Hierarchical Markov Modeling For Generative Music

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Sample sequence

a b c d b c a

Statistical modeling approach

a b c d b c a

```
P(b|a) = 1.0
P(c|b) = 1.0
P(d|c) = 0.5
P(a|c) = 0.5
P(b|d) = 1.0
```

d b c d b c a ...

Structural modeling approach

a b c d b c a

$0 = b c

a $0 d $0 a

Informational approach

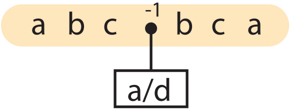
a b c d b c a

Total information = 3x7=21 bits

Information decay

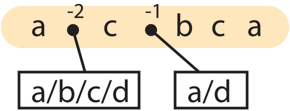
Uncertainize sequence by replacing symbols with choices.

MSI = (21-1)/8 = $\boxed{2.5}$



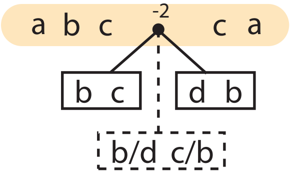
More decay

MSI = (21-3)/9 = $\boxed{2.0}$



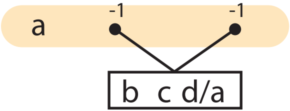
Choices involving subsequences

MSI = (21-2)/8 = 2.4



Profitable uncertainization

MSI = (21-2)/6 = $\boxed{3.2}$



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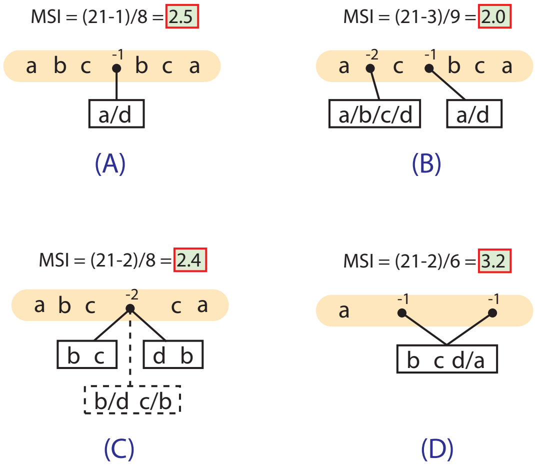
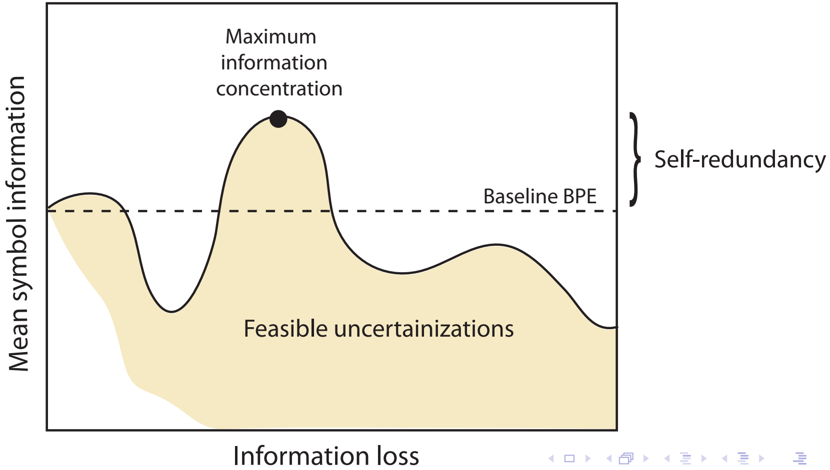

MSI = (21-1)/8 = 2.5
a b c -1 b c a
a/d
(A)
MSI = (21-3)/9 = 2.0
a -2 c -1 b c a
a/b/c/d
a/d
(B)
MSI = (21-2)/8 = 2.4
a b c -2 c a
b c
d b
b/d c/b
(C)
MSI = (21-2)/6 = 3.2
a -1 -1
b c d/a
(D)

Conservative uncertainization

Uncertainization which maximizes MSI is informationally *conservative*.



Recursive refinement

Uncertainizations are themselves sequences.

So recursive processing is possible.

This generates a hierarchical series of informational refinements of the original sequence.

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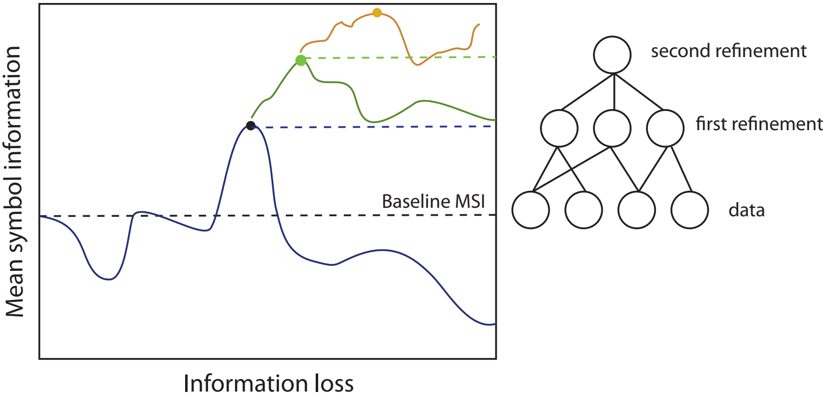
Refinements are like informational principle components.

Structure produced is a symbol hierarchy modeling levels of organization in the sequence



Consider a sequence s to be a symbolic specification for a sequence of distributions.

So 'x y z' seen as specifying distributions

P(x)=1.0 P(y)=0.0 P(z)=0.0
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Total information content of s:

$$I(s) = \sum_i \log |s_i| - H(s_i)$$

where s_i denotes i'th distribution.

Mean information

Mean information (MSI):

$$\bar{I}(s) = \frac{I(s)}{|s|}$$

Sequence u^s uncertainizes s if it specifies the same sequence of distributions with greater total entropy.

Information loss produced by an uncertainization:

$$L(u^s) = \sum_i H(U_i^s)$$

Mean information in an uncertainization:

$$\bar{I}(U^s) = \frac{I(s) - L(U^s)}{|U^s|}$$

Conservative uncertainization

Informationally optimal ('conservative') uncertainizations of s:

$$\dot{U}^s = \operatorname*{argmax}_{u \in U(S)} \bar{I}(u)$$

Refinement (*globally* optimal uncertainization):

$$r(s) = \operatorname*{argmax}_{u \in \dot{U}(S)} \bar{I}(u)$$

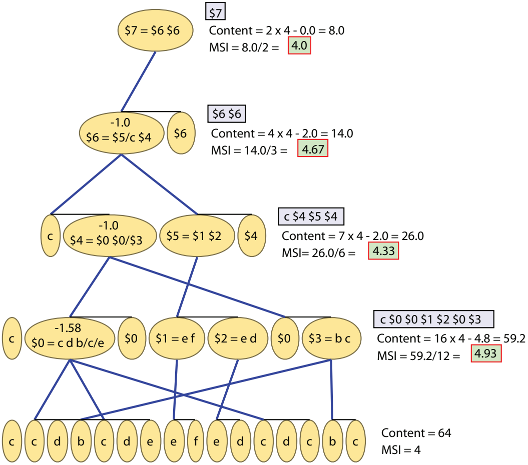
Second refinement:

$$r(r(s))$$

Third refinement:

$$r(r(r(s)))$$

Tree representation



Markov hierarchies

A conventional Markov model can be derived for each refinement.

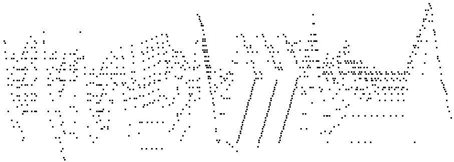
Since refinements are hierarchically related, so are the models.

Hence 'Markov hierarchy'.

Significant improvement in generative use. Top-down symbol expansion is 'guided' by the Markov models.

Markov models generate 'chains'
Markov hierarchies generate 'cascades'

Learning models for complex musical sequences (e.g., sonatas).



Initial sequence from Chopin's 3rd Sontata

Replex generted from 5th refinement

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- Applied to music, this can be the means of generating variations of existing pieces.
- Symbol structures can be treated as knowledge representation.
- But what exactly is the 'knowledge' represented?

Resources

For music applications, the web page www.christhornton.eu/replex-music.html