

Adaptive Systems: Pre-Lab exercise.

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The following exercise will facilitate your work during the Adaptive Systems Lab Class on week 8. It involves some minimal maths in order to understand the role of different parameters in a simple Continuous-Time Recurrent Neural Network (CTRNN).

The state of a neuron in a CTRNN evolves as follows:

$$\tau_i \frac{dy_i}{dt} = -y_i + \sum_j w_{ji} z_j + I_i; \quad z_j = \frac{1}{1 + \exp[-(y_j + b_j)]},$$

where, y_i represents the membrane potential of neuron i , τ_i the decay constant, b_i the bias, z_i the firing rate, w_{ij} the strength of synaptic connection from node i to node j , and I_i an incoming current (e.g., the degree of sensory perturbation for sensory nodes). The firing rate z_i is given by a sigmoid function of the membrane potential plus bias; this function tends to 0 for large negative values of y_i and to 1 for large positive values, with a transition centred around $y_i = -b_i$.

Consider a single node with only an incoming current I and one self connection w . Suppose initially that $I = 0$ and $w \geq 0$

1. Find the equilibrium condition for the state and firing rate of the neuron. Use a graphic method to explore different possibilities.
2. What is the maximum and minimum number of stable and unstable equilibrium points? (Stability can be analyzed simply by considering small displacements around an equilibrium point and the effect on the time derivative of y .)
3. What is the effect of varying the bias b ?
4. What is the effect of varying the decay constant τ ?
5. What happens if $I \neq 0$?
6. What is the effect of varying w from very small to very large positive values?
7. Calculate the critical value of w at which the number of stable equilibrium points may change, (it may help to assume $b = -0.5$). Discuss practical implications.
8. Repeat analysis for $w < 0$. What's the main difference?