On the Benefit of Expert Services in Mathematics Education Systems

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Students learn mathematics more effectively if the traditional rote learning which aims at memorizing facts and procedures is supplemented with exploratory learning which aims at the acquisition of problem solving skills and meta-cognition by active exploration. If a first phase of *narrow learning* is followed by a phase of *explorative learning*, the general performance and the ability to transfer solutions to new situations and to apply the acquired knowledge in novel situations and contexts reportedly improve substantially.

Most mathematics education systems still stick to the tradition and narrowly guide the students on one single path to the solution of a problem. Computational systems have the potential to support exploratory learning in several ways. We are currently developing an interactive mathematics textbook that supports both strictly guided learning and exploration. In particular, the integration of expert services into learning systems can provide hands-on experience. A more detailed presentation of the subject can be found in [2].

The Need for Expert Services

The learning by exploratory problem solving can be supported by integrating *expert services* (ESs). An ES is a stand-alone system such as a computer algebra system or proof planner which can perform subtasks of the problem solving process. ESs may be domain-specific but in any case they can use domain-specific knowledge such as methods and control-rules in proof planning.

A proof planner is a system that (semi-)automatically produces abstract representations of proofs, that is, outlines of proofs that can be more and more detailed if required. Such a system composes a plan from proof patterns and proof steps represented by *methods* corresponding to frequently used mathematical techniques such as induction, diagonalization, or estimations. Our proof planner ΩMEGA [1] can use *strategies* such as proving or disproving a conjecture or proving by analogy. Such strategies produce a whole subproof when they are executed. Moreover, a proof planner has a meta-reasoning facility which heuristically selects or prefers certain methods. ΩMEGA offers (meta-)reasoning about failed proof attempts that goes beyond the usual backtracking facilities. For example, it suggests that if the planner knew a lemma of a certain form a certain method would be applicable.

The knowledge acquisition for proof planners can be very time consuming and difficult. For instance, it is a research topic to discover frequent patterns of failed attempts and to find out how to avoid these failures. However, when the knowledge (methods, control knowledge, and strategies) is isolated, it can be explicitly communicated and explained to the user.

The benefit of integrating ESs into learning systems is twofold. (1) ESs allow the user to focus on a particular skill to solve a specific problem. For example, the user of the system should not fail to calculate the limit of a function just because he cannot find the factorization of a polynomial; a computer algebra system fulfills this task. Similarly, the user should not fail to find the overall proof just because he cannot prove a subgoal. It may suffice from a pedagogical point of view that the user suggests some intermediate goals that should be provable and a proof planner fills the gaps in the proof plan. (2) ESs allow the user to explore a problem interacively. They can check the correctness of his calculations or derivations. Moreover, they can help with feedback on where it is promising to explore and where dead ends are reached.

The Use of Expert Services

An expert service has to provide information about why the solution of a problem is correct or incorrect. For example, a proof planner should provide the derivation that justifies a proof. This information can be used to explain the proof if it was computed by the ES. Such an explanation is possible at different levels of detail. If the user interactively used the ES for a step in the solution process this information is a useful feedback for the user.

A proof planning system can assist a user in planning and finding a proof, in disproving a conjecture, or in constructing mathematical objects with certain properties. Moreover, it can help the user to learn from the analysis of failed proof attempts and from dead end situations.

In our interactive textbook, the proof planning system supports the user in several ways. First, the system supports the user by book-keeping and checking the correctness of a user's choice. Next, it keeps the user focused on the main goal of the lesson by taking over tasks inside a proof that are routine to the user. Moreover, it supports the user by providing some heuristic or legal pre-selection and ordering of methods. This is possible at different levels of detail and abstraction depending on the user's expertise. In the future, the production of plain error messages or of linguistically processed error dialogs shall support or stimulate the user's meta-cognition, for example, by presenting the heuristic knowledge used for intelligent backtracking.

The use of expert services such as proof planners in mathematics education could pave the way toward the inclusion of proofs in school curricula.

References

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