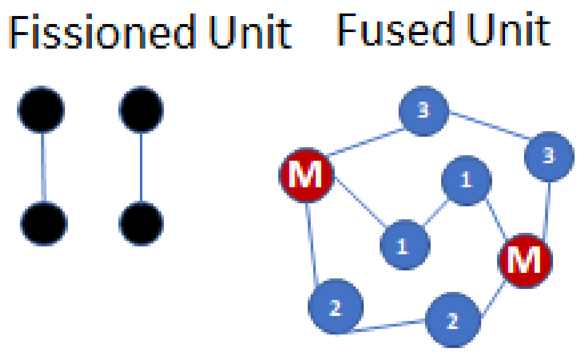

Fig. 4. Visual depictions of fissioned and fused units.

and γ is the fitness rank of the jth member (1 to 4, the fitter the particle, the higher the rank). Eq. 12 is an adapted version of the equation used to model animal group behavior in [6], and it allows members of a unit to individually gather information and compare findings with other unit members to reach the final decision. The QR equation essentially mimics how decentralised animal groups without a "leader" form a consensus. The decision is finalised by counting the number of unit members with $QR_j > \overline{QR}$, where $\overline{QR}$ is the average quorum response for the unit. Subsequently, the following rules are applied to determine the final decision

1) If more than 2 unit members have greater QR values than $\overline{QR}$, fission or fusion behaviour is adopted.
2) If more than 2 unit members have lower QR values than $\overline{QR}$, fission or fusion is not adopted.
3) If the number of unit members with QR values greater and less than $\overline{QR}$ are the same or if all QR values of unit members are the same, we use the QR value of the unit master in place of $\overline{QR}$ and proceed according to the first two rules.

As mentioned previously, many animals exhibit fission-fusion behaviour by temporarily splitting up and merging hence, fission-fusion behaviour is a strategy employed by social animals to reorganise their groups to increase or reduce potential loss of fitness. Fig 4 shows the visual depictions of a unit with fission and fusion behaviour.

A. *Fission-Fusion Behaviour*

As briefly mentioned, in this study, we conceptualise each unit as a fission-fusion society like those that exist in nature. Fission-fusion societies split up into smaller social groups or merge to form larger groups. This type of behaviour is usually adopted after a group decision, if it is the optimal option for all individuals to behave in smaller or larger social units to, for instance, forage, mate or exhibit predatory behaviour. In our behavioural model, units that adopt fission behaviour are divided into two equal sub-units and as oppose to the unit structure, particles within these sub-units do not have master-slave roles. The units that have not adopted fission-fusion behaviour remain as part of the heterogeneous unit population and control their movements according to the original HIDMS-PSO rules (Sect. III, and see the pseudocode). The fission behaviour provides four different exemplars to guide particles and their velocities are calculated using the following equation:

$$\vec{v}_i^{(t+1)} = \omega^{(t)}\vec{v}_i^{(t)} + c_1\vec{r}_1(p\vec{best}_i - \vec{x}_i^{(t)}) + c_2\vec{r}_2(\vec{x}_{fission} - \vec{x}_i^{(t)}) \tag{13}$$

Where $x_{fission}$ is the position of a particle randomly selected from: the most diverse fissioned sub-unit, a random fissioned sub-unit, the first fissioned and the last fissioned sub-units. As oppose to the fission behaviour, the fusion behaviour combines a maximum of two units while maintaining master-slave roles (as shown in Fig 4). The fusion behaviour allows the fused units to influence each other at individual level resulting in a potentially more diverse units at higher level. The particles within fused units use the following equation to update their velocity

$$\vec{v}_i^{(t+1)} = \omega^{(t)}\vec{v}_i^{(t)} + c_1\vec{r}_1(p\vec{best}_i - \vec{x}_i^{(t)}) + c_2\vec{r}_2(\vec{x}_{fusion} - \vec{x}_i^{(t)}) \tag{14}$$

Where x_{fusion} is randomly chosen as either the position of the conspecific particle (e.g. slave type) within the unit, the most diverse particle relative to its master within the fused unit, or the particle's personal best position.

By combining both behaviours, units regain diversity, or at least significantly slow down loss of diversity. This progressively extends to the overall population, resulting in particles escaping from local optima. Since fission-fusion behaviour is triggered as a result of loss of diversity in a unit, in both behaviours, the motivation is to guide particles using different sources, as defined in Eqs. 13 and 14, to avoid stagnation and potentially improve diversity of each individual to prepare them to reunite and form new units with sufficient diversity to carry out the search.

The FFQ-HIDMS-PSO algorithm uses the same parametric settings as the standard HIDMS-PSO; for a detailed description of the parameters, refer to the original study [5]. Besides the standard PSO parameters c_1, c_2 and ω, the HIDMS-PSO employ an additional parameter RG to reshape unit structures at specific intervals (see the pseudocode).

IV. Experimental Results and Discussions

This section presents the experimental design and results. The first subsection describes the experimental setup, benchmark suites, and statistical analysis and the latter presents the results of the three experiments conducted on the CEC17 and CEC05 benchmark suites.

A. *Experimental Setup*

The present study conducted three experiments to examine the performance of the proposed method, FFQ-HIDMS-PSO, using the CEC'05 and CEC'17 benchmark test suites. In the first experiment, the performance of the proposed algorithm is tested using the CEC'17 test suite. The CEC'17 test suite consists of 30 and CEC'05 consists of 25 continuous optimisation test functions. For the first and second experiments, we replicated the experiments conducted in [5] and for the third experiment, study [27] (which uses a different set of comparator algorithms) was replicated to produce comparable results. The results of the proposed algorithm is compared with 11 state-of-the-art evolutionary methods including two inertia weight PSO algorithms with different parametric settings ($\omega = 0.9 \rightarrow 0.4, c_1, c_2 = 2$ and $\omega = 0.4, c_1, c_2 = 2$), and evolutionary algorithms (including the bat algorithm (BA) [7], grey wolf optimiser (GWO) [8], butterfly optimisation algorithm (BOA) [9], whale optimisation algorithm (WOA) [10], moth flame optimisation (MFO) [11], artificial bee colony (ABC) [12], invasive weed optimisation (IWO) [13], flower pollination algorithm (FPA) [14] and cuckoo search algorithm (CS) (p=0.25) [15]. In both the second and third experiments, the proposed algorithm's performance was tested using the CEC'05 test suite and results were compared with a total of 12 state-of-the-art PSO variants including χPSO (ring with neighborhood radius $n_r = 2, \phi = 4.1, \chi = 0 : 72984, c_1, c_2 = 2.05$) [16], BBPSO [17], DMSPSO ($\omega = 0.729, c_1, c_2 = 1.49445, V_{max} = 0.5 * Range$) [18], FIPS [19], UPSO [20], CLPSO ($\omega = 0.9 \rightarrow 0.2, c_1, c_2 = 1.49445, V_{max} = 0.2 * Range$) [21], HIDMS-PSO ($\omega = 0.99 \rightarrow 0.29, c_1 = 2.5 \rightarrow 0.5, c_2 = 0.5 \rightarrow 2.5$) [5], HPSO-TVAC [22], FDR-PSO [23], HCLDMS-PSO ($\omega = 0.99 \rightarrow 0.29, c_1 = 2.5 \rightarrow 0.5, c_2 = 0.5 \rightarrow 2.5, pm = 0.1, V_{max} = 0.5 * Range$)) [24], HCLPSO [25] and MNHPSO-JTAC [26]. In the first experiment, the population size was set to 100 for all metaheuristics, and 40 for the two PSO variants and the proposed algorithm. In the second and third experiments, the population size was set to 40 for all methods [5]. For the first and second experiments, each problem was tested 30 times, and 100 times in the third experiment; 300,000 functions evaluations at 30 dimensions and 500,000 function evaluations at 50 dimensions. For detailed parameter values on the comparative methods, refer to studies [5] [27]. The mean errors are recorded for each problem and the results are shown in Tables I-VI. The average and final ranks of the mean performance are listed in Tables VII-IX. The Wilcoxon signed rank test conducted on the final ranks of the three experiments reveal that, for the first experiment conducted on the CEC'17 suite, the result is significant between the proposed algorithm and all comparison methods at both 30 and 50 dimensions at $p < 0.05$. For the second experiment conducted on the CEC'05 suite, at 30 dimensions, the result is significant between the comparison methods and the proposed algorithm except for HIDMS-PSO and HCLDMS-PSO and at 50 dimensions, the result is significant between the comparison methods and the proposed algorithm except for HIDMS-PSO. For the third experiment conducted on the CEC'05 suite, the result is significant between the comparison methods and the proposed algorithm except for BBPSO and CLPSO for the problem size of 30 dimensions, and at 50 dimensions, the result is significant between the proposed algorithm and all comparison methods at $p < 0.05$. Due to length restrictions of this paper, experimental results are partially included. External supplementary material is provided for complete results of experiments that can be accessed from users.sussex.ac.uk/fv47/FFQ-HIDMS-PSO.pdf.

B. Results

The results for the first experiment conducted on the CEC17 test suite at 30 dimensions show that the proposed algorithm (FFQ-HIDS-PSO) outperformed comparison methods for problems F5, F7, F8, F9, F11, F12, F16, F17, F20, F21, F22, F23, F24,F27, F29 and F30. The HIDMSPSO algorithm achieved the best mean performance for problems F3 and F28. CS outperformed comparison algorithms for problems F14, F14, F15, F18 and F19. For problems F1, F4, F6, F10, F25 and F26, ABC attained the best mean performance. BA, GWO, BOA, WOA, MFO, FPA, IWO, PSO1 and PSO2 did not outperform any of the comparison algorithms on any problems at 30 dimensions. The second experiment conducted on the CEC17 test suite for the problem size of 50 dimensions reveals that FFQ-HIDMS-PSO attained the best mean performance for problems F1, F5, F7, F8, F9, F10, F11, F12, F13, F16, F17, F20, F21, F22, F23, F24, F25, F26, F29 and F30. For problems F3, F4 and F6, HIDMS-PSO achieved the best mean performance. The CS algorithm outperformed the comparison methods for problems F14, F15, F18, F19 and F28. Lastly, BA, GWO, BOA, WOA, MFO, FPA, IWO, PSO1 and PSO2 did not outperform any of the comparison algorithms on any problems at 50 dimensions. The results for the second experiment conducted on the CEC05 test suite at 30 dimensions reveal that FFQ-HIDMS-PSO outperformed comparison state-of-the-art PSO variants for problems F5, F6, F10, F14, F19, F22, and F25. HCLDMS-PSO attained the best mean performance for problems F6, F17, F18, F20, F21 and F24. For problems F2, F7, F8, F11, F12 and F23, the HIDMS-PSO algorithm outperformed the comparison methods. HCLPSO achieved the best performance for problems F9, F13 and F15. MNHPSO-JTVAC outperformed comparison methods for a single problem (F3) while HPSO-TVAC attained the best performance for F1 and F16. The same experiment conducted at 50 dimensions reveal that the proposed algorithm outperformed the comparison methods for problems F4, F5, F10, F14, F16, F18, F19, F20, F21, F22, F24, and F25. The HIDMS-PSO algorithm attained the best mean performance for problems F2, F6, F7, F8, F11, F12 and F23. For problems F9, F13, F15 and F17, the HCLPSO algorithm outperformed the comparison methods. HPSO-TVAC and MNHPSO-JTAC each achieved the best performance for a single problem: F1 and F3, respectively. HCLDMS-PSO and FDR-PSO did not outperform any of the algorithms for any problems at 50 dimensions. The results for the third experiment conducted on the CEC05 test suite at 30 dimensions reveal that FFQ-HIDMS-PSO attained the best mean performance for problems F4, F5, F10, F11, F14, F17, F19, F20, F22 and F25. CLPSO outperformed comparison methods for problems F1, F6, F8, F9, F13, F15, F18, F21 and F23. For problems F1, F2, F3, F12 and F16, CLPSO obtained the best performance and DMSPSO achieved the best mean performance for a single problem of F7. χPSO, BBPSO and CLPSO attained an equal performance for problem F24. FIPS and UPSO did not outperform any of the algorithms for any problems at 30 dimensions. The same experiment conducted at 50 dimensions reveal that FFQ-HIDMS-PSO achieved the best performance for problems F4, F5, F10, F16, F17, F18, F19, F20 and F25. The CLPSO algorithm attained the best performance for problems F1, F9, F15, F21, F23 and F24. For problems F1, F2, F3, F6, F12, and F23, BBPSO achieved the best mean performance, UPSO outperformed the comparison algorithms for problems F8, F11, F13 and F14 and DMSPSO outperformed the comparison algorithms in a single case for problem F7. FIPS and XPSO did not outperform any of the algorithms for any problems at 50 dimensions. An additional experiment was conducted to observe and compare the rate of population diversity for the standard HIDMS-PSO algorithm and the new proposed variant. Fig 5 shows average value of population diversity over 20 consecutive runs for both algorithms. The empirical evidence clearly indicates that the proposed algorithm, FFQ-HIDMS-PSO, is capable of avoiding the depletion of population diversity. It is also worth noting that, in addition to a communication model mentioned in previous sections, HIDMS-PSO employs a non-uniform mutation operator at each iteration which significantly contributes to evading stagnation [5]. However, both of those mechanisms are not included in FFQ-HIDMS-PSO indicating the effectiveness of the mechanisms proposed in this study.

V. Conclusions

The present study proposed an extension of the state-of-the-art HIDMS-PSO algorithm that incorporates bio-inspired fission-fusion behaviour and a quorum decision mechanism, FFQ-HIDMS-PSO. The original algorithm was trimmed down by discarding the mutation operator and the communication model that was proposed in the original study. The new algorithm was equipped with bio-inspired fission-fusion behaviour and the quorum decision mechanisms provide the new algorithm with "diversity aware" units capable of adopting a suitable behaviour through quorum decision to regain a unit's diversity and boost the overall population diversity. The empirical evidence suggests that the new algorithm is superior in maintaining the population diversity throughout the search in comparison to the original HIDMS-PSO algorithm. The proposed algorithm was tested with three distinct experiments on the CEC17 and CEC05 test suites at 30 and 50 dimensions against 12 state-of-the-art metaheuristics and 12 state-of-the-art PSO variants. The proposed algorithm has shown a superior performance by outperforming all 24 algorithms in all conducted experiments. The present can be extended by further improving or applying the proposed algorithm to practical engineering problems.

Algorithm 1: FFQ-HIDMS-PSO

population size $n, C = 0.15, w_{max} = 0.99, w_{min} = 0.2$;
randomly define each particle's velocity υ and position x;
$c_1 = 2.5 - (1 : T_{max} * 2/T_{max}$;
$c_2 = 0.5 - (1 : T_{max} * 2/T_{max}$;
$\omega_1 = \frac{w_{max}+(w_{min}-w_{max})}{1+exp\left(-5\left(\frac{2t}{T_{max}}-1\right)\right)}$;
$RG_{min} = T_{max} * 0.01; \ \ RG_{max} = T_{max} * 0.1$;
$RG = RG_{max}; \ \ FF_{period} = T_{max} * 0.01$;
for *t=1:*T_{max} **do**
  if *mod(t,RG)==0* **then**
    vertically shuffle slave particles
  end
  if *mod(t,*FF_{period}*)==0* **then**
    Randomly reform new units from fissioned and fused particles
  end
  if *mod(t,round(*$\frac{FF_{period}}{10}$*))==0* **then**
    Calculate δ for all units using Eq. 11
    if *any unit's* $\delta < \overline{\delta}$ **then**
      Randomly select fission or fusion behaviour
      Calculate QR for each unit member using Eq. 12
      Use the rules in section 3 to determine the final decision
    end
  end
  for *i=1:n* **do**
    if $f(x_i) >= \overline{f(x)}$ **then**
      $\omega = \omega_1^{(t)} + C$; if $\omega > 0.99, \omega = 0.99, end$
    else
      $\omega = \omega_1^{(t)} - C$; if $\omega < 0.20, \omega = 0.20, end$
    end
    if i^{th} *particle belongs to a unit that is not currently fissioned/fused* **then**
      if *randi([0 1])==0 (inward-strategy)* **then**
        if i^{th} *particle is a master* **then**
          behaviour = randi([1 3]);
          if *behaviour == 1* **then**
            update υ_i and x_i using Eqs. 4 and 2
          else if *behaviour == 2* **then**
            update υ_i and x_i using Eqs. 5 and 2
          else if *behaviour == 3* **then**
            update υ_i and x_i using Eqs. 6 and 2
          end
        else
          update υ_i,x_i using Eqs. 7 and 2
        end
      else
        if i^{th} *particle is a master* **then**
          behaviour = randi([1 3]);
          if *behaviour == 1* **then**
            update υ_i,x_i using Eqs. 8 and 2
          else if *behaviour == 2* **then**
            update υ_i,x_i using Eqs. 9 and 2
          else if *behaviour == 3* **then**
            update υ_i,x_i using Eqs. 10 and 2
          end
        else
          update υ_i,x_i using the Eqs. 11 and 2
        end
      end
    else if i^{th} *particle belongs to a fisioned sub-pop* **then**
      update υ_i,x_i using the Eqs. 13 and 2
    else if i^{th} *particle belongs to a fused sub-pop* **then**
      update υ_i,x_i using the Eqs. 14 and 2
    end
    Evaluate the fitness of x_i
    Update the $pbest_i$ and $gbest$
  end
  $RG = round(RG_{max} - (RG_{max} - RG_{min}) * \frac{t}{T_{max}})$
end

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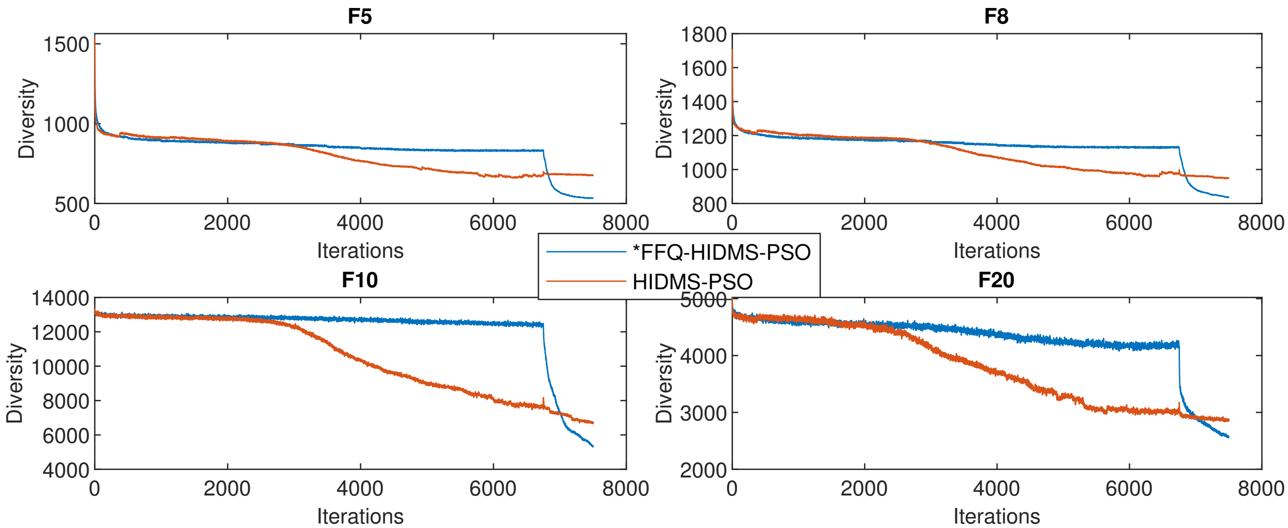

Fig. 5. Comparison of population diversity for HIDMS-PSO and FFQ-HIDMS-PSO for CEC17 test suite problems F5, F8, F10 and F20.

TABLE I
THE MEAN ERROR RESULTS OBTAINED FOR THE FIRST EXPERIMENT CONDUCTED USING THE CEC2017 TEST SUITE FOR PROBLEM SIZE OF 30 DIMENSIONS.

	F1	F3	F4	F5	F6	F7	F8	F9	F10
BA	7.3E+10	2.2E+05	2.1E+04	5.1E+02	1.1E+02	1.5E+03	4.3E+02	2.1E+04	8.8E+03
GWO	1.1E+09	2.9E+04	1.5E+02	8.7E+01	4.0E+00	1.6E+02	7.7E+01	5.4E+02	2.8E+03
BOA	3.0E+10	6.7E+04	2.5E+03	3.3E+02	6.4E+01	5.1E+02	2.9E+02	6.9E+03	7.7E+03
WOA	2.1E+06	1.6E+05	1.5E+02	2.7E+02	6.6E+01	5.1E+02	1.9E+02	7.7E+03	4.8E+03
MFO	8.1E+09	7.7E+04	5.1E+02	1.8E+02	2.5E+01	3.5E+02	1.7E+02	5.1E+03	4.1E+03
ABC	1.3E+02	1.2E+05	3.4E+01	8.8E+01	0.0E+00	1.0E+02	8.9E+01	8.2E+02	2.3E+03
FPA	1.1E+11	1.8E+06	3.6E+04	6.2E+02	1.3E+02	2.5E+03	5.6E+02	3.1E+04	9.1E+03
CS	1.9E+04	4.5E+04	7.5E+01	1.4E+02	5.0E+01	1.6E+02	1.3E+02	4.6E+03	3.7E+03
IWO	3.0E+03	6.4E+03	8.8E+01	4.1E+02	7.2E+01	2.0E+03	3.5E+02	7.6E+03	4.7E+03
PSO^1	1.3E+11	3.9E+08	4.4E+04	6.8E+02	1.4E+02	2.7E+03	6.1E+02	3.8E+04	9.6E+03
PSO^2	1.3E+11	3.9E+08	4.4E+04	6.8E+02	1.4E+02	2.7E+03	6.1E+02	3.8E+04	9.6E+03
HIDMS-PSO	2.6E+03	2.5E-10	6.2E+01	5.3E+01	9.6E-03	9.2E+01	5.2E+01	3.6E+00	2.9E+03
*FFQ-HIDMS-PSO	1.8E+03	1.0E+01	8.9E+01	2.6E+01	2.1E-01	6.6E+01	2.5E+01	1.1E-01	3.1E+03

TABLE II
THE MEAN ERROR RESULTS OBTAINED FOR THE FIRST EXPERIMENT CONDUCTED USING THE CEC2017 TEST SUITE FOR PROBLEM SIZE OF 50 DIMENSIONS.

	F1	F3	F4	F5	F6	F7	F8	F9	F10
BA	1.7E+11	8.2E+07	6.3E+04	9.5E+02	1.3E+02	3.3E+03	9.7E+02	7.5E+04	1.6E+04
GWO	4.6E+09	7.0E+04	4.3E+02	1.7E+02	1.1E+01	3.0E+02	2.0E+02	3.7E+03	5.6E+03
BOA	4.3E+10	2.2E+05	9.9E+03	6.2E+02	7.9E+01	1.1E+03	6.5E+02	2.8E+04	1.4E+04
WOA	7.1E+06	7.8E+04	2.8E+02	4.2E+02	7.6E+01	9.9E+02	4.1E+02	1.9E+04	9.1E+03
MFO	3.2E+10	1.7E+05	2.6E+03	4.2E+02	4.5E+01	9.0E+02	3.8E+02	1.5E+04	7.9E+03
ABC	9.2E+08	6.6E+05	1.2E+03	5.0E+02	3.0E+01	5.7E+02	5.0E+02	3.0E+04	1.5E+04
FPA	2.3E+11	1.9E+08	9.0E+04	1.1E+03	1.4E+02	4.7E+03	1.1E+03	9.2E+04	1.6E+04
CS	1.4E+05	1.6E+05	7.7E+01	2.9E+02	6.2E+01	3.4E+02	2.8E+02	1.6E+04	7.0E+03
IWO	6.9E+03	2.6E+04	1.2E+02	7.4E+02	7.8E+01	3.5E+03	7.2E+02	2.0E+04	7.7E+03
PSO^1	1.3E+09	9.6E+03	2.5E+02	2.3E+02	2.0E+01	2.8E+02	2.3E+02	5.8E+03	6.5E+03
PSO^2	1.2E+10	5.8E+04	9.3E+02	2.0E+02	1.2E+01	2.7E+02	2.0E+02	3.6E+03	6.1E+03
HIDMS-PSO	4.6E+03	1.7E-03	7.3E+01	1.1E+02	7.1E-02	1.8E+02	1.1E+02	4.2E+01	5.5E+03
*FFQ-HIDMS-PSO	1.4E+03	1.9E+03	1.1E+02	4.9E+01	6.2E-01	1.2E+02	4.5E+01	3.2E+00	5.3E+03

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TABLE III

THE MEAN ERROR RESULTS OBTAINED FOR THE SECOND EXPERIMENT CONDUCTED USING THE CEC2005 TEST SUITE FOR PROBLEM SIZE OF 30 DIMENSIONS.

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
HIDMS-PSO	1.400E-12	1.075E-03	1.127E+06	1.736E+03	2.980E+03	6.963E+01	4.696E+03	2.069E+01	4.956E+01	6.523E+01
HPSO-TVAC	5.495E-14	4.782E-02	1.745E+06	2.997E+03	5.459E+03	1.092E+02	4.696E+03	2.099E+01	3.638E+01	9.984E+01
FDR	4.970E+02	1.361E+03	1.622E+07	2.796E+03	3.623E+03	2.373E+06	4.696E+03	2.099E+01	2.737E+02	1.980E+02
HCLDMS-PSO	3.297E-12	3.453E+01	2.940E+06	2.214E+03	2.847E+03	6.333E+01	4.696E+03	2.084E+01	3.718E+01	3.549E+01
HCLPSO	1.262E+01	2.196E+01	3.688E+06	2.147E+03	2.393E+03	2.891E+05	4.696E+03	2.094E+01	4.017E+00	6.669E+01
MNHPSO-JTVAC	5.874E-14	9.344E-03	9.784E+05	3.575E+03	5.366E+03	9.910E+01	4.696E+03	2.100E+01	2.454E+01	1.007E+02
*FFQ-HIDMS-PSO	1.177E-03	7.955E+01	3.440E+06	3.175E+02	1.267E+03	1.670E+02	4.696E+03	2.091E+01	2.109E+01	3.091E+01

TABLE IV

(THE MEAN ERROR RESULTS OBTAINED FOR THE SECOND EXPERIMENT CONDUCTED USING THE CEC2005 TEST SUITE FOR PROBLEM SIZE OF 50 DIMENSIONS.

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
HIDMS-PSO	2.5E-09	2.8E+01	3.8E+06	2.5E+04	6.8E+03	1.2E+02	6.2E+03	2.1E+01	1.2E+02	1.3E+02
HPSO-TVAC	1.0E-13	1.9E+02	4.4E+06	3.1E+04	1.6E+04	1.7E+02	6.2E+03	2.1E+01	1.1E+02	1.9E+02
FDR	1.3E+03	1.1E+04	7.2E+07	2.6E+04	8.2E+03	9.9E+06	6.2E+03	2.1E+01	5.6E+02	4.3E+02
HCLDMS-PSO	6.9E-07	2.8E+03	1.1E+07	2.2E+04	7.5E+03	2.4E+02	6.2E+03	2.1E+01	1.1E+02	9.5E+01
HCLPSO	8.0E+00	2.0E+03	1.4E+07	2.5E+04	6.3E+03	1.8E+05	6.2E+03	2.1E+01	1.8E+01	1.2E+02
MNHPSO-JTVAC	1.2E-13	9.6E+01	2.9E+06	2.7E+04	1.4E+04	1.3E+02	6.2E+03	2.1E+01	8.3E+01	1.6E+02
*FFQ-HIDMS-PSO	4.3E-02	1.8E+03	1.2E+07	5.3E+03	3.5E+03	4.0E+02	6.2E+03	2.1E+01	4.4E+01	4.8E+01

TABLE V

THE MEAN ERROR RESULTS OBTAINED FOR THE THIRD EXPERIMENT CONDUCTED USING THE CEC2005 TEST SUITE FOR PROBLEM SIZE OF 30 DIMENSIONS.

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
χPSO	9.7E+00	1.6E+01	1.0E+07	1.8E+03	8.1E+03	1.2E+03	6.8E+03	2.1E+01	6.5E+01	8.7E+01
BBPSO	0.0E+00	9.3E-03	1.3E+06	2.3E+03	5.3E+03	2.8E+01	4.7E+03	2.1E+01	5.6E+01	7.6E+01
DMSPSO	3.1E+02	7.8E+02	5.6E+06	8.6E+02	4.3E+03	2.7E+07	4.3E+03	2.1E+01	4.8E+01	8.0E+01
FIPS	5.3E+02	1.5E+04	1.9E+07	2.1E+04	1.2E+04	2.5E+07	7.5E+03	2.1E+01	5.4E+01	1.5E+02
UPSO	1.3E+03	7.6E+03	5.3E+07	1.9E+04	1.3E+04	1.2E+07	7.5E+03	2.1E+01	7.8E+01	1.6E+02
CLPSO	0.0E+00	3.8E+02	1.2E+07	5.4E+03	4.0E+03	1.8E+01	4.7E+03	2.1E+01	0.0E+00	8.0E+01
*FFQ-HIDMS-PSO	1.1E-03	8.1E+01	3.4E+06	4.3E+02	1.3E+03	1.9E+02	4.7E+03	2.1E+01	2.4E+01	3.2E+01

TABLE VI

THE MEAN ERROR RESULTS OBTAINED FOR THE THIRD EXPERIMENT CONDUCTED USING THE CEC2005 TEST SUITE FOR PROBLEM SIZE OF 50 DIMENSIONS.

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10
χPSO	9.7E+00	7.8E+02	2.0E+07	2.8E+04	1.1E+04	6.4E+06	6.2E+03	2.1E+01	1.8E+02	1.8E+02
BBPSO	0.0E+00	2.9E+02	3.7E+06	3.0E+04	1.3E+04	5.8E+01	6.2E+03	2.1E+01	1.3E+02	1.8E+02
DMSPSO	3.9E+02	9.7E+02	1.3E+07	1.3E+04	5.5E+03	1.8E+07	6.1E+03	2.1E+01	9.9E+01	1.7E+02
FIPS	1.7E+03	2.6E+04	5.9E+07	3.4E+04	1.6E+04	8.0E+07	1.0E+04	2.1E+01	1.5E+02	3.9E+02
UPSO	7.1E+02	4.2E+03	5.3E+07	1.4E+04	1.2E+04	2.7E+06	7.4E+03	2.1E+01	6.5E+01	1.4E+02
CLPSO	0.0E+00	1.0E+04	4.9E+07	3.4E+04	9.7E+03	8.7E+01	6.2E+03	2.1E+01	0.0E+00	2.2E+02
*FFQ-HIDMS-PSO	4.6E-02	1.8E+03	1.2E+07	5.4E+03	3.5E+03	4.5E+02	6.2E+03	2.1E+01	4.6E+01	5.7E+01

TABLE VII

RANKS OF MEAN PERFORMANCE FOR THE FIRST EXPERIMENT.

Algorithm	Avg(30D)	Final(30D)	Avg(50D)	Final(50D)
*FFQ-HIDMS-PSO	1.90	1	1.45	1
HIDMS-PSO	2.45	2	2.17	2
ABC	2.93	3	8.45	10
CS	3.90	4	4.17	3
GWO	5.21	5	5.00	4
MFO	6.52	6	8.10	9
IWO	6.83	7	7.14	7
WOA	7.24	8	7.83	8
BOA	8.07	9	9.69	11
BA	10.07	10	12.03	12
FPA	10.93	11	12.93	13
PSO^1	12.00	12	5.48	5
PSO^2	12.00	12	6.55	6

TABLE VIII

RANKS OF MEAN PERFORMANCE FOR THE SECOND EXPERIMENT.

Algorithm	Avg(30D)	Final(30D)	Avg(50D)	Final(50D)
*FFQ-HIDMS-PSO	2.64	1	2.32	1
HCLDMS-PSO	2.84	2	3.44	3
HIDMS-PSO	2.92	3	2.88	2
HCLPSO	3.60	4	3.48	4
MNHPSO-JTVAC	4.32	5	4.04	5
HPSO-TVAC	4.36	6	4.56	6
FDR	6.24	7	6.32	7

TABLE IX

RANKS OF MEAN PERFORMANCE FOR THE THIRD EXPERIMENT.

Algorithm	Avg(30D)	Final(30D)	Avg(50D)	Final(50D)
*FFQ-HIDMS-PSO	2.08	1	2.16	1
CLPSO	2.32	2	3.32	2
BBPSO	2.80	3	3.48	4
XPSO	3.76	4	4.24	6
DMSPSO	4.16	5	3.44	3
FIPS	5.84	6	6.40	7
UPSO	6.04	7	4.16	5

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