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What Constitutes An Effective Representation?

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Abstract. This paper presents a taxonomy of 19 cognitive criteria for judging what constitutes effective representational systems, particularly for knowledge rich topics. Two classes of cognitive criteria are discussed. The first concerns access to concepts by reading and making inferences from external representations. The second class addresses the generation and manipulation of external representations to fulfill reasoning or problem solving goals. Suggestions for the use of the classification are made. Examples of conventional representations and Law Encoding Diagrams for the conceptual challenging topic of particle collisions are provided throughout.

Keywords External representation, mental representation, cognition, knowledge domains, Law Encoding Diagrams, algebraic expressions, tables, particle collisions, problem solving

1 Introduction

What constitutes an effective representation? Here *representations* include abstract (non-figurative) encodings and presentations of information, such as tables, formal notations, maps, diagrams and interfaces to computers. The title question is important because the design of representations may have a dramatic impact on cognitive processes at different times scales – from perception on the order seconds, to problem solving over minutes, learning lasting hours and days, and discovery taking years. For example, isomorphic representations of the Tower of Hanoi problem can increase problem solution times by up to 16 times [20]. An empirical study [7] on the mechanics problem from Larkin & Simon's [21] seminal paper showed a six-fold benefit for diagrams over sentential representations. A computational study [12] on the topic of particle collisions showed how diagrams (such as that in Fig 1A, below) might have been instrumental to the discovery of certain conservation laws in physics. So, this paper addresses the title question from a cognitive perspective, with a particular focus on representations for knowledge rich topics.

The question is challenging in cognitive terms. A cognitive answer must integrate: (a) considerations of the nature of external representations (ER); (b) considerations of the nature of the internal mental representations (IR); (c) investigate the rich and complex relations between the two – how ERs and IRs work together to encode knowledge. ERs may in themselves be complex [15, 17]. IRs are also complex [22] and must be examined in relation to the information processing capabilities of the human cognitive architecture [23], including visual perception, mental imagery, propositional (verbal/logical) reasoning and spatial reasoning, which involve memory encoding and retrieval processes at many levels [28].

To be clear on terminology: an ER is a particular physically rendered instance of a representation in the external environment; an IR comprises the information associated with the representation in the internal mental environment. Here, *representation* will refer to the combination of the ER and IR, and the term *representational system*, RS, will be used when this needs to be explicit. An RS is a *representing world* that encodes knowledge about the *represented world* of the target concepts and ideas with which the user of a RS is engaged.

Many answers to the title question have been obtained from specific perspectives using a myriad of approaches, including: task analysis (e.g., [6, 8]); computational models (e.g., [12], [24]); empirical studies (e.g., [6, 8], [14], [29]); eye-movement studies (e.g., [24]); theoretical analyses (e.g., [26, 27], [16]). These studies span all levels of cognition from perception, reasoning, problem solving (e.g., [21], [24], [30]), learning (e.g., [5, 6]), and discovery (e.g., [12]). Thus, a single coherent answer to the title question does not appear feasible or even seem appropriate. Green's *Cognitive Dimensions* [16] is a particularly extensive set of heuristic "tools" for identifying poor notations and interfaces. So, this paper aims to collate the previously identified characteristics, or criteria, to propose additional criteria, and to present them in a cognitively motivated classification. The classification emphasizes (a) general classes of representationally related cognition and (b) many levels of cognitive processing. Regarding the first of these, the classification identifies two major classes of criteria. (1) How readily a RS provides *access* to concepts – what in the relation between an ER and IR makes reading and interpretation easy? (2) The *generativity* of a RS concerns the ease of producing and manipulating an ER to achieve task goals – what about the nature of RSs can make the transformation of ERs easier? Each class is present in a section below, but first sample RSs for a knowledge rich topic will be introduced as a source of illustrative examples.

2 Sample topic and representations: particle collisions

The classification is motivated by, and draws upon, the author's experience in the design and evaluation of *Law Encoding Diagrams*, LEDs, for educational domains [4-60] and to serve as graphical computer interfaces for complex problem solving [10, 120]. A LED is a special RS because it directly encodes the conceptual structure of a topic in the graphical format of its ERs using geometric, spatial and topological rules, such that each instantiation of an ER represents one state-of-affairs in the topic. Thus, LEDs provide useful theoretical leverage to study representational issues, because they combine characteristics of abstract general notations (c.f., formulas) and concrete particular displays of data (c.f., line graphs). The topic of particle collisions will provide a thoroughgoing set of examples. This will include, tables and algebraic equations, which are the conventional representations for this topic in physics texts, which will be compared with a LED.

A basic 1D head-on elastic collision between two bodies, body-1 and body-2, which have masses m_1 and m_2, may be characterized by the velocities before impact, U_1 and U_2, and the velocities after impact, V_1 and V_2. (Units of kg and m/s may, respectively, be assumed.) Table 1 shows a selection of cases; in each row it assigns values across the variables. Further, it displays derived quantities of momentum, M, and energy, E, that were computed elsewhere. In valid cases momentum is conserved and so it is equal before and after impact: $M_{pre} = M_{post}$. For elastic collisions (1-2E, 5C), energy, E, is also conserved, $E_{pre} = E_{post}$, but for inelastic cases some is lost in the collision: $E_{pre} > E_{post}$ (6A/B).

Table 1. Data and derived quantities for particle collisions (2F is not a valid case).

Case	m_1	m_2	U_1	U_2	V_1	V_2	M_{pre}	M_{post}	E_{pre}	E_{post}
1	5	3	2	-2	-1	3	4	4	16	16
2A	1	1	1	-1	-1	1	0	0	1	1
2B	1	1	1	0	0	1	1	1	0.5	0.5
2C	1	1	3	1	1	3	4	4	5	5
2D	2	0.1	0.1	-2	-0.1	2	0	0	0.21	0.21
2E	1.9	0.1	1	-1	0.8	2.8	1.8	1.8	1	1
2F	5	3	2	2	-1	3	16	4	16	16
5C	5	3	1	-3	-2	2	-4	-4	16	16
6A	5	3	2	-2	-0.5	2.167	4	4	16	7.667
6B	5	3	2	-2	0.5	0.5	4	4	16	1

Expressing a case in a purely algebraic representation requires six equations: e.g.,

$$m_1=5,\ m_2=3,\ U_1=2,\ U_2=-2,\ V_1=-1,\ V_2=-3 \qquad 1$$

Physics texts typically analyze 1D elastic collisions in terms of the momentum and energy conservation laws, respectively:

$$m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2 \qquad 2$$

$$\tfrac{1}{2}m_1U_1^2 + \tfrac{1}{2}m_2U_2^2 = \tfrac{1}{2}m_1V_1^2 + \tfrac{1}{2}m_2V_2^2\ . \qquad 3$$

With some algebraic manipulation of Equations 2 and 3 it is possible to eliminate the mass terms, to obtain the "velocity difference" equation:

$$U_1 - U_2 = V_2 - V_1\ . \qquad 4$$

To model elastic collisions an energy loss parameter or a coefficient of restitution are introduced as multiplicative factors to one side of equation 2 or 4, respectively.

A typical textbook problem is to compute values of V_1 and V_2 given the other variables. This requires many algebraic manipulation steps, the simultaneous solution of equations (2) and (3), the application of the standard formula for quadratic equations, and the substitution of values from (1) into the resulting solution formulas.

Fig 1A shows one example of the diagrams that Huygens and Wren presented to the Royal Society of London in 1669 as models of 1D elastic collusions. It is a LED. The diagram has been redrawn in Fig 1B, with arrows for the velocities and line (segments) for the masses. In previous work, LEDs like these have been shown to enhance the learning [2, 3] and have been deployed in a computer-based discovery learning environment [4]. Fig 1C shows how the LEDs will be drawn here: they will be called *H&W diagrams*. This particular format allows the LEDs to be extended to cover sequences of collisions, inelastic impacts and 2D collisions (see below). Fig 1 shows the same collision as case 1 in Table 1 and Fig 2 shows other examples that correspond to the cases in Table 1 with the same numbers.

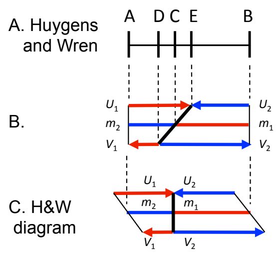


Fig 1. Huygens, Wren and H&W Diagrams.

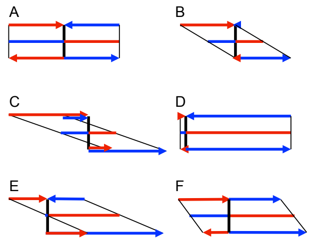


Fig 2. H&W diagrams (F is invalid).

Simple syntactic rules based on the relation of the arrows and lines to the central vertical *origin* and the parallelogram determine the structure of H&W diagrams. Most of the semantic rules for interpreting the diagrams are obvious but one should note that the mass lines are on the opposite side to their respective velocity arrows. Also, the slope of the parallelogram represents the overall momentum of the system. In Fig 2A the overall momentum is zero, but increasing the mass of body-1 or decreasing the speed of body-2 will positively increase the overall momentum, Fig 2E and Fig 2B, respectively.

H&W diagrams can be derived from Equations 2 and 3 (and vice versa). For instance, Equation 4 encodes the idea that the parallelogram has a constant width.

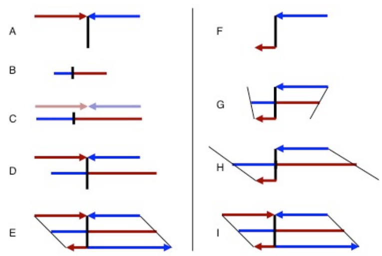


Fig 3. Quantitative problem solution.

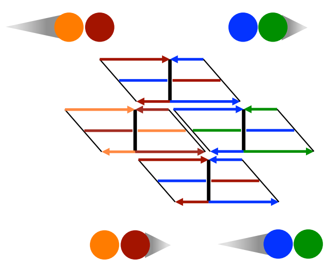
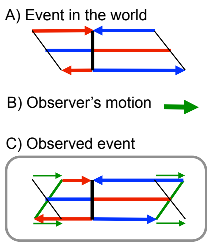
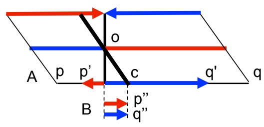


Fig 4. Modelling sequence of impacts

Fig 3 shows how the typical textbook problem mentioned above is solved. To find the final velocities, arrows for the initial velocities are first drawn to some chosen scale (Fig 3A). Lines for the masses are drawn end to end to an arbitrary scale (B) and this line is rescaled to match the initial velocities (C). They are then aligned at the origin (D) and the parallelogram is completed with the final velocities (E), allowing their values to be read-off to scale. Other combinations of given variables may require some iteration of the diagrams. For example, given one initial and one final velocity (Fig 3F) one must produce a parallelogram (I) by finding the correct length of the mass lines that is not too small (G) nor too large (H).

H&W diagrams may be composed to model sequences of collisions, such as two pairs of balls approaching at different speeds in a Newton's cradle, Fig 4 – the middle balls rebound (row 1), collide with outer balls and head back to the centre (row 2), where they again rebound (row 3). Moving frames of reference is a core notion in physics that H&W diagrams usefully visualize. Fig 5A is a given collision, Fig 5B gives the relative motion of an observer (say, on a train), and Fig 5C shows what the observe sees (through the window). Although the same constant velocity (green arrow) has been deducted from all the velocities, it is clear that the H&W diagram is valid and would be so for any observer's velocity. Thus, the conservation laws are the same for all observers in uniform motion.





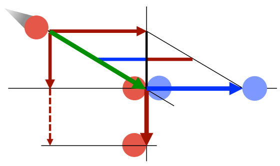


Fig 5. Moving frame of reference. Fig 6. Inelastic collision Fig 7. 2D collisions

Fig 6 shows how H&W diagrams can be extended to model in-elastic collisions using the fact that the diagonal of the parallelogram represents the overall momentum of the system. The thick diagonal line bisecting the origin, o, runs parallel to the sides of the parallelogram. In the extreme case when the maximum amount of energy is lost, the bodies coalesce and c gives the velocities of the bodies after collision, p'' and q'' at B in Fig 6. Between that extreme, c, and the fully elastic case, p and q, the overall momentum remains constant, thus any change to the momentum of one body must be compensated by the other, so the position of the arrow heads p' and q' from c must be in the same proportion as p and q are from c: i.e., p'c:q'c::pc:qc – e.g., A in Fig 6. At c these ratios are both zero. Can all the energy of the system be lost? The general form of H&W diagrams reveals this can only occur when the overall momentum is zero, when the diagram is a rectangle and both final velocities tend to zero.

The modelling of two-dimensional impacts is illustrated in Fig 7. When a moving ball, left (red), hits a stationary ball, right (blue), the departure angle between the balls is always 90°. Why? The diagram's orientation has been chosen so that the head-on and sliding components of the impact are horizontal and vertical, respectively. The sliding contact means the vertical motion is unchanged. The horizontal motion is simply modelled by Fig 2B with all the motion of the first ball being transferred to the second, so after impact each ball has a motion just associated with one component of the initial motion, which are perpendicular by definition. To model 2D cases where both bodies are moving, the H&W diagram for a moving frame of reference, Fig 5, can be used to decompose the situation into one similar to Fig 7 and some uniform motion for the whole system.

The range of examples reveals H&W diagram's ability to model many types of collision situation and to support reasoning about important concepts of the topic. The contrast between H&W diagrams and the conventional representations will illustrate the effectiveness criteria in the following sections.

3 Access to concepts: from ER to IR

The first class of effectiveness criteria concerns how readily concepts can be accessed in the IR from a given ER by the reading and interpreting the ER, without changes to its written or drawn form. Ready access to concepts is critical to the comprehensibility, memorability and learnability of a topic's content. Access is good when the cognitive demands, or work needed, to read and interpret information encoded in an ER is low. Further, with easy access related information will be readily retrieved from memory as the ER may provide rich cues for recall. An effective RS enables recognition of concepts and facilitates their interpretation. Poor access has negative consequences spanning all cognitive levels. It may reduce the rate at which meaningful cases that can be considered, increase the likelihood of interpretation errors and may hamper the spotting of errors when made. In learning contexts, poor access will increase the signal to noise ratio of positive learning episodes to negative ones [6]. Access will be considered in three sub-classes.

Elementary encoding. This group of criteria considers how particular ways to encode concepts in ERs may affect the access of the concept in the IR.

A1.1. One token for each type. Consider the elementary concepts of a topic, including properties, variables, entities and values. Access will be better when there is a one-to-one match between an elementary concept, or type, and a single symbol, or token, in the ER. Such mappings make the least cognitive demands because they avoid the work associated with managing complex associations between symbols and concepts, such as the need to exhaustively search for all occurrences of a symbol in the ER for a given variable [21]. In H&W diagrams, one graphical property encodes each type of elementary concept, but Equations 2 and 3 include two occurrences of letters for each velocity, two letters for each mass, and eight subscript numbers to denote the bodies. The original Huygens and Wren diagrams are poor, because many variables are mapped to different sections of one line (Fig 1A).

A1.2. Reflects structure of concepts. Beyond elementary concepts, similar reasoning applies to the claim that the structure of expressions should reflect the topic's conceptual structure [29]; hierarchically related concepts should be encoded by hierarchically organized representations [1] and more generally they should be isomorphic [17]. Equations 1 and 2 clearly show how momentum and energy terms are sums of products of the variables. In general, however, equations tend to hide conceptual structures [6, 80]. (See [9] for an alternative RS for algebra that has one ER symbol for each variable and that shows the hierarchal relations among variables graphically.) Finding the conservation laws from data in Table 1 is a challenging inductive discovery problem [12]. The shape of H&W diagrams supports reasoning about momentum, but inferences about energy requires inferences about relative lengths of the mass lines and velocity arrows, without explicit support in the ER.

A1.3. Directly depicts structure of cases. In additional to a topic's conceptual structure, it is desirable that ERs for individual cases reflect the concrete organization or physical structure of each case. H&W diagrams clearly do this, but the algebraic representation tends to hide such structure; for example, they do not explicitly encode the fact that the spatial ordering of the bodies is fixed.

In terms of Green's Cognitive Dimensions framework [16], criteria A1.2 and A1.3 are aspects of *closeness of mapping* and *consistency*.

A1.4. Exploits spatial indexing. Spatial indexing of information, rather than symbolic encoding, can make accessing concepts easier, by facilitating searches for information and the recognition of operators [21]. In H&W diagrams conceptually related information is often spatially co-located, and tables exploits spatial coordination in their columns and rows, but equations' linear concatenation of symbols tends to separate pieces of information that need to be related.

A1.5. Iconic expressions. Expressions for important concepts should be iconic; they should consist of distinctive shapes or patterns that are clearly recognizable and particularly memorable. H&W diagrams are iconic at several levels: each pair of arrows forms a distinctive pattern, that are combined as unique parallelogram configurations (e.g., Figs 1C, 2A-E), and in turn assemblies of H&W diagrams may themselves be iconic (e.g., Fig 4). Whether a pattern is iconic depends on the user's level of experience with the RS and in some domains expertise is based on the acquisition of large number of perceptual chunks [28]. Scientists and engineers can instantly recognize that terms in Equation 2 represents quantities of kinetic energy, but novices may perceive the expressions as arbitrary concatenations of symbols. Iconic expressions is one aspect of *visibility* in the Cognitive Dimensions framework [16]

Reading and inference operations. This group of criteria considers transfer of information in the ER to the IR and mentally transforming expressions of the IR.

A2.1. Prefer low cost forms of processing. Simply, ERs that invoke IRs and processes that have low cognitive demands will facilitate access to concepts. The Cognitive Dimensions framework [16] recommends avoiding *hard mental operators*, in general. More specifically, perceptual operators are easier than using visual imagery, and visual imagery is less demanding than verbal logical reasoning, in gross terms. For example, many important concepts can be accessed rapidly by visual inspection of H&W diagrams, but it is harder to imagine changing the shape of a H&W diagram in one's mind's eye (e.g., given Fig 2A imagine Fig 2C). It is harder still to use Equations 2 and 3 to mentally reason propositionally about the impact of changing some variables with others held constant. *Computational off-loading* [25] may be interpreted as the potential of some ERs to allow perceptual inferences to be substituted for purely mental forms of reasoning.

A2.2. Prefer low cost operators. For a particular form of processing (whether perceptual, imagistic or verbal/propositional) some types of operator will be less demanding than others. For example, Cleveland & McGill [14] empirically established an order of effectiveness for simple perceptual operators used to judge quantities. In mental imagery, translation and rotation operations are likely to be easier than composing irregular shapes [18]. Verbal reasoning about chance situations is superior when probabilities are interpreted as frequencies rather than as decimal numbers [13].

A2.3. Invoke more structured IRs. Cognitive scientists explain human ability on complex information processing tasks using a variety of types of IR, including associative semantic networks, condition-action production rules, semantic networks with inheritance, and schemas (or frames) [22, 28]. Cognitive benefits naturally arise from the use of IRs that are more systematic, arguably in the order just given, because more precise and rich indexing of information will aid contextualization and access to concepts. Thus, RSs that recruit IRs with good structure appear preferable. For example, schemas are IRs that possess particular *slots* which may be *filled* by certain types of information. This establishes specific relations among the pieces of information. Interpreted tables may be comprised of generic schemas that coordinate values in the columns and rows, but provide less in the way of topic specific relations. An equation may invoke a schema with slots for the left and right sides of an equation and that encodes the concept that they are equal. H&W diagrams may encourage users to develop a particularly effective schema – see next criteria.

A2.4. Support for diagrammatic configuration schemas. Experts' proficiency in certain forms of problem solving may be attributable to their organization of information as a special form of schemas, *diagrammatic configuration schemas*, DCS, [19]. A DCS uses a diagram of a specific situation to coordinate what inferences can be made about the situation under given sets of constraints. This rich encoding allows experts to efficiently solve problems by rapidly planning effective sequences of operations, by decomposing the ER into characteristic patterns associated with DCSs and using the constraints to identify feasible inferences. Users familiar with H&W diagrams may possess DCS as IRs, because the rules governing the diagrams can be encoded as inference and applicability conditions. Such an encoding is unlikely for the algebraic representation, because the algebraic inference rules are diverse and generic and therefore not tightly and specifically associated with Equations 2 and 3.

Conceptual transparency. This class of accessibility criteria considers the design of RSs when a full characterization of the conceptual structure of the topic is available. It differs from those above (esp. A1.2) by embracing the complexity and depth of ideas in knowledge rich topics. The *conceptual transparency* criteria (elsewhere called *semantic transparency* criteria [6, 8, 10]) consider how to make the full richness and range of important and distinctive concepts of a topic directly accessible as simple patterns in ERs. Such concepts include: the primary symmetries, invariants, laws and major regularities of the topic; alternative conceptions or ontological perspectives, such as taxonomic, causal processes and formal constraints; types of cases, including prototypical, special, extreme and limiting cases; valid versus invalid relations and cases [10]. Importantly, when diverse concepts are readily accessible simultaneously, they can provide mutual supportive contexts for each other's interpretations [10]. So, the challenging demand of conceptual transparency is to use what is known about the nature of a topic's content to encode it in a manner that allows it to be easily accessed and interrelated. The following criteria promote such encodings.

A3.1. A format for each class of primary concepts. For conceptual transparency, a RS should provide a distinctive *representational format* for each of the *primary classes of concepts* of the topic [10]. A representational format is a particular type of graphical or notational scheme for encoding information, such as a spatial coordinate system, a set of visual properties, or formal rules applied to concatenations of symbols. Important types of concepts include: (a) properties and their values; taxonomic relations; (b) structural concepts; (c) temporal concepts; (d) behavioural concepts; (e) functional concepts; (f) formal relations (logical, mathematical). A topic might not include all of these classes. H&W diagrams has largely separate representational formats for each primary class of concepts: (a) velocity and mass and their values are represented by types and sizes of lines; (b) the structure of collisions is represented by the topology of the arrows (their relative left-right placement); (c) time is represented by relative vertical position; (d) collision behaviour is represented by the configuration of the arrows; (f) formal relations are encoded by the geometric rules of the diagram. The algebraic representation does not satisfy this criterion well, as types of alphanumeric symbols and algebraic relations span several classes of concepts.

A3.2. Coherent encoding of primary concepts in a format. For conceptual transparency, the format for each primary class of concepts should simultaneously differentiate and integrate the concepts in the class, so that the concepts can be readily distinguished but also to provide mutual contexts for each other's interpretation [10]. For example, all properties/quantities in H&W diagrams are line segments, but scalars are plain lines and vector are arrows, with the orientation of the arrows giving directions of motion. Equivalent information in equations is distributed across conventions on alphanumeric symbols and the assignment of numerical values to variables.

A3.3. Provide an overarching interpretive scheme. For conceptual transparency a RS should have an overarching interpretive scheme to coherently combine the formats of the primary class of concepts [10]. The arrangement of the origin and parallelogram in H&W diagrams constitutes such an overarching interpretive scheme, whereby different properties, quantities, structure, times, behaviors, functions and formal relations are well integrated. Concatenation of symbols under algebraic rules provides little in the way of a topic-specific overarching interpretive scheme.

H&W diagrams largely satisfy A3.1-3 so they possess greater conceptual transparency than the equations. Both show the spatial and temporal symmetry of the collisions. The form of the equations is invariant across the identity of variables (body-1/body-2) and order of terms (pre/post collision). Valid H&W diagrams are produced when the whole diagram is reflected about the origin, or reflected vertically with the directions of the arrows reversed. However, the H&W diagrams simply encodes the notion of moving frames of reference (Fig 5), but it is demanding problem to show that adding a constant to U_1 and U_2 in Equations 2 and 3 necessarily changes both V_1 and V_2 by the same amount. Prototypical (Fig 1A/C), special (Fig 1B/D) and limiting (Fig D/E) cases are distinctive H&W diagrams, but such cases are less apparent in Table 1. Invalid H&W diagrams often standout and what is wrong is often obvious (e.g., Fig 1F), but without the momentum values this case is not obvious in Table 1 nor is the source of the problem (a missing minus sign).

Representations with conceptual transparency may suffer less from the problem *diffuseness* as identified in the Cognitive Dimensions framework [16]. Shimojima [26] identifies three semantic properties of diagrams that appear to promote their access to information in the ER, specifically the potential for *free rides*, *consistency-checking* and *derivative meanings*. These three potentially beneficial properties may be interpreted in terms of conceptual transparency. So, criteria A3.1-3 may provide a means by which to design representations possessing the properties, for topics that are more knowledge rich than the cases examined by Shimojima [26].

4 Generating ERs

This part of the classification concerns the production of ERs through their modification or generation from scratch in order to revise or obtain new concepts. Given a new set of data one might add a row to Table 1 or draw a H&W Diagram; or to solve a problem one might write a new equation derived from equations 1 and 2, or sketch a sequence of H&W diagrams. The ease of producing ERs will substantially impact the effectiveness of RS at multiple cognitive levels. Reasoning, problem solving, learning and discovery may be enhanced when generating ERs requires little effort and can be done so reliably. A RS in which it is complicated to do things, and in which great care is needed to avoid errors, is undesirable. Obviously, when a new ER has been generated the concepts contained within it are accessed, so the processes of generating and accessing ERs are symbiotic. The term *operation* is used for elementary manipulations of an ER and *procedure* denotes sequences of operations to achieve goals of ER transformation tasks. Unfortunately, it appears there is little prior work on the effective generation of ERs, per se. Two classes of criteria are considered.

Syntactic plasticity. A RS is a medium for modelling ideas in a topic, much like materials are use to make physical models. A plastic material (e.g., clay) is good for creating a sculpture as it can be readily molded: it is not too brittle like chalk nor so fluid it flows in an unconstraint fashion like syrup. By analogy a RS should be syntactically plastic, with desired ERs being easy to produce, guided by the syntactic rules of the RS [10]. Producing ERs can, more formally, be treated as a form of problem solving and Newell & Simon's classical theory of problem solving applied [23]. The target ER is the *goal state* that is to be reached from an *initial state* of some given ER (or none) by the *search* of the *space* of possible partial ERs that can be generated using the RS's syntactic *operators*. *Tests* are applied at each production step to see if the goal has been reached. The search process may be conceptualized as a tree, the trunk being the initial state and leaves at the end of branches being completed ERs, one (or more) of which might be the desired goal. *Search heuristics* [23] guide navigation through the tree (*problem state space*). The following criteria consider the effectiveness of RSs in terms of the character of their problem states spaces, the demands of searching the tree.

G1.1. Simple operations. A RS should possess simple operators that are easy to execute and that involve small amounts of cognitive effort. Drawing most components of a H&W diagram simply involves producing lines to scale, but in some cases a succession of sketches is needed to find the right line proportions (Fig 3). Moving a whole term from one side of an equation to another is a simple operation (e.g., m_2u_2 to the right of Equation 2), but many algebraic operations are more demanding, such as factoring a quadratic equation.

G1.2. Limited types of operators. A RS that has a small *set* of operators will tend to have a problem state space with a lower branching factor at each node: the tree will be narrower overall and thus tend to be simpler to search in general. Few operator types means fewer options to be consider at each inference step, which reduces the likelihood of selecting unproductive operators. The possible drawing operations for H&W diagrams is highly constrained, whereas a myriad of algebraic manipulations may be applied to a formula.

G1.1 and G1.2 and can be applied individually when all else is equal. Typically, however, comparisons between RSs will likely consider the trade-offs between them.

G1.3. Short procedures. A RS with short procedures requires fewer executions of operators to complete each procedure. The problem state space will be shallower overall, so potential solution states are reached more quickly. Short procedures present less opportunity for error and are obviously less effortful to execute. For example, checking whether case 1 in Table 1 is valid given the masses and velocities requires few steps using H&W diagram (Fig 3) but requires the substitution of all the values in Equation 1 into Equation 2 and a series of computations, and the same with Equation 3. The full solution procedure for Equations 1 and 2 was outlined above. Modelling a series of collisions with H&W diagrams involves the simple composition of whole diagrams (Fig 4) but may demand processing multiple simultaneous equations under the algebraic approach.

G1.4. Uniform procedures. A representation should have similar procedures to handle most problems, so the chances of picking unfruitful strategies are lessened and so that few strategies and heuristics need to be learnt. If the shape of problem trees are limited to a small number of forms, the cost of choosing one, and the chances of picking an inappropriate one, are reduced. Solving problems with H&W diagrams involves variations on the construction of the diagram and complex situations may be resolved using moving frames of reference (Fig 5) to decompose cases into iconic diagrams like those in Fig 2.

A RS lacking syntactic plasticity may be considered to be *viscous* in the terms of the Cognitive Dimensions framework [16] and is likely to be *error prone*, to have *hidden dependencies* and involve *premature commitments*.

Conceptual-syntactic compatibility. Conceptual transparency and syntactic plasticity may complement each other to increase the effectiveness of a RS.

G2.1. Meaningful syntactic constraints. Generating ERs may be more effective in RSs that have conceptual transparency, because valid manipulations of the ER will likely correspond to meaningful variations of states of affairs in the topic. Any syntactically valid change to a H&W diagram yields a real collision, whereas valid algebraic manipulations often produce expressions whose interpretation are obscure relative to the topic content. When the syntax and encoding of concepts coincide in this way the actual problem context may directly support the selection of appropriate procedures to achieve task sub-goals. It may also highlight incorrect syntactic operations or invalid expressions, because the actual problem situation can be used as a test that a partial solution is sensible. This is like *progressive evaluation* in the Cognitive Dimensions framework [16].

G2.2. ER construction parallels topic processes. Extending the previous criteria, a RS will be more effective when processes to construct ERs coincide with the natural processes of the topic, and not just meaningful states of affair. For example, the assembly of H&W diagrams into larger diagrams for sequences of collisions mirrors the occurrence of impacts in such situations (e.g., Fig 4). Writing equations to assign values to variables and writing multiple versions of Equations 1 and 2 for those variables does not directly reflect the impact sequence. Again, benefits accrue in relation to the easier selection and application of procedures.

G2.3. Separation of modeling, interpretation and calculation. A RS should permit the separation of *situation modelling*, *interpretation* and *calculation* into distinct phases of problem solving [6]. Situation modeling involves the construction of an ER that interrelates the given information about the problem, including relevant laws: a H&W diagram is such a model. Interpretation identifies the target configurations in the ER associated with problem goal; for instance identifying a certain pattern of lines. Calculation finds the desired relation or computes the required quantity from the target configuration; for instance, the ratios of lines. The separation of these phases is has the benefit of disentangling considerations of what is known about the problem situation (modelling) from inferences needed to find a solution (interpretation). In contrast, the algebraic approaches typically involves a single phase of analysis prior to calculation, which depends on the selection of an appropriate solution strategy based on an abstract understanding of the nature of the problem prior to any solution activity. With no modelling phase, important information about the structure of the problem situation is not systematically examined so clues about appropriate strategies may be missed. For example, in one study, graduate physic students were asked to solve the textbook problem mentioned above [2]. All struggled to pick an appropriate solution strategy on the first attempt. When they eventually followed the typical strategy, some correctly derived the quadratic expression for velocities and applied the standard quadratic solution formula. However, from the two values obtained some picked a value that was one of the initial velocities without realizing so, which reveals they had little overall sense of the overall nature of the situation and problem they were attempting to solve.

5 Discussion

What constitutes an effective representation? Table 2 summarizes the 19 identified criteria, classified into two main classes and five sub-classes. The classification is more comprehensive than previous analyses taking a cognitive orientation. Whereas

A) Access to concepts: from ER to IR

Elementary encoding
A1.1. One token for each type
A1.2. Reflects structure of concepts
A1.3. Directly depicts structure of cases
A1.4. Exploits spatial indexing
A1.5. Iconic expressions

Reading and inference operations
A2.1. Prefer low cost forms of processing
A2.2. Prefer low cost operators.
A2.3. Invoke more structured IRs
A2.4. Support for diagram config. schemas

Conceptual transparency
A3.1. A format for each class of 1° concepts
A3.2. Coherent encoding of primary concepts
A3.3. Overarching interpretive scheme

G) Generating ERs

Syntactic plasticity
G1.1. Simple operations
G1.2. Limited types of operators
G1.3. Short procedures
G1.4. Uniform procedures

Conceptual-syntactic compatibility
G2.1. Meaningful syntactic constraints
G2.2. ER construction parallels topic process
G2.3. Separation of modeling, interpretation and calculation

Table 2. The classification of characteristics of effective representations

previous accounts have tended to focus largely on either (a) access to concepts in ERs [26] or (b) generation of ERs [16], the present classification covers both and also begins to consider how effective RSs obtain benefits when they are combined. The classification also reveals that factors that may positively impact the efficacy of a RS occur at many cognitive levels. No claim is made that the classification is complete. Nor is it claimed that the criteria are mutually exclusive in all respects, because some address related themes at different levels. At minimum Table 2 may serve as a check-list for those investigating or designing RSs. One may gain an overall sense of whether one RS is better than another, or identify the particular areas of strength and weakness of a RS. In this respect, this work follows Green's [16] approach with the Cognitive Dimensions framework.

How should one use the classification for RSs design? From the author's experience of designing RSs for knowledge rich topics (e.g., [6], [8], [9]) and graphical interfaces for complex problems (e.g., [10], [11]), the conceptual transparency criteria (A3.1-3) should be given priority, because the elementary encoding criteria (A1.1-5) and the reading and inference criteria (A2.1-4) tend also to be satisfied when one aims for conceptual transparency. Conceptual transparency focuses on the coherent encoding of conceptual structures in systematic representational formats at the level of individual classes of concepts and of the topic as a whole, which appears to yield representations that are simple and rational (e.g., [8], [9], [11]). Further, conceptually transparent RSs tend to satisfy the conceptual-syntactic compatibility criteria (G2.1-3) and thereby naturally meet many of the syntactic plasticity criteria (G1.1-4).

What constitutes a fair basis for applying the criteria to compare RSs? This is a fundamental issue that Larkin & Simon [21] recognized in their foundational paper on RSs. They asserted that two RSs must be *informationally equivalent* before one can compare their respective computational demands. Information inferable in one representation must also be inferable in the other. So, one basis for making comparisons between RSs across a range of tasks in *knowledge rich topics* is to ensure that they are *conceptually equivalent* [8]. This notion asserts that all the ideas that are required for a full range of tasks must be expressible in both RSs. Of course, when the coverage of concepts are not equivalent one could limit the comparison just to tasks that are within the scope of the RSs under consideration, but this a rather arbitrary approach that may introduce biases. Therefore, above the cognitive level considered here, the *conceptual coverage* of RSs with respect to target topics may be taken as another perspective for identifying a wider class of efficacy criteria. It is at this level that the greater generality of algebraic equations compared to H&W diagrams would be addressed.

What about relations between RSs? In real world circumstances RSs are not used in isolation. The table and equations are often found together and instruction with H&W diagrams is likely to refer to equations. This suggests that a further set of criteria is needed to address the effectiveness of coordinating information between RSs and the transformation of ERs in one RS into associated ERs in other RS.

To conclude, consider H&W diagrams one final time. Although Huygens and Wren's diagrams (Fig 1A) have been considered elsewhere [2-4], their extension via H&W diagrams to model series of collisions, moving frames of references, inelastic collisions and 2D impact (Fig 4-7) is novel. The contrast between H&W diagrams and the conventional representations is clear both in terms of access to concepts and also the generation of ERs. The previous research on RSs has largely focused on the former class of criteria but the contrast between the RSs in the examples here emphasizes the need to consider the processes of manipulating ERs and also the variety of diagrammatic operators that may be used to transform ERs. Further work is needed to classify and understand the full potential of such diagrammatic operators.

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